Rüchardt's Method

One common method of measuring the ratio of heat capacities of gases, $\gamma = \frac{C_p}{C_v}$, is Rüchardt's method. This method uses the oscillation of a mass supported by the pressure of the gas.

Traditionally, this is done with a steel bearing oscillating in a precisely-fitted glass tube attached to a gas reservoir. Exact measurements of the frequency are difficult by this method, however, so we will be using a ground-glass syringe[1] and either a magnetic pickup or an accelerometer instead.

Mathematical Background

For an adiabatic process,

$$PV^\gamma = \text{ constant} \equiv K \ .$$

We sum the forces on the system:

$$PA - P_0A - mg = \frac{K}{V^\gamma}A - P_0A - mg = ma \tag{1}$$

where at the equilibrium position, $ma = 0$.

We will take small oscillations δx about the equilibrium position x, noting that $V = A(x + \delta x)$, and expand the first term in equation 1 using the binomial expansion:[1]

$$\frac{KA}{V^\gamma} = \frac{KA}{A^\gamma(x+\delta x)^\gamma} = \frac{KA}{(Ax)^\gamma\left(1+\frac{\delta x}{x}\right)^\gamma} \approx \frac{KA}{V_0^\gamma}\left(1-\gamma\frac{\delta x}{x}\right) \ . \tag{2}$$

We note that the equilibrium pressure in the syringe is

$$P_e \equiv \frac{K}{V_0^\gamma} \ .$$

Substitution of this definition and equation 2 into equation 1 gives us

$$P_eA\left(1-\gamma\frac{\delta x}{x}\right) - P_0A - mg = ma$$

$$P_eA - \frac{P_eA\gamma\delta x}{x} - P_0A - mg = ma$$

[1]Thanks to Matt Smith for this improved derivation.

but our equilibrium condition states that

$$P_e A - P_0 A - mg = 0 \ ,$$

so

$$ma = -\frac{P_e A \gamma \delta x}{x}$$

or since $x = V/A$,

$$a = \frac{d^2x}{dt^2} = -\left[\frac{\gamma A^2 P_e}{mV}\right]\delta x \ . \tag{3}$$

Equation 3 is the equation for simple harmonic motion, with ω^2 equal to the term in brackets. This gives us an equation relating γ to measurable quantities:

$$f_o = \frac{\omega}{2\pi} = \frac{A}{2\pi}\sqrt{\frac{\gamma P_e}{mV}} \ . \tag{4}$$

One thing to note is that equation 4 is for undamped motion. The plunger is damped by the viscosity of the gas in the syringe, so the measured frequency f will be lower than f_o. For under-damped simple harmonic motion with viscous damping, such as for this system, we can expect that the excursion X of successive maxima on one side of the oscillation would go as

$$X = X_o e^{-\lambda n} \tag{5}$$

where n is some oscillation number. In this case, the measured frequency f is related to the undamped frequency f_o by[2]

$$\frac{f_o}{f} = \sqrt{1 + \left(\frac{\lambda}{2\pi}\right)^2} \ . \tag{6}$$

Procedure

1. Make sure the syringe is well-braced within its support structure.

2. Measure all relevant experimental parameters. When considering the equilibrium pressure P_e in the syringe, remember that the pressure is not just air pressure.

3. Remove the cap from the bottom of the syringe by twisting gently. Adjust the syringe to the desired gas volume and replace the cap. Note that the volume will decrease gradually over time due to leakage

[2]Please note that the equivalent equation in [1] is erroneous. See [2] or [3].

past the plunger, so be sure to record the volume that was actually used, rather than the volume you set the apparatus to fifteen minutes before you took the data!

4. Adjust the coil support so that the magnet at the top of the plunger is centered on, and just at the bottom of, the coil.
5. Set the digital oscilloscope so that it is prepared to save a single sweep of the coil signal. "Pluck" the plunger lightly. You may have to try this a few times to get a good signal on the 'scope.
6. Use the cursors on the 'scope to measure and record amplitude and time values for as many successive peaks as possible.
7. Calculate γ. I would suggest using curve fits to calculate f and λ from the data. Be sure to include estimates of uncertainty in your results!

Repeat the measurement for several gases. Supplies may vary, but we typically have helium, argon, carbon dioxide, nitrogen, and methane available somewhere around the department.

Safety notes

- The syringe and plunger are delicate, they are a matched set, and they are expensive. Be gentle!
- When assembling the device, clamp the aluminum base to a table *before* putting the syringe into the base. When putting it away, take the syringe out *before* unclamping the base.
- Do not touch the ground-glass (frosted) section of the plunger. Fingerprint oil is a significant source of drag for this experiment.
- Do *not* fill the syringe directly from a high-pressure source with the plunger in place, as it is difficult to retrieve the plunger from the ceiling intact.
- Small oscillations are sufficient. (Required, even; consider the expansion used in the derivation!) "Plucking" the plunger as one would pluck a guitar string is adequate for good data.
- The thumbscrews on the coil holders should be finger-tightened only. Do not use an allen wrench.

References

[1] J. L. Hunt. Accurate experiment for measuring the ratio of specific heats of gases using an accelerometer. *American Journal of Physics*, 53(7):696–697, July 1985.

[2] J. B. Marion and S. T. Thornton. *Classical Dynamics.* Harcourt, Brace, Jovanovich, 3 edition, 1988.

[3] H. Lamb. *Dynamics.* Cambridge University Press, 2nd edition, 1961.