First to Third

<u>What is this about?</u>

The runner at first takes off as the batter rips a solid single to right. He's thinking he can make it to third base. What path should he take? He knows that the shortest distance between two points is a straight line, but does that mean he should run directly toward second, make a 90˚ turn and head directly for third? Usually runners take a rounded path from first to third. We'll learn which path takes the least time.

<u>What do I need?</u>

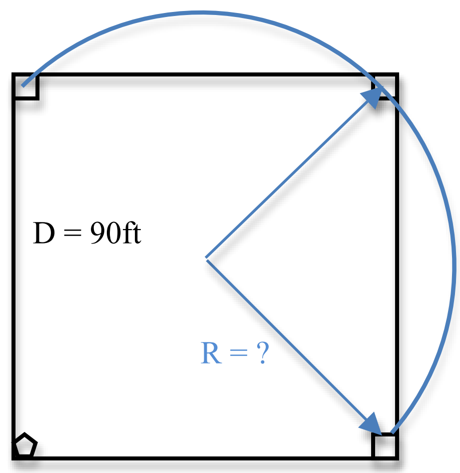
All you will need is a calculator. You will find it helpful to be familiar with the use of the kinematic equations.

<u>What will I be doing?</u>

We will need to know some information about how fast athletes can run. We'll also need to know how quickly they can speed up (accelerate) and slow down (decelerate). Then we can calculate the time it takes to run directly to second and then to third as well as the time to take a rounded path.

<u>What do I think will happen?</u>

Take a minute and write down a description of what you think will happen and why you think it. Which do you think takes less time, the direct path or the circular path? Explain your thinking.



<u>What really happened?</u>

Can you use your knowledge of triangles to figure out the radius of the circular path shown at the right. Remember that distance between the bases is ninety feet.

You can use the Pythagorean Theorem,

$$R^2 + R^2 = D^2 \Rightarrow 2R^2 = D^2.$$

So,

$$R = \frac{D}{\sqrt{2}} = \frac{90}{\sqrt{2}} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Did you get R = 63.6ft?

Do you remember that the ratio between the circumference of a circle and the radius? You're right, it's two pi = 2π = 2(3.14). Use pi to find the distance the runner has to travel along the circular path that seems to be halfway around the whole circle.

$$C = 2\pi R \Rightarrow d_{cir} = \tfrac{1}{2} 2\pi R = \pi R = \_\_\_\_\_\_\_\_\_\_\_\_$$

Did you get d_{cir} = 200ft?

So, the distance along a straight path is twice ninety feet or d_{str} = 180ft while the distance on the rounded path is d_{cir} = 200ft. Why do runners usually take a curved path, if it is so much longer? Can you think of some reasons?

Did you realize that a runner on the circular path doesn't have to slow down to change direction at second base while the runner on the straight path needs to make a ninety-degree turn? I seems like we will need to know the maximum speed, the acceleration, and deceleration of a typical runner so we can choose the best path.

A sprinter can run 100 yards in about ten seconds, so their speed is about 300ft/10s or 30ft/s. This is an average speed, so let's say their top speed is 32ft/s. Studies of sprinters show that starting from rest, they can speed up by about 16ft/s every second so their acceleration is about 16ft/s^2. The quickest way to stop at a base is to slide. A well-trained base runner can slide into a base at just the right speed that their momentum after they hit the base causes them to stand up. This is called a "pop-up slide." Let's estimate that during the slide they decelerate at about 64ft/s^2. In summary,

$$v = 32.0\text{ft/s} \quad a_{+} = 16.0\text{ft/s}^2 \quad a_{-} = 64.0\text{ft/s}^2.$$

Let's find the time for the runner to speed up to 32ft/s and find the distance they have traveled during this time. Drag out your kinematic equations!

$$v = v_o + at \Rightarrow v = 0 + a_+ t_{speedup} \Rightarrow t_{speedup} = \frac{v}{a_+} = \underline{\hspace{3cm}}$$

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow x_{speedup} = 0 + 0 + \tfrac{1}{2}a_+ t_{speedup}^2 = \tfrac{1}{2}a_+ t_{speedup}^2 = \underline{\hspace{3cm}}$$

Did you find that the runner traveled 32.0 feet while speeding up for 2.00 seconds?

The runner on the curved path can run top speed for the rest of the way to third base. Let's find how long it will take at top speed. Since the runner had 200ft to travel and he has already covered 32ft. The runner is now moving at a constant speed of 32ft/s so they are no longer accelerating,

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow x = x_o + vt \Rightarrow t_{topspeed} = \frac{x - x_o}{v} = \underline{\hspace{4cm}}$$

Did you find that the runner traveled 200-32.0=168 feet at 32.0ft/s for 5.25 seconds?

Now we know the total time it takes to run the curved path by adding the time to speed up to the time to run the rest of the distance,

$$t_{curved} = t_{speedup} + t_{topspeed} = 2.00 + 5.25 = 7.25\text{s}.$$

Now we need to find the time along the straight path. The runner will have to start at first, speed up for 2.00s traveling 32ft in the process, then run at 32ft/s for a bit, then pop-up slide into second, then speed up again, and finally run the rest of the way to third at top speed.

CURVED PATH	**distance (ft)**	**time (s)**
to speed up	32.0	2.00
at top speed	168	5.25
TOTAL	**200**	**7.25**

Let's find the distance covered and the time it takes for the slide assuming the runner is moving at 32ft/s, then decelerates at 64.0ft/s^2 and just comes to rest at second base.

$$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow 0 = v_{topspeed}^2 - 2a_{-}x_{slide} \Rightarrow x_{slide} = \frac{v_{topspeed}^2}{2a_{-}} = \underline{\hspace{5cm}}$$

$$v = v_o + a_o t \Rightarrow 0 = v_{topspeed} - a_{-}t_{slide} \Rightarrow t_{slide} = \frac{v_{topspeed}}{a_{-}} = \underline{\hspace{4.5cm}}$$

The sliding distance is 8.00ft and this takes 0.500s. So, if the speed up from first covers 32ft and the slide is 8.00ft long, that leaves 90-32-8=50ft to cover at the top speed of 32ft/s. We need the time this takes,

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow x = v_{topspeed}t_{topspeed} \Rightarrow t_{topspeed} = \frac{x}{v_{topspeed}} = \underline{\hspace{5cm}}$$

Did you get 1.56s? So, at this point the runner has sped up for 2.00s, ran at top speed for 1.56s, and completed a pop-up slide into second in 0.500s. Now, he needs to speed up again as he heads toward third. This takes 2.00s and covers 32ft, the same as before. The remaining distance to third, 90-32=58ft must be covered at top speed of 32ft/s,

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow x = v_{topspeed}t_{topspeed} \Rightarrow t_{topspeed} = \frac{x}{v_{topspeed}} = \underline{\hspace{5cm}}$$

This takes 1.81s. So, we can find the total time from first to third by adding up the time to speed up from first, the time at top speed between first and second, the time for the pop-up slide, the time to speed up toward third, and the time at top speed toward third.

STRAIGHT PATH	**distance (ft)**	**time (s)**
to speed up from 1st	32.0	2.00
at top speed	50.0	1.56
to slide into 2nd	8.00	0.500
to speed up from 2nd	32.0	2.00
at top speed to 3rd	58.0	1.81
TOTAL	**180**	**7.87**

$$t_{straight} = t_{speedup} + t_{topspeed} + t_{slide} + t_{speedup} + t_{topspeed}$$
$$t_{straight} = 2.00 + 1.56 + 0.500 + 2.00 + 1.81 = 7.87\text{s}$$

Taking the straight path takes more than half a second longer!

What did I learn?

While the shortest distance between first and third is the straight-line path, the shortest time is along a curved path. This is due to the extra time it takes to slow down, stop at second base, change direction toward third and speed up again. A runner on a curved path can stay at top speed right through second base. That is why you will always see professional ball players "round the bag."

What else should I think about?

We didn't worry about our runners slowing down as the approached third base. Can you figure out how the total times would change is the runner slide into third?

Catch it in the Web!

- Baseball And Softball Base Running Speed Calculator
  (http://www.csgnetwork.com/baseballbaserunspdcalc.html)
  This calculator is designed to show the speed of a batter (becoming a baserunner), going from home plate to first base.

- Math and Science Baseball Activities from the Event-Based Science Institute
  (http://www.ebsinstitute.com/Baseball/EBS.crm1sa.htm)
  This is a fun experiment you can do to compare your base-running times with the running times of Major League baseball players.