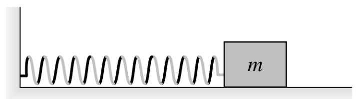
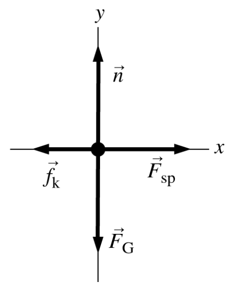
Problem set 8 key

9.59. Model: The box starts from rest.

Visualize: Use the work–kinetic energy theorem





Known
$k = 250$ N/m
$\Delta x = 0.12$ m
$m = 0.25$ kg
$\mu_k = 0.23$

Solve: First compute the total work done on the box during the launch.

$$W = \int_{x_0}^{x_1}(F_{sp} - f_k)dx = \int_{x_0}^{x_1}(kx - \mu_k n)dx = \left[\tfrac{1}{2}kx^2 - \mu_k mgx\right]_{x_0}^{x_1}$$

$$= \left[\tfrac{1}{2}(250\text{ N/m})x^2 - (0.23)(0.25\text{ kg})(9.8\text{ m/s}^2)x\right]_0^{0.12} = 1.73\text{ J}$$

Now use the work–kinetic energy theorem.

$$1.73\text{ J} = \tfrac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2(1.73\text{ J})}{0.25\text{ kg}}} = 3.7\text{ m/s}$$

Assess: The friction decreased the launch speed only a bit.

9.64. Model: Use the model of static friction, kinematic equations, and the definition of power.

Solve: (a) The rated power of the Porsche is $217\text{ hp} = 161{,}882\text{ W}$ and the gravitational force on the car is $(1480\text{ kg})(9.8\text{ m/s}^2) = 14{,}504\text{ N}$. The amount of that force on the drive wheels is $(14{,}504)(2/3) = 9670\text{ N}$. Because the static friction of the tires on road pushes the car forward,

$$F_{max} = f_{s,max} = \mu_s n = \mu_s mg = (1.00)(9670\text{ N}) = ma_{max}$$

$$a_{max} = \frac{9670\text{ N}}{1480\text{ kg}} = 6.53\text{ m/s}^2$$

(b) Only 70% of the power generated by the motor is applied at the wheels.

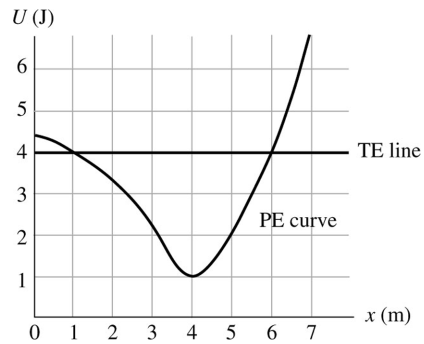
$$P = Fv_{max} \quad \Rightarrow \quad v_{max} = \frac{P}{F} = \frac{(0.70)(161{,}882\text{ W})}{9670\text{ N}} = 11.7\text{ m/s}$$

(c) Using the kinematic equation, $v_{max} = v_0 + a_{max}(t_{min} - t_0)$ with $v_0 = 0$ m/s and $t_0 = 0$ s, we obtain

$$t_{min} = \frac{v_{max}}{a_{max}} = \frac{11.7\text{ m/s}}{6.53\text{ m/s}^2} = 1.79\text{ s}$$

Assess: An acceleration time of 1.79 s for the Porsche to reach a speed of ≈ 26 mph from rest is reasonable.

10.24. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle is released from rest at $x = 1.0$ m. That is, $K = 0$ at $x = 1.0$ m. Since the total energy is given by $E = K + U$, we can draw a horizontal total energy (TE) line through the point of intersection of the potential energy curve (PE) and the $x = 1.0$ m line. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: (a) We have $E = 4.0$ J and this energy is a constant. For $x < 1.0, U > 4.0$ J and, therefore, K must be negative to keep E the same (note that $K = E - U$ or $K = 4.0 \text{ J} - U$). Since negative kinetic energy is unphysical, the particle cannot move to the left. That is, the particle will move to the right of $x = 1.0$ m.

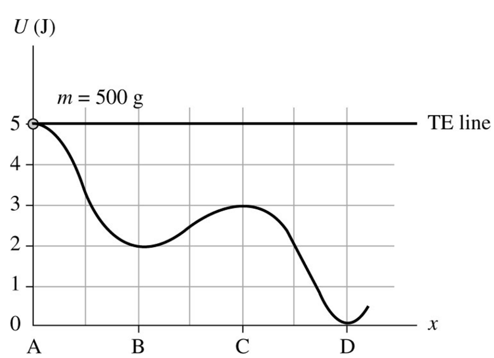
(b) The expression for the kinetic energy is $E - U$. This means the particle has maximum speed or maximum kinetic energy when U is minimum. This happens at $x = 4.0$ m. Thus,

$$K_{max} = E - U_{min} = (4.0 \text{ J}) - (1.0 \text{ J}) = 3.0 \text{ J} \qquad \frac{1}{2}mv_{max}^2 = 3.0 \text{ J} \Rightarrow v_{max} = \sqrt{\frac{2(3.0 \text{ J})}{m}} = \sqrt{\frac{8.0 \text{ J}}{0.020 \text{ kg}}} = 17.3 \text{ m/s}$$

The particle possesses this speed at $x = 4.0$ m.

(c) The total energy (TE) line intersects the potential energy (PE) curve at $x = 1.0$ m and $x = 6.0$ m. These are the turning points of the motion.

10.25. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle with a mass of 500 g is released from rest at A. That is, $K = 0$ at A. Since $E = K + U = 0 \text{ J} + U$, we can draw a horizontal TE line through $U = 5.0$ J. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: The kinetic energy is given by $E - U$, so we have

$$\frac{1}{2}mv^2 = E - U \Rightarrow v = \sqrt{2(E-U)/m}$$

Using $U_B = 2.0$ J, $U_C = 3.0$ J, and $U_D = 0$ J, we get

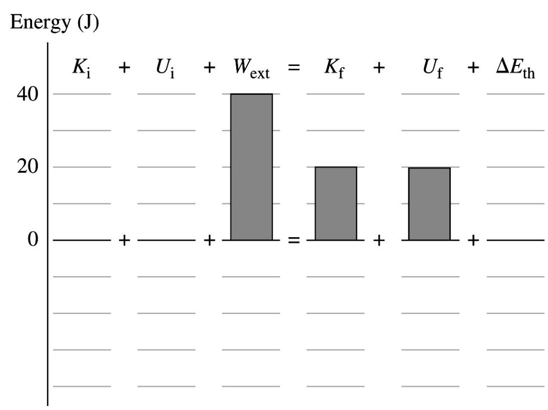
$$v_B = \sqrt{2(5.0 \text{ J} - 2.0 \text{ J})/0.500 \text{ kg}} = 3.5 \text{ m/s} \qquad v_C = \sqrt{2(5.0 \text{ J} - 3.0 \text{ J})/0.500 \text{ kg}} = 2.8 \text{ m/s}$$
$$v_D = \sqrt{2(5.0 \text{ J} - 0 \text{ J})/0.500 \text{ kg}} = 4.5 \text{ m/s}$$

10.32. Model: Use the negative derivative of the potential energy to determine the force acting on a particle.
Solve: The y-component of the force is

$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}(4y^3) = -12y^2$$

At $y = 0$ m, $F_y = 0$ N; at $y = 1$ m, $F_y = -12$ N; and at $y = 2$ m, $F_y = -48$ N.

10.40. Visualize: The tension of 20.0 N in the cable is an external force that does work on the block $W_{ext} =$ $(20.0 \text{ N})(2.00 \text{ m}) = 40.0$ J, increasing the gravitational potential energy of the block. We placed the origin of our coordinate system on the initial resting position of the block, so we have $U_i = 0$ J and $U_f = mgy_f =$ $(1.02 \text{ kg})(9.8 \text{ m/s}^2)(2.00 \text{ m}) = 20.0$ J. Also, $K_i = 0$ J, and $\Delta E_{th} = 0$ J. The energy bar chart shows the energy transfers and transformations.

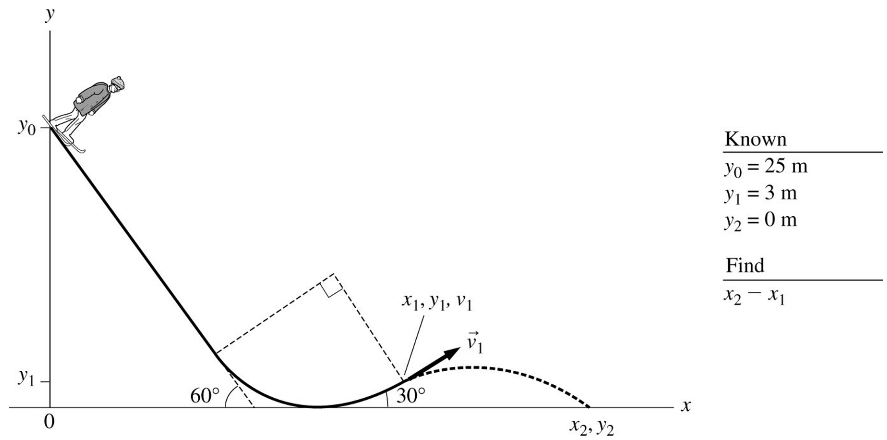


Solve: The conservation of energy equation is

$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th} \quad \Rightarrow \quad 0 \text{ J} + 0 \text{ J} + 40.0 \text{ J} = \tfrac{1}{2}mv_f^2 + 20.0 \text{ J} + 0 \text{ J}$$
$$v_f = \sqrt{(20.0 \text{ J})(2)/(1.02 \text{ kg})} = 6.26 \text{ m/s}$$

10.44. Model: Since there is no friction, the sum of the kinetic and gravitational potential energy does not change. Model Julie as a particle.

Visualize:



We place the coordinate system at the bottom of the ramp directly below Julie's starting position. From geometry, Julie launches off the end of the ramp at a 30º angle.

Solve: Energy conservation: $K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

Using $v_0 = 0$ m/s, $y_0 = 25$ m, and $y_1 = 3$ m, the above equation simplifies to

$$\frac{1}{2}mv_1^2 + mgy_1 = mgy_0 \Rightarrow v_1 = \sqrt{2g(y_0 - y_1)} = \sqrt{2(9.8\ \text{m/s}^2)(25\ \text{m} - 3\ \text{m})} = 20.77\ \text{m/s}$$

We can now use kinematic equations to find the touchdown point from the base of the ramp. First we'll consider the vertical motion:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \frac{1}{2}a_y(t_2 - t_1)^2 \quad 0\ \text{m} = 3\ \text{m} + (v_1\sin 30°)(t_2 - t_1) + \frac{1}{2}(-9.8\ \text{m/s}^2)(t_2 - t_1)^2$$

$$\Rightarrow (t_2 - t_1)^2 - \frac{(20.77\ \text{m/s})\sin 30°}{(4.9\ \text{m/s}^2)}(t_2 - t_1) - \frac{(3\ \text{m})}{(4.9\ \text{m/s}^2)} = 0$$

$$(t_2 - t_1)^2 - (2.119\ \text{s})(t_2 - t_1) - (0.6122\ \text{s}^2) = 0 \Rightarrow (t_2 - t_1) = 2.377\ \text{s}$$

For the horizontal motion:

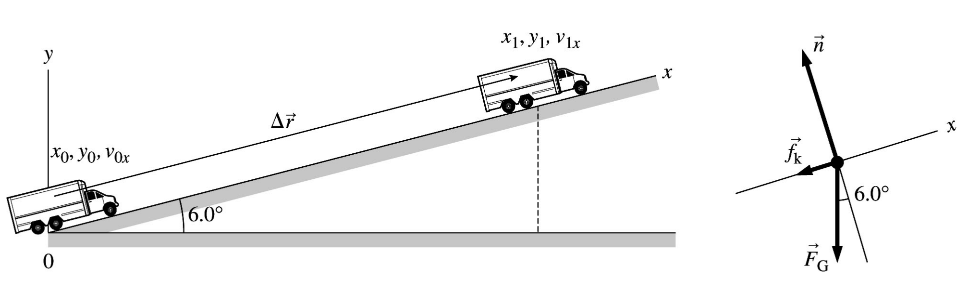
$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

$$x_2 - x_1 = (v_1\cos 30°)(t_2 - t_1) + 0\ \text{m} = (20.77\ \text{m/s})(\cos 30°)(2.377\ \text{s}) = 43\ \text{m}$$

Assess: Note that we did not have to make use of the information about the circular arc at the bottom that carries Julie through a 90° turn.

10.50. Model: Identify the truck and the loose gravel as the system. We need the gravel inside the system because friction increases the temperature of the truck and the gravel. We will also use the model of kinetic friction and the conservation of energy equation.

Visualize:



Known
$m = 15{,}000$ kg
$\mu_k = 0.40$
$v_{0x} = 35$ m/s
$v_{1x} = 0$ m/s
$x_0 = 0$ m
$y_0 = 0$ m
$y_1 = x_1 \sin(6.0°$

Find
x_1

We place the origin of our coordinate system at the base of the ramp in such a way that the x-axis is along the ramp and the y-axis is vertical so that we can calculate potential energy. The free-body diagram of forces on the truck is shown.

Solve: The conservation of energy equation is $K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}$. In the present case, $W_{ext} = 0$ J, $v_{1x} = 0$ m/s, $U_{g0} = 0$ J, $v_{0x} = 35$ m/s. The thermal energy created by friction is

$$\Delta E_{th} = f_k(x_1 - x_0) = (\mu_k n)(x_1 - x_0) = \mu_k mg\cos(6.0°)(x_1 - x_0)$$
$$= (0.40)(15{,}000 \text{ kg})(9.8 \text{ m/s}^2)\cos(6.0°)(x_1 - x_0) = (58{,}478 \text{ J/m})(x_1 - x_0)$$

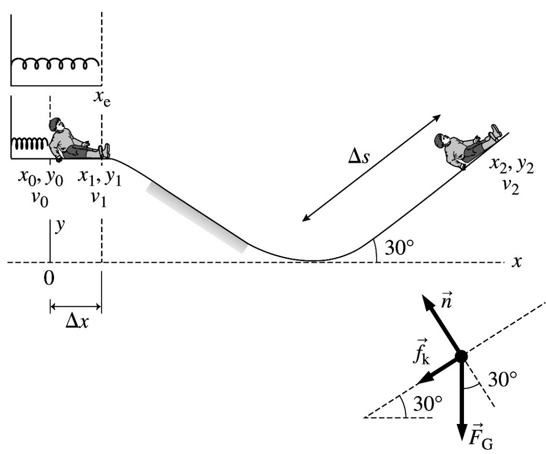
Thus, the energy conservation equation simplifies to

$$0 \text{ J} + mgy_1 + (58{,}478 \text{ J/m})(x_1 - x_0) = \tfrac{1}{2}mv_{0x}^2 + 0 \text{ J} + 0 \text{ J}$$
$$(15{,}000 \text{ kg})(9.8 \text{ m/s}^2)(x_1 - x_0)\sin(6.0°) + (58{,}478 \text{ J/m})(x_1 - x_0) = \tfrac{1}{2}(15{,}000 \text{ kg})(35 \text{ m/s})^2$$
$$(x_1 - x_0) = 124 \text{ m} = 0.12 \text{ km}$$

Assess: A length of 124 m at a slope of $6°$ seems reasonable.

10.54. Model: Assume an ideal spring, so Hooke's law is obeyed. Treat the physics student as a particle and apply the law of conservation of energy. Our system comprises the spring, the student, and the ground. We also use the model of kinetic friction.

Visualize: We place the origin of the coordinate system on the ground directly below the end of the compressed spring that is in contact with the student.



Known
$x_0 = 0$ m $v_0 = 0$ m/s
$x_1 - x_0 = 0.50$ m $k = 80{,}000$ N/m
$y_0 = y_1 = 10$ m $m = 100$ kg
$\mu_k = 0.15$ $v_2 = 0$ m/s

Find
v_1 $\Delta s = y_2/\sin 30°$

Solve: (a) The energy conservation equation gives

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$
$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_1 - x_e)^2 + 0 \text{ J} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0 \text{ J}$$

Since $y_1 = y_0 = 10$ m, $x_1 = x_e$, $v_0 = 0$ m/s, $k = 80{,}000$ N/m, $m = 100$ kg, and $(x_1 - x_0) = 0.5$ m,

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}k(x_1 - x_0)^2 \quad \Rightarrow \quad v_1 = \sqrt{\frac{k}{m}}(x_1 - x_0) = \sqrt{\frac{80{,}000\ \text{N}/m}{100\ \text{kg}}}(0.50\ \text{m}) = 14\ \text{m/s}$$

(b) Friction creates thermal energy. Applying the conservation of energy equation once again:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_2^2 + mgy_2 + 0\ \text{J} + f_k\Delta s = 0\ \text{J} + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\ \text{J}$$

With $v_2 = 0$ m/s and $y_2 = \Delta s\sin(30°)$, the above equation is simplified to

$$mg\Delta s\sin(30°) + \mu_k n\Delta s = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

From the free-body diagram for the physics student, we see that $n = F_G\cos(30°) = mg\cos(30°)$. Thus, the conservation of energy equation gives

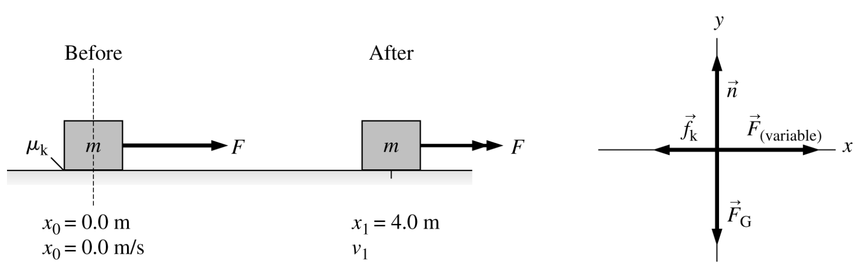
$$\Delta s[mg\sin(30°) + \mu_k mg\cos(30°)] = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

Using $m = 100$ kg, $k = 80{,}000$ N/m, $(x_1 - x_0) = 0.50$ m, $y_0 = 10$ m, and $\mu_k = 0.15$, we get

$$\Delta s = \frac{mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2}{mg[\sin(30°) + \mu_k\cos(30°)]} = 32\ \text{m}$$

Assess: $y_2 = \Delta s\sin(30°) = 16$ m, which is greater than $y_0 = 10$ m. The higher value is due to the transformation of the spring energy into gravitational potential energy.

10.56. Model: Model the block as a particle.
Visualize: The system is the block.



Solve: A preliminary calculation: use Newton's second law in the y direction to see that $n = mg$ so $f_k = \mu_k mg$. Now find the work done on the block by the forces on it.

$$W_{net} = \int F_{net}dx = \int_0^4((20 - 5x) - \mu_k mg)dx = \left[20x - \tfrac{5}{2}x^2 - (0.25)(2.6\ \text{kg})(9.8\ \text{m/s}^2)x\right]_0^4 = 14.52\ \text{J}$$

Use the work–kinetic energy theorem: $W_{net} = \Delta K$ remembering that $K_i = 0$.

$$v = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2(14.52\ \text{J})}{2.6\ \text{kg}}} = 3.3\ \text{m/s}$$

Assess: This seems like a reasonable speed for a 2.6 kg block after pulling it 4.0 m.

10.57. Solve: (a) The equilibrium positions are located at points where $\dfrac{dU}{dx} = 0$.

$$\frac{dU}{dx} = 0 = 1 + 2\cos(2x) \Rightarrow \cos(2x) = -\frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right)$$

Note that $-\frac{1}{2}$ is in radians and x is in meters. The function $\cos^{-1}\left(-\frac{1}{2}\right)$ may have values $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Thus there are two values of x,

$$x_1 = \frac{\pi}{3} \text{ and } x_2 = \frac{2\pi}{3}$$

within the interval $0 \text{ m} \le x \le \pi \text{ m}$.

(b) A point of stable equilibrium corresponds to a local minimum, while a point of unstable equilibrium corresponds to a local maximum. Compute the concavity of $U(x)$ at the equilibrium positions to determine their stability.

$$\frac{d^2U}{dx^2} = -4\sin(2x)$$

At $x_1 = \frac{\pi}{3}$, $\frac{d^2U}{dx^2}(x_1) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$. Since $\frac{d^2U}{dx^2}(x_1) < 0$, $x_1 = \frac{\pi}{3}$ is a local maximum, so $x_1 = \frac{\pi}{3}$ is a point of unstable equilibrium.

At $x_2 = \frac{2\pi}{3}$, $\frac{d^2U}{dx^2}(-x_2) = -4\left(-\frac{\sqrt{3}}{2}\right) = +2\sqrt{3}$. Since $\frac{d^2U}{dx^2} > 0$, $x_2 = \frac{2\pi}{3}$ is a local minimum, so $x_2 = \frac{2\pi}{3}$ is a point of stable equilibrium.

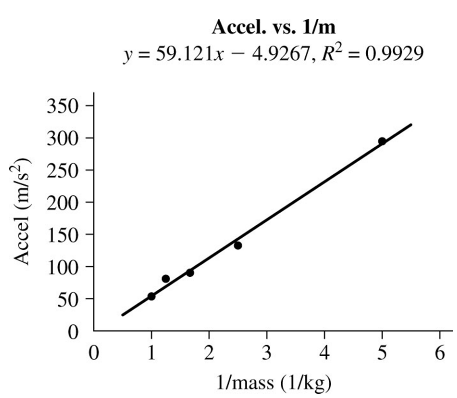
10.61. Model: Since there is a potential energy then the force is conservative.
Visualize: The force is the negative derivative of the potential energy.
Solve:

$$F_x = -\frac{dU_x}{dx} = -\frac{d\left(Ax^2 + B\sin\left(\frac{\pi}{L}x\right)\right)}{dx} = -2Ax - B\left(\frac{\pi}{L}\right)\cos\left(\frac{\pi}{L}x\right)$$

(a) Evaluate this expression at $x = 0$: $F_x(0) = -\pi B/L$.
(b) Evaluate the expression at $x = L/2$: $F_x(L/2) = -AL$.
(c) Evaluate the expression at $x = L$: $F_x(L) = -2AL + \pi B/L$.

6.43. Visualize: We'll use $v_f^2 = v_i^2 + 2a\Delta s$ to find the acceleration of the balls, which will be inversely proportional to the mass of the balls. $\Delta s = 15$ cm and $v_i = 0$ in each case.

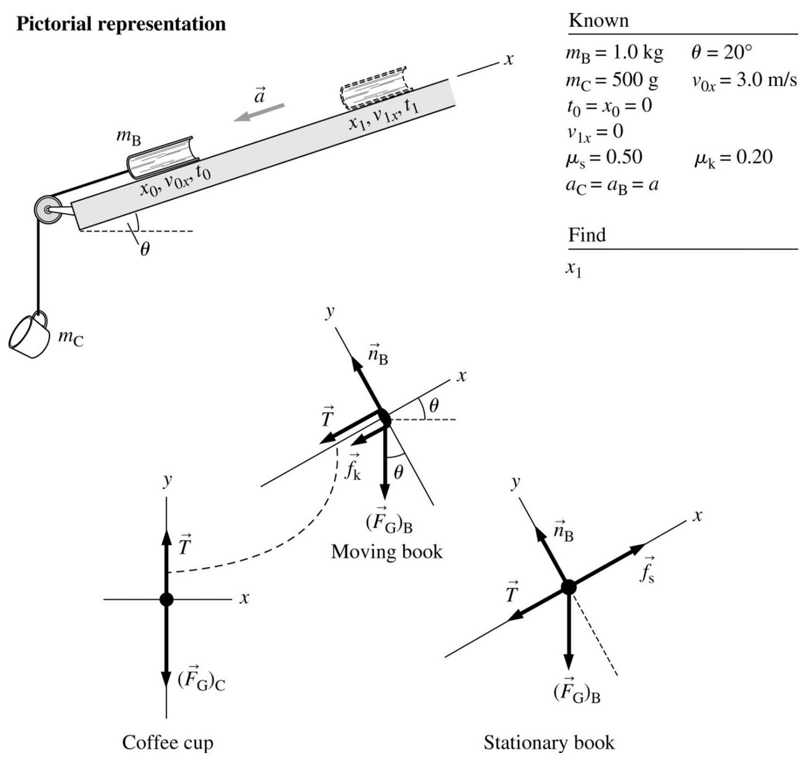
Solve: Newton's second law relates mass, acceleration, and net force: $a = F\frac{1}{m}$. If we graph a vs. $\frac{1}{m}$ then the slope of the straight line should be the size of the piston's force.



We see that the linear fit is very good. The slope is $59.12 \text{ N} \approx 59 \text{ N}$; this is the size of the piston's force.

Assess: We are glad to see that the intercept of our line looks very small, even though we don't have a ball the inverse of whose mass is zero.

7.41. Model: Use the particle model for the book (B) and the coffee cup (C), the models of kinetic and static friction, and the constant-acceleration kinematic equations.
Visualize:



Solve: (a) Using $v_{1x}^2 = v_{0x}^2 + 2a(x_1 - x_0)$, we find

$$0 \text{ m}^2/\text{s}^2 = (3.0 \text{ m/s})^2 + 2a(x_1) \quad \Rightarrow \quad ax_1 = -4.5 \text{ m}^2/\text{s}^2$$

To find x_1, we must first find a. Newton's second law applied to the book and the coffee cup gives

$$\sum(F_{\text{on B}})_y = n_B - (F_G)_B \cos(20°) = 0 \text{ N} \quad \Rightarrow \quad n_B = (1.0 \text{ kg})(9.8 \text{ m/s}^2)\cos(20°) = 9.21 \text{ N}$$

$$\sum(F_{\text{on B}})_x = -T - f_k - (F_G)_B \sin(20°) = m_B a_B \quad \sum(F_{\text{on C}})_y = T - (F_G)_C = m_C a_C$$

The last two equations can be rewritten, using $a_C = a_B = a$, as

$$-T - \mu_k n_B - m_B g \sin(20°) = m_B a \quad T - m_C g = m_C a$$

Adding the two equations gives

$$a(m_C + m_B) = -g[m_C + m_B \sin(20°)] - \mu_k (9.21 \text{ N})$$

$$(1.5 \text{ kg})a = -(9.8 \text{ m/s}^2)[0.500 \text{ kg} + (1.0 \text{ kg})\sin 20°] - (0.20)(9.21 \text{ N}) \quad \Rightarrow \quad a = -6.73 \text{ m/s}^2$$

Using this value for a, we can now find x_1 as follows:

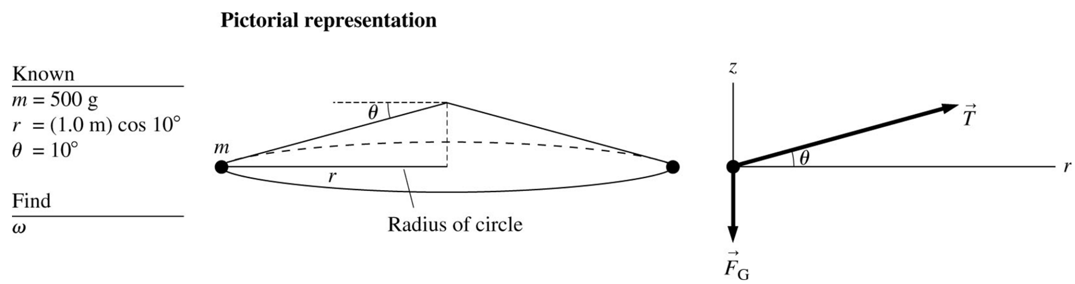
$$x_1 = \frac{-4.5 \text{ m}^2/\text{s}^2}{a} = \frac{-4.5 \text{ m}^2/\text{s}^2}{-6.73 \text{ m/s}^2} = 0.67 \text{ m}$$

(b) The maximum static friction force is $(f_s)_{max} = \mu_s n_B = (0.50)(9.21 \text{ N}) = 4.60$ N. We'll see if the force f_s needed to keep the book in place is larger or smaller than $(f_s)_{max}$. When the cup is at rest, the string tension is $T = m_C g$. Newton's first law for the book is

$$\sum(F_{\text{on B}})_x = f_s - T - w_B \sin(20°) = f_s - m_C g - m_B g \sin(20°) = 0$$
$$f_s = (M_C + M_B \sin 20°)g = 8.25 \text{ N}$$

Because $f_s > (f_s)_{max}$, the book slides back down.

8.41. Model: Use the particle model for the rock, which is undergoing uniform circular motion.
Visualize: L is the hypotenuse of the right triangle. The radius of the circular motion is $r = L\cos\theta$.



Solve:
(a) Apply Newton's second law in the z- and r-directions.

$$\sum F_z = T\sin\theta - mg = 0 \Rightarrow T = \frac{mg}{\sin\theta}$$

$$\sum F_r = T\cos\theta = m\omega^2 r = m\omega^2 (L\cos\theta) \Rightarrow T = m\omega^2 L$$

Set the two expressions for T equal to each other and solve for ω.

$$\frac{mg}{\sin\theta} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{L\sin\theta}}$$

(b) Insert $L = 1.0 \text{ m}$ and $\theta = 10°$.

$$\omega = \sqrt{\frac{g}{L\sin\theta}} = \sqrt{\frac{9.8 \text{ m/s}^2}{(1.0 \text{ m})\sin 10°}} = 7.51 \text{ rad/s}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 72 \text{ rpm}$$

Assess: Notice that the mass canceled out of the equation so the 500 g was unnecessary information. In other words, the answer, 72 rpm, would be the same regardless of the mass.
The dependencies of ω on g, L, and θ seem to be in the right directions.