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Accurate experiment for measuring the ratio of specific heats of gases using an accelerometer

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I. INTRODUCTION

Undergraduate laboratory exercises to measure $\gamma = c_p/c_v$, the ratio of the specific heats of gases, have generally been unsatisfactory. The method of Clement and Desormes,[1] an old standby, is notoriously inaccurate and presents difficulties when one desires to change the gas under study. Another method, which has the advantage of using relatively small volumes of completely contained gas, is that of Ruchardt.[1,2] In this method a precision steel ball is dropped into a matching precision glass tube connected to the gas volume. The ball performs damped oscillations in the tube (if it does not stick) and the student is required to obtain the frequency from the four or five appreciable oscillations. The volume of the gas must actually be rather large in order that the frequency be low enough to measure—a requirement that is not consistent with adiabatic processes. The method is, in principle, the more interesting one not only because it links the physics of oscillations to thermodynamics, but also as it was the inspiration for the elegant experiments of Clark and Katz[3] in the precision measurement of γ.

The present experiment is a simple development of the Ruchardt method using electronic detection of the oscillations and permitting the investigation of γ as a function of degrees of freedom of the gas molecule. In addition it uses very small volumes of gas and easily yields values of γ within 3% of the theoretical prediction.

II. APPARATUS

Modern glass syringes,[4] when clean and dry, run together as frictionlessly as the usual ball and tube. In addition they are volume calibrated to better than 1%. A 50-ml syringe was mounted vertically in a metal stand (Fig. 1) and closed at the needle end with a matching miniature stopcock.

An accelerometer[5] of mass 50 g and output 50 mV/g (g is the gravitational acceleration) was mounted on the top of the syringe plunger. The output of the accelerometer was displayed on the vertical input of an oscilloscope. The triggering and sweep of the oscilloscope were set so that, when the plunger was depressed by a few mm and released, one sweep was triggered displaying five to ten damped oscillations. The pattern was photographed with an oscilloscope camera. Such a trace for air is shown in Fig. 2. The output could easily be digitized and used as input to a microcomputer.

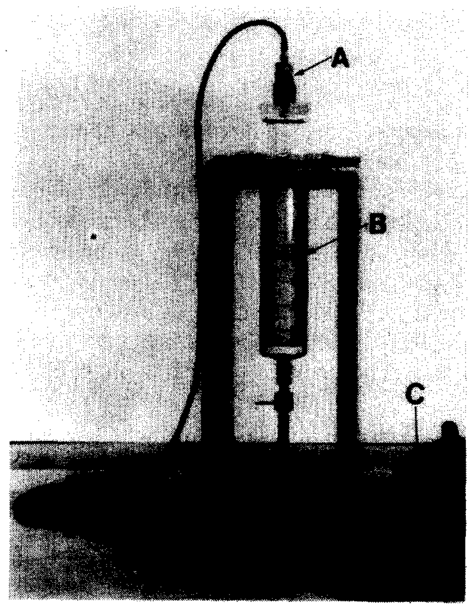

Fig. 1. Specific heat apparatus showing A, Accelerometer; B, syringe; and C, gas bladder.

III. DISCUSSION

All the quantities in the equation

$$f_0 = (s/2\pi)[(\gamma/m)(P/V)]^{1/2} \quad (1)$$

for the frequency of oscillation of the piston can be measured to within 1%. In Eq. (1), s is the cross-section area of the piston, m is its mass, V is the volume of gas enclosed in

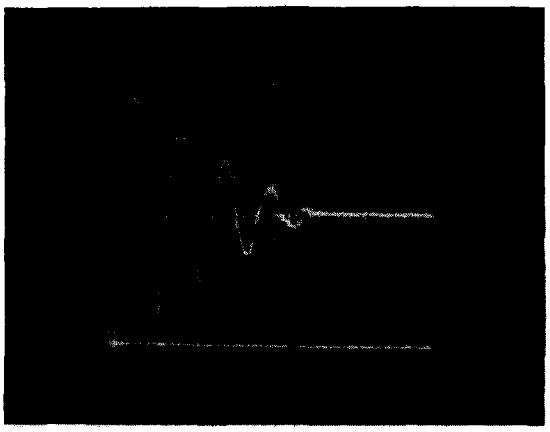

Fig. 2. Oscilloscope trace for air. The sweep speed is 50 ms per div.

the syringe, and P is its pressure. The quantity f_0 is the undamped frequency which can be obtained from the oscilloscope trace using the damped frequency and the logarithmic decrement, λ (see Ref. 6), of the damped oscillation using

$$(f/f_0)^2 = [1 + (\lambda/\pi)^2]^{1/2}. \quad (2)$$

In this formula λ is the natural logarithm of the ratio of successive amplitudes of the oscillatory pattern.

By means of miniature stopcocks and fine rubber tubing, various gases can be introduced from a bladder into the syringe: helium ($\gamma = 1.67$), air ($\gamma = 1.40$), and natural gas ($\gamma = 1.33$). By ensuring that the oscillation amplitudes are small (1 to 2 mm) experimental values of γ can always be obtained within 3% of the expected value.

[1] M. W. Zemansky, *Heat and Thermodynamics* (McGraw-Hill, New York, 1957), 4th ed., p. 126.
[2] E. Ruchardt, Phys. Z. **30**, 58 (1929).
[3] A. L. Clark and L. Katz, Can. J. Res. **18A**, 23 (1940).
[4] An example is the 50-ml Multifit Syringe, supplied by Fisher Scientific, Catalog No. 14-823-10F.
[5] Vibro-Meter Corp., Cambridge, MA, 02138; accelerometer 508B.
[6] H. Lamb, *Dynamics* (Cambridge University, Cambridge, 1951).

SOLUTION TO THE PROBLEM ON PAGE 644

The general equation of motion is obtained from Newton's second law

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} = m\frac{dv}{dt} + rv, \quad (1)$$

where $m = m_0 + rt$.

(i) For a constant force, $-F_0$, Eq. (1) may be integrated directly

$$V = (1 - aT)/(1 + T), \quad (2)$$

where $V = v/v_0$, $T = rt/m_0$, and $a = F_0/rv_0$. Integrating again

$$X = (1 + a)\ln(1 + T) - aT, \quad (3)$$

where $X = rx/m_0v_0$. For $a = 0$, one obtains the $F = 0$ results.

(ii) For a sliding friction force, $F = -\mu mg$, one obtains

$$V = (1 - aT - aT^2/2)/(1 + T), \quad (4)$$

where $V \gtrsim 0$ and

$$X = (1 + a/2)\ln(1 + T) - a(T/2 + T^2/4), \quad (5)$$

where $a = \mu m_0 g/rv_0$.

(iii) For a viscous force proportional to the velocity, $F = -bv$, one obtains

$$V = (1 + T)^{-(1+a)}, \quad (6)$$

$$X = [1 - (1 + T)^{-a}]/a, \quad (7)$$

where $a = b/r$.

(iv) For a viscous force proportional to the square of the velocity, $F = -cv^2$, one obtains

$$V = 1/(1 + T + aT), \quad (8)$$

$$X = \ln(1 + T + aT)/(1 + a), \quad (9)$$

where $a = cv_0/r$.

(v) For a Hooke's law force, $F = -m_0\omega_0^2 x$,

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + m_0\omega_0^2 x = 0. \quad (10)$$

Equation (10) may be rewritten as

$$\frac{d^2X}{du^2} + \left(\frac{1}{u}\right)\frac{dX}{du} + X = 0, \quad (11)$$

where $u = a(1 + T)^{1/2}$ and $a = (2m_0\omega_0/r)$. Equation (11) is Bessel's equation of order 0. The solution is a linear combination of $J_0(u)$ and $N_0(u)$ which satisfies the initial conditions

$$X = \pi[J_0(a)N_0(u) - N_0(a)J_0(u)], \quad (12)$$

$$V = (\pi a^2/2u)[N_0(a)J_1(u) - J_0(a)N_1(u)]. \quad (13)$$

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