Speed of Light

Background

The first known measurement of the speed of light was attempted by Galileo, in the early 1600's. In this experiment, Galileo and an assistant stood on hilltops — about a mile apart — with shuttered lanterns. Galileo would first open the shutter in his lantern. The assistant would open the shutter in *his* lantern as soon as he saw the light from Galileo's lantern. From the time between opening his own lantern and seeing the light from his assistant's lantern, Galileo was able to determine that light moved pretty fast.

Experimental equipment has advanced somewhat since Galileo's time, although the basic idea of this lab is pretty much the same. Instead of hand-opened lanterns, we will use a pulsed diode laser with a lock-in amplifier[1] to measure the phase shift of the pulses in the reflected beam.

Lock-In Amplification

The integral of the product of two sine functions,

$$\int_{-\infty}^{\infty} \sin(\omega_1 t)\sin(\omega_2 t)\, dt$$

is usually zero. This is because the two functions are each positive (and negative) half the time, so their product is positive (and negative) half the time and the integral is then zero. The exception to this is if $\omega_1 = \omega_2 \equiv \omega$, in which case the integral becomes

$$\int_{-\infty}^{\infty} \sin^2(\omega t)\, dt$$

which is always positive.

Any signal can be expressed as the sum of some carefully-chosen set of sine waves.

$$\mathcal{S}(t) = \sum_i A_i \sin(\omega_i t)$$

So if we took an arbitrary signal and multiplied it by a signal at the frequency we wanted to see, and integrated, most of the components of the signal would

[1] Learning to use a lock-in amplifier is the real reason for measuring the speed of light in this lab. You already know the speed of light exactly, as a matter of fact: it's used to define the meter!

go away, leaving only the component with the reference frequency.

$$\begin{aligned}\int_{-\infty}^{\infty} \mathcal{S}(t)\sin(\omega_o t) &= \int_{-\infty}^{\infty} \sum_i A_i \sin(\omega_i t)\sin(\omega_o t)\ dt \\ &= 0 + 0 + \cdots + \int_{-\infty}^{\infty} \sin^2(\omega_o t)\ dt + 0 + \cdots\end{aligned}$$

This is, in essence, what is done by a lock-in amplifier.

Of course, there are some complications. For starters, it takes awhile to integrate over all time. Instead, we integrate over some time constant τ. This results in a somewhat broader peak: the integration does not go completely to zero for values of ω near ω_o. The peak is still centered at ω_o, but the width is inversely related to τ. There is a trade-off: you can get fast results with a short time constant, or precise results with a long one. In general, use as long a value of τ as you can, and at least make sure it's much greater than $1/\omega$.

Another complication is that the *phase* of the signal affects the result. We've ignored phase so far in this discussion, but consider what happens if the desired frequency component in the signal is out of phase with the reference signal by 90°.

$$\int_{-\tau}^{\tau} \sin\left(\omega t + \frac{\pi}{2}\right)\sin(\omega t)\ dt = \int_{-\tau}^{\tau} \cos(\omega t)\sin(\omega t)\ dt = 0$$

It turns out that we can use this to our advantage, though, by use of a dual-channel lock-in. One channel multiplies the signal by $\sin(\omega t)$ and integrates: the other multiplies by $\cos(\omega t)$ and integrates. The resulting two components correspond to the y and x components of the signal in phasor representation. We can then use this to obtain not only the original signal amplitude R, but also the phase difference ϕ between the signal and the reference.

$$R = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\frac{y}{x}$$

If phase measurement is an important consideration in your experiment, try to set the phase angle to about 45° so as to minimize the effect of small errors in x or y.

Experiment

- The lock-in you will be using is a dual-channel lock-in with a built-in reference signal. Set this signal to as high a frequency as possible, and connect it to the input of the diode laser. Set the amplitude of the built-in reference to 5 V.

- Arrange the laser and reflector in the hall so that the reflected signal returns to the detector, which should be connected to the input of the lock-in amplifier. Make sure you can accurately measure any *change* in the distance traveled by the laser beam.

- Configure the lock-in amplifier so that it displays the amplitude and the phase of the returning beam. Adjust the input sensitivity so that the returning signal is neither too small to be measured well, nor so large that it overloads the amplifier. Adjust the time constant to something on the order of seconds.

- Adjust the phase of the built-in reference so that the phase of the returning signal is roughly $45°$. Note that this phase by itself tells you nothing, since you do not know how many cycles off the reflection is, or how much of a phase delay there is in the electronics. However, you can measure the *change* in phase with changing distance.

- Measure the phase for a wide range of distances. Note that you may need to adjust the input sensitivity at different distances. Do *not*, however, adjust the phase or frequency!

- Use the modulation frequency of the pulsed beam to convert information about the change in phase of the returning beam to information about time. Plot distance vs. time, and find the speed of light from the slope of the graph. Report c and your experimental uncertainty in c.