Sixty Baseball Physics Problems

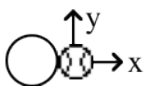
(many with correct solutions!)

Table of Contents

1. 1D Kinematics (4)

Problem 1.1:

A baseball is thrown at 40.0m/s. Mike Trout hits it back in the opposite direction at a speed of 65.0m/s. The ball is in contact with the bat for 1.20ms. Find (a)the acceleration of the ball assuming it is constant and (b)the distance the ball travels while it is in contact with the bat.



$x_o = 0$
$x = ?$
$v_o = -40.0m/s$
$v = +65.0m/s$
$a = ?$
$t = 1.20ms$

(a)Use the kinematic equation without the final position,

$$v = v_o + at \Rightarrow a = \frac{v - v_o}{t} = \frac{65-(-40)}{1.2x10^{-3}} \Rightarrow \boxed{a = 8.75x10^4 m/s^2}.$$

(b)Use the kinematic equation without the acceleration,

$$\frac{x - x_o}{t} = \frac{1}{2}(v_o + v) \Rightarrow x = \left(\frac{v_o + v}{2}\right)t$$

$$x = \left(\frac{-40+65}{2}\right)(1.2m) \Rightarrow \boxed{x = 15.0mm}.$$

Problem 1.2:

A ball is thrown upward with a speed of 15.0m/s. Find (a)the maximum altitude and (b)the time it takes to get there.

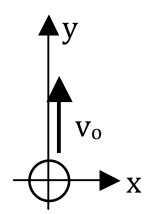
(a)Use the kinematic equation without the time,

$$v^2 = v_o^2 + 2a(y - y_o) \Rightarrow 0 = v_o^2 + 2ay \Rightarrow y = -\frac{v_o^2}{2a}.$$

Plugging in the numbers, $y = -\frac{15^2}{2(-9.8)} \Rightarrow \boxed{y = 11.5m}$.

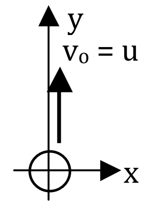
(b)Use the kinematic equation without the final position,

$$v = v_o + at \Rightarrow t = -\frac{v_o}{a} = -\frac{15}{-9.8} \Rightarrow \boxed{t = 1.53s}.$$



$y_o = 0$
$y = ?$
$v_o = 15.0m/s$
$v = 0$
$a = -9.80m/s^2$

Problem 1.3:
A baseball is thrown upward with a speed, u. (a)Show that the time it takes to reach maximum height is equal to the time it takes to fall back down. (b)Find the velocity when it gets there.



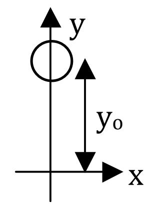
$y_o = 0$
$y = ?$
$v_o = u$
$v = 0$
$a = -g$
$t = ?$

Let's break this up into two problems, the upward motion and the downward motion.
(a) Use the kinematic equation without the final position,

$$v = v_o + at \Rightarrow 0 = u - gt \Rightarrow \boxed{t_{up} = \frac{u}{g}}.$$

For the downward motion we will need the maximum height, so using the kinematic equation without the time,

$$v^2 = v_o^2 + 2a(y - y_o) \Rightarrow 0 = u^2 - 2gy \Rightarrow y = \frac{u^2}{2g}.$$



$y_o = \frac{u^2}{2g}$
$y = 0$
$v_o = 0$
$v = ?$
$a = -g$
$t = ?$

For the downward motion, the final height for the ball becomes the initial position and the initial velocity is zero. Using the kinematic equation without the final velocity,

$$y = y_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow 0 = \frac{u^2}{2g} + 0 - \tfrac{1}{2}gt^2 \Rightarrow \tfrac{1}{2}gt^2 = \frac{u^2}{2g} \Rightarrow$$

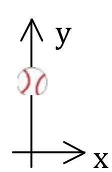
$$\boxed{t_{down} = \frac{u}{g}}.$$

Therefore, $t_{up} = t_{down}$.
(b)Using the kinematic equation for the final speed,

$$v = v_o + at = -gt = -g\left(\frac{u}{g}\right) \Rightarrow \boxed{v = -u}.$$

The final speed is the same as the initial speed.

Problem 1.4:
A student throws a baseball vertically up to her roommate at the top of the stairs 4.00m above. The ball is in the air for 1.50s. Find (a)the initial velocity of the ball and (b)the velocity of the ball just before it is caught.



$y_0 = 0$
$y = 4.00m$
$v_o = ?$
$v = ?$
$a = -9.80m/s^2$

(a)Using the kinematic equation without the final velocity,

$$y = y_o + v_o t + \tfrac{1}{2}at^2$$

Noting that $y_o = 0$ and solving for the initial velocity,

$$v_o = \frac{y}{t} - \frac{at}{2} = \frac{4.00}{1.50} - \frac{(-9.80)(1.50)}{2} \Rightarrow \boxed{v_o = 10.0m/s}$$

(b)Using the kinematic equation,

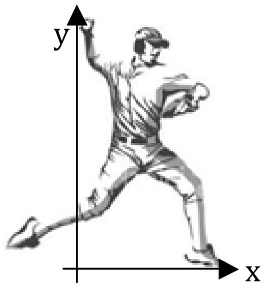
$$v = v_o + at = 10.0 + (-9.80)(1.50) \Rightarrow \boxed{v = -4.68m/s}$$

This must mean that the ball was caught on the way down, not on the way up!

2. 2D Kinematics (8)

Problem 2.1:

The pitcher throws a pitch from the mound toward home plate 18.4m away. The ball is released horizontally from a height of 2.00m with a velocity of 42.0m/s. Find (a)the time it takes for the ball to reach home plate and (b)the height above the ground when it gets there.



$x_o = 0$ $\quad y_o = 2.00m$
$x = 18.4m$ $\quad y = ?$
$v_{ox} = 42.0m/s$ $\quad v_{oy} = 0$
$v_x = 42.0m/s$ $\quad v_y = ?$
$a_x = 0$ $\quad a_y = -9.80m/s^2$

(a)Use the kinematic equation along the x-direction to get the time,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2.$$

Plugging in the things that are zero and solving for t,

$$x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{18.4}{42.0} \Rightarrow \boxed{t = 0.438s}.$$

(a)Use the kinematic equation without v_y to get the final height,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2.$$

The initial velocity is zero so plugging the numbers in,

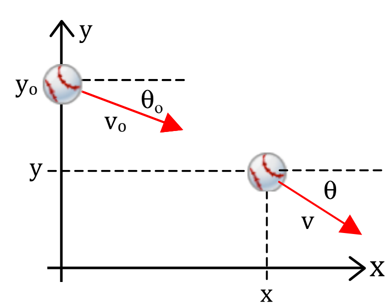
$$y = y_o + \tfrac{1}{2}a_y t^2 = 2.00 - \tfrac{1}{2}(9.80)(0.438)^2 \Rightarrow \boxed{y = 1.06m}.$$

Problem 2.2:

The last pitch of the 2012 World Series was thrown by Sergio Romo to Miguel Cabrera. When it was 48.6ft (14.8m) home plate, it was 5.233ft (1.60m) above the ground and was moving at 88.9mph (39.7m/s) at a downward angle of 1.41˚. When it got to home plate it was 2.88ft (0.878m) above the ground traveling at 81.2mph (36.3m/s) at a downward angle of 4.26˚. Find (a)the horizontal acceleration of the ball, (b)the vertical acceleration of the ball, and (c)the time to get to home plate.

Given:

$x_o = 0$ $\quad y_o = 1.60m$
$x = 14.8m$ $\quad y = 0.878m$
$v_o = 39.7m/s$ $\quad v = 36.3m/s$
$\theta_o = 1.41^\circ$ $\quad \theta = 4.26^\circ$
$v_{ox} = v_o \cos\theta_o = 39.7m/s$ $\quad v_{oy} = v_o \sin\theta_o = 0.977m/s$
$v_x = v \cos\theta = 36.2m/s$ $\quad v_y = v \sin\theta = 2.70m/s$



Find: ax = ?, ay = ? and t = ?

(a)Using the kinematic equation without the time,

$$v_x^2 = v_{ox}^2 + 2a_x(x - x_o) \Rightarrow a_x = \frac{v_x^2 - v_{ox}^2}{2(x - x_o)}.$$

Plugging in the values, $a_x = \frac{(36.2)^2 - (39.7)^2}{2(14.8 - 0)} \Rightarrow a_x = -8.97\,\frac{m}{s^2}$.

(b)Similarly along the vertical direction, $v_y^2 = v_{oy}^2 + 2a_y(y - y_o) \Rightarrow a_y = \frac{v_y^2 - v_{oy}^2}{2(y - y_o)}$.

Plugging in the values, $a_y = \frac{(2.7)^2 - (0.977)^2}{2(0.878 - 1.6)} \Rightarrow a_y = -4.39\,\frac{m}{s^2}$.

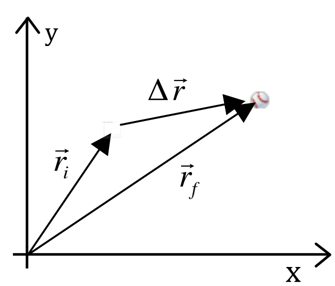
(c)Using the kinematic equation, $v_x = v_{ox} + a_x t \Rightarrow t = \frac{v_x - v_{ox}}{a_x} = \frac{36.2 - 39.7}{-8.97} \Rightarrow$

$\boxed{t = 0.390s}$.

Notice that there is horizontal acceleration and the vertical acceleration is not g. Why?

Problem 2.3:

A baseball is tossed into the air. After 0.30s it is at the position (2.2m, 2.5m). After 1.3s it is at the position (9.6m, 4.5m). (a)Draw these two position vectors and (b)the displacement vector. (c)Find the magnitude and direction of the displacement vector.



Given: $\vec{r}_i$ = (2.2m, 2.5m) and $\vec{r}_f$ = (9.6m, 4.5m).

Find: Δr = ? and θ = ?.

(c) The definition of displacement is $\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i$.

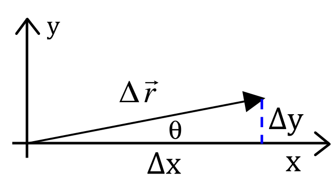
So, $\Delta x \equiv x_f - x_i$ and $\Delta y \equiv y_f - y_i$.

$\Delta x = 9.6 - 2.2 = 7.4m$ and $\Delta y = 4.5 - 2.5 = 2.0m$.

Now, we'll find the magnitude and direction using the Pythagorean Theorem and the definition of tangent.

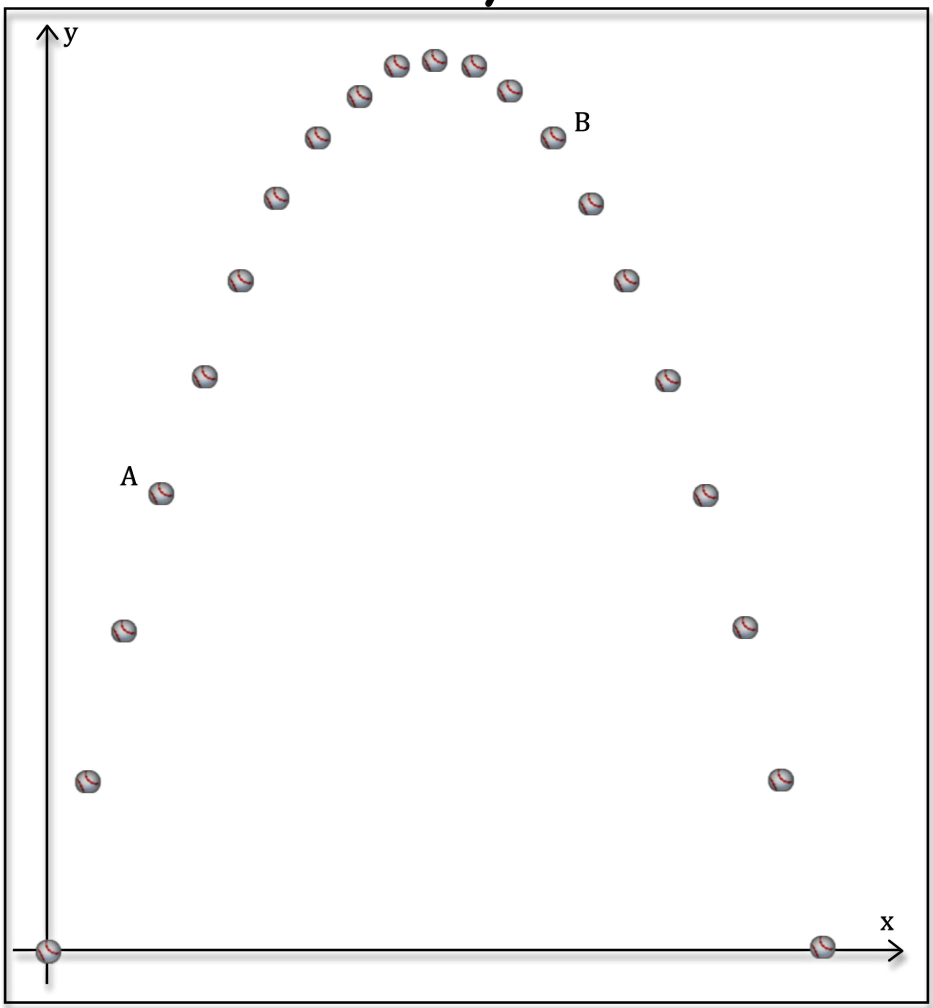
$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(7.4)^2 + (2.0)^2} \Rightarrow \boxed{\Delta r = 7.7m}$.

$\theta = \arctan\frac{\Delta y}{\Delta x} = \arctan\frac{2}{7.4} \Rightarrow \boxed{\theta = 15^\circ}$.



Problem 2.4:

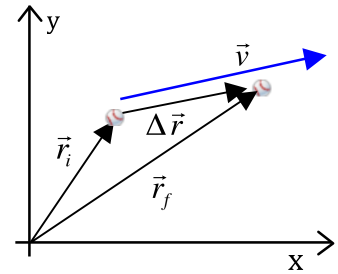
Displacement and Velocity



1. Draw the displacement vector for the ball from one image before A to one image after A. Label it $\Delta\vec{r}_A$.
2. Draw the displacement vector for the ball from one image before B to one image after B. Label it $\Delta\vec{r}_B$.
3. Explain why the velocity vector at A must point in the direction of the displacement vector at A (This is also true at B).
4. Sketch the velocity vectors at A and B.

Problem 2.5:

The position vector of the ball at 0.20s has the components (1.47m, 1.76m) and the position vector at 0.40s is given (2.94m, 3.14m). During this interval, find (a)the components of the average velocity vector, (b)the average speed of the ball, and (c)the direction of the average velocity vector.



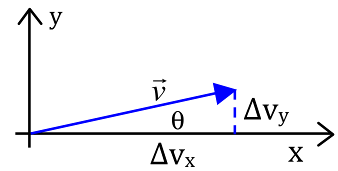
Given: $\Delta t = 0.4 - 0.2 = 0.20$s, $x_o = 1.47$m, $x = 2.94$m, $y_o = 1.76$m, and $y = 3.14$m.

Find: $v_x = ?$, $v_y = ?$, $v = ?$, and $\theta = ?$.

(a)Using the definition of average velocity, the components of the average velocity are the components of the displacement per time,

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x - x_o}{\Delta t} = \frac{2.94 - 1.47}{0.2} \Rightarrow \boxed{v_x = 7.35\,\tfrac{m}{s}}$$ and

$$v_y = \frac{\Delta y}{\Delta t} = \frac{y - y_o}{\Delta t} = \frac{3.14 - 1.76}{0.2} \Rightarrow \boxed{v_y = 6.90\,\tfrac{m}{s}}.$$



(b)Using the Pythagorean Theorem or the definition of speed,

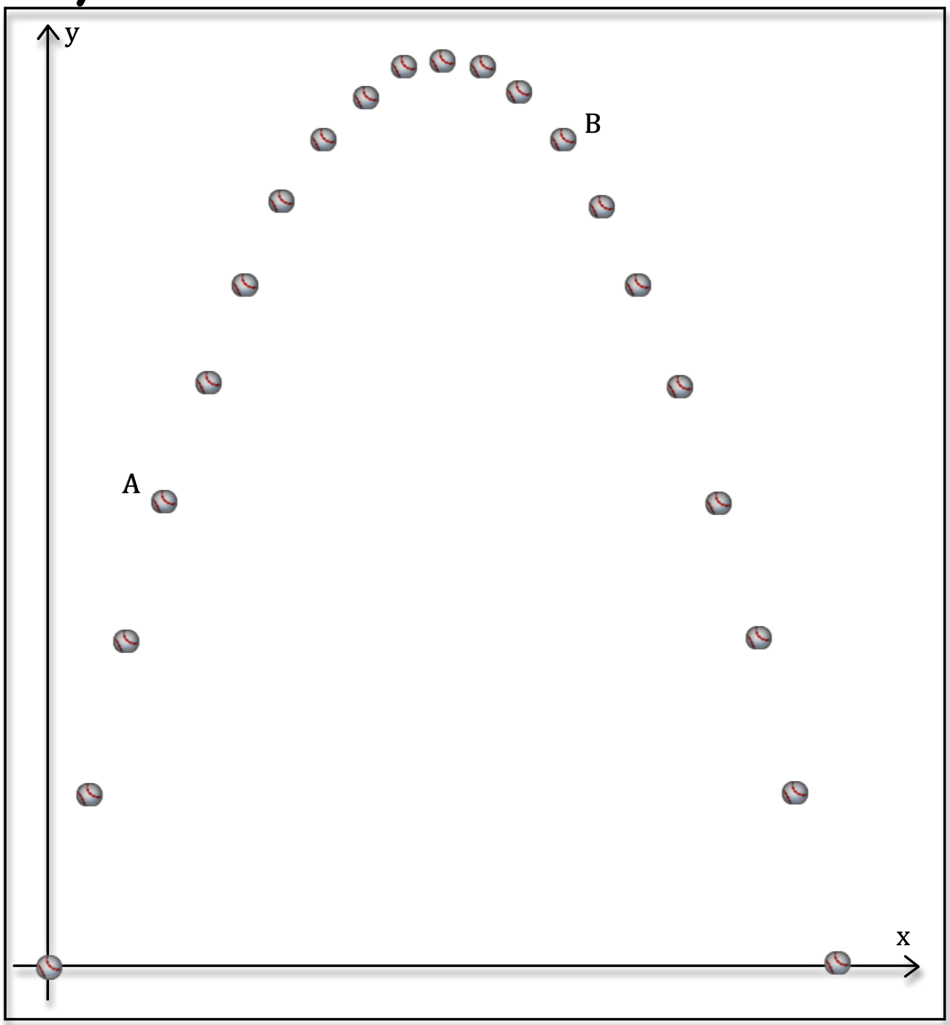
$$v \equiv |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(7.35)^2 + (6.9)^2} \Rightarrow \boxed{v = 10.1\,\tfrac{m}{s}}.$$

(c)Using the definition of the tangent, $\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \arctan\frac{v_y}{v_x} = \arctan\frac{6.90}{7.35} \Rightarrow$

$$\boxed{\theta = 43.2^\circ}.$$

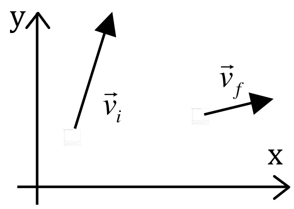
Problem 2.6:

Velocity and Acceleration



1. Draw the velocity vector for the ball at A. Label it $\vec{v}_A$.
2. Draw the velocity vector for the ball at B. Label it $\vec{v}_B$.
3. Redraw the two velocity vectors with their tails at the origin.
4. Draw the change in velocity vector. Label it $\Delta\vec{v}$.
5. Explain why $\Delta\vec{v}$ must point in the direction of the acceleration vector.
6. Explain why the acceleration vector points directly downward.

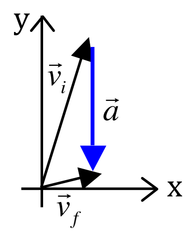
Problem 2.7:



At t = 0.30s a baseball has a velocity of (7.35m/s, 6.86m/s). At t = 1.3s its velocity is (7.35m/s, -2.94m/s). Find the magnitude and direction of the average acceleration vector.

Given: v_{ox} = 7.35m/s, v_{oy} = 6.86m/s, v_x = 7.35m/s, v_y = -2.94m/s, and Δt = 1.3-0.3 = 1.00s.

Find: $a = |\vec{a}|$ and θ = ?.



(a)Using the definition of average acceleration to find the components,

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_x - v_{ox}}{\Delta t} = \frac{7.35 - 7.35}{1} \Rightarrow a_x = 0 \text{ and}$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{v_y - v_{oy}}{\Delta t} = \frac{-2.94 - 6.86}{2} \Rightarrow a_y = -9.80\,\tfrac{m}{s^2}.$$

Since there is no acceleration along x the magnitude is just,

$\boxed{a = -9.8\,\tfrac{m}{s^2}}$ and the direction is along –y or downward (θ = -90°).

So even though the ball is moving horizontally, it is still obeying the Rule of Falling Bodies.

Problem 2.8:

A baseball has a velocity of $35.0\hat{i} + 12.0\hat{j} + 25.0\hat{k}$ in m/s. Its velocity is $32.0\hat{i} + 10.0\hat{j} + 5.00\hat{k}$ after 2.00s. Find (a)the average acceleration vector and (b)the magnitude of the average acceleration.

(a)Using the definition of average acceleration,

$$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(32\hat{i} + 10\hat{j} + 5\hat{k}) - (35\hat{i} + 12\hat{j} + 25\hat{k})}{2} \Rightarrow \boxed{\vec{a} = -1.50\hat{i} - 1.00\hat{j} - 10.0\hat{k}}$$

in m/s^2.

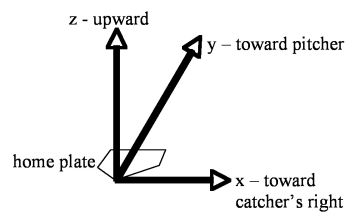
(b)Using the Pythagorean Theorem,

$$a \equiv |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(-1.5)^2 + (-1)^2 + (-10)^2} \Rightarrow \boxed{a = 10.2m/s^2}.$$

3. 3D Kinematics (4)

On September 14, 2008 a very unusual Major League Baseball game was played between the Chicago Cubs and the Houston Astros. Due to Hurricane Ike moving through Houston the game was played in neither teams home stadium. It was at a neutral site, Miller Park in Milwaukee. The uniqueness of the game expanded in the ninth inning when Carlos Zambrano struck out Darin Erstad to complete a no-hitter, the first no-no at a neutral site in major league history. Below is a table containing the PicthFX kinematic data in three dimensions for the final pitch of the game and a graphic illustrating the coordinate system.

	x-direction	y-direction	z-direction
initial position (ft)	-2.434	50.0	5.428
initial velocity (ft/s)	3.344	-125.384	-4.389
acceleration (ft/s^{S})	-13.031	25.891	-29.975



Problem 3.1:
Find the initial speed of the pitch in mph.

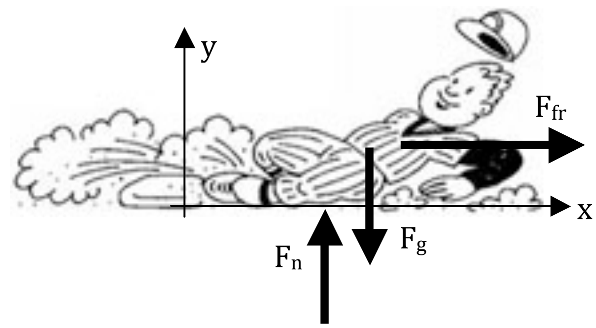
Problem 3.2:
Use the data from the y-direction to find the time for the pitch to get from the pitcher to the back of home plate. You will have to solve a quadratic equation. One answer will be reasonable and the other is not.

Problem 3.3:
Use the time of flight to find the (a)x-component of the velocity, (b)y-component of the velocity, (c)z-component of the velocity, and (d)speed (in mph) of the pitch when it gets to the back of home plate.

Problem 3.4:
Find the (a)x-position, (b)y-position, and (c)z-position of the ball at the back of home plate. Your right! The answer to part b is zero. If you can find a video of this pitch on-line you can check your answer.

4. Forces (7)

Problem 4.1:



An 85.0kg base runner tries to steal second base. When he is 3.00m from the base he begins his slide at a speed of 9.00m/s. He comes to rest just as he touches the base. Find (a)his acceleration, (b)the average frictional force exerted by the ground on the sliding runner, and (c)the coefficient of friction between the runner and the ground.

given:
$x_o = 3.00m$
$x = 0$
$v_o = -9.00m/s$
$v = 0$
$a = ?$
$m = 85.0kg$
$F_{fr} = ?$
$\mu = ?$

(a)Use the kinematic equation without time,

$$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow 0 = v_o^2 - 2ax_o \Rightarrow a = \frac{v_o^2}{2x_o} = \frac{(-9.00)^2}{2(3.00)} \Rightarrow$$

$\boxed{a = 13.5m/s^2}$.

(b)Applying the Second Law along the x-direction,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} = ma = (85.0)(13.5) \Rightarrow \boxed{F_{fr} = 1150N}.$$

(c)Applying the Second Law along the y-direction,

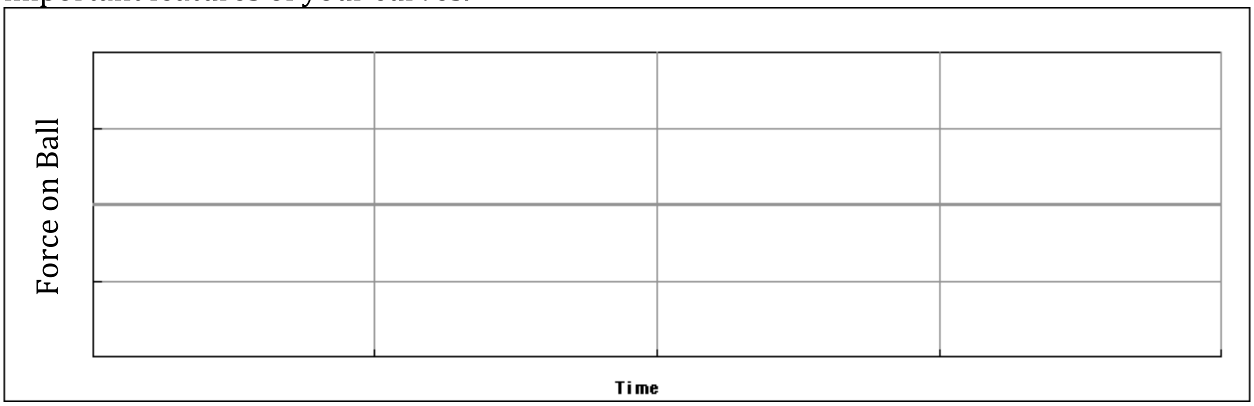
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g = mg.$$

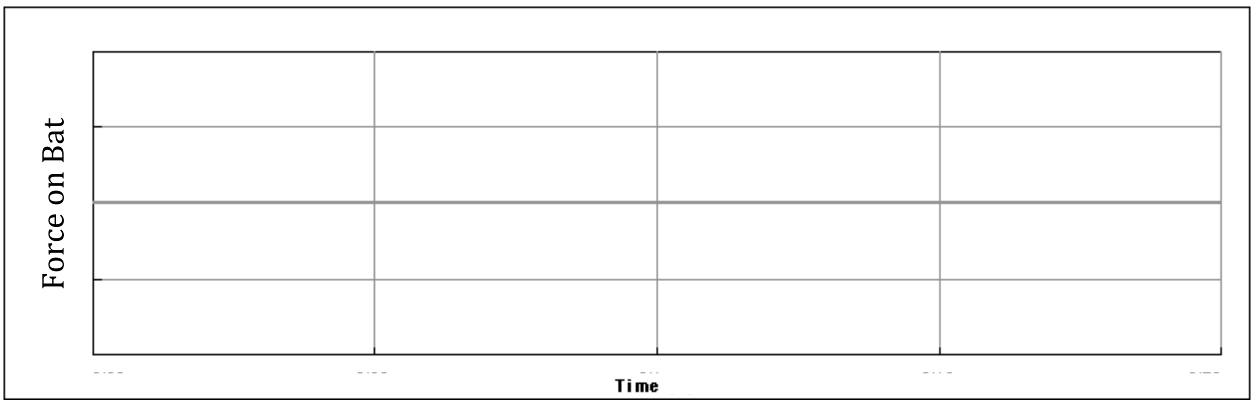
Using the definition of COKF,

$$\mu_k \equiv \frac{F_{fr}}{F_n} = \frac{ma}{mg} = \frac{a}{g} = \frac{13.5}{9.80} \Rightarrow \boxed{\mu_k = 1.38}.$$

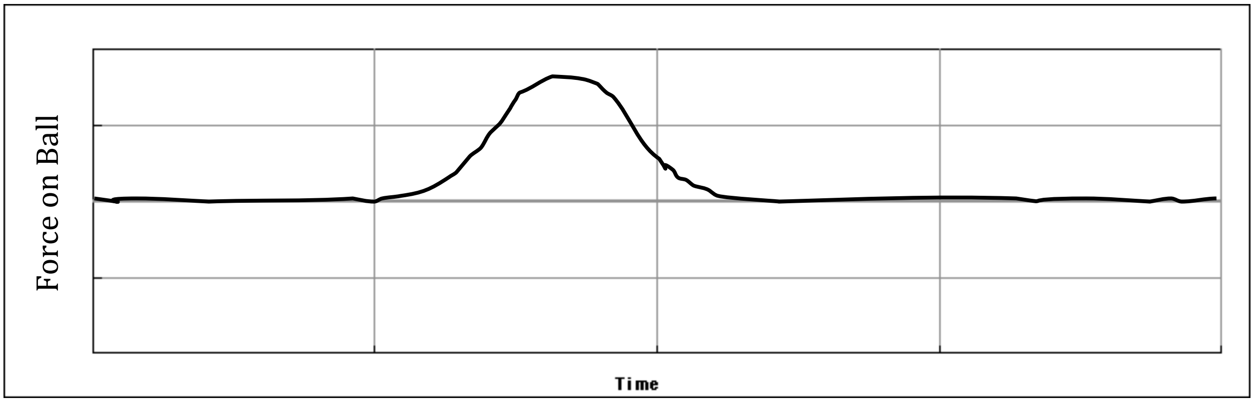
Problem 4.2:

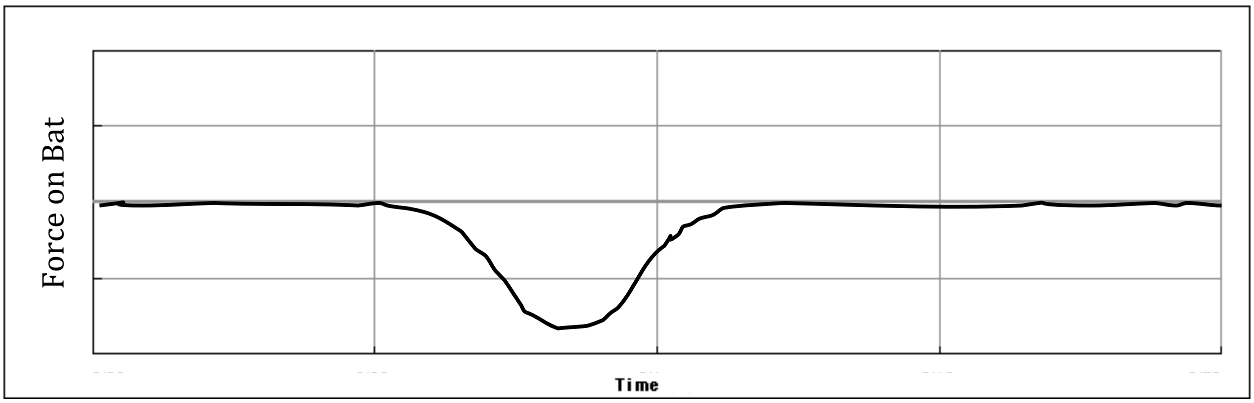
A 0.145kg baseball collides with a 1.10kg baseball bat. (a)On the upper graph below sketch the force felt by the ball as a function of time. (b)On the lower graph sketch the force felt by the bat as a function of time. (c)Explain your thinking as well as the important features of your curves.





The key idea is Newton's Third Law. At every instant during the short time of the collision, the forces are equal and opposite. Other features of the graph include the fact that the force is probably not constant during the collision and the force lasts for only a short time.



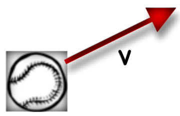


Problem 4.3:

A force is a force, of course, of course…

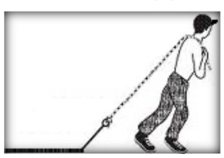
For each situation below, draw the forces that act on the object and sketch the free body diagram.

1. A well hit ball flying through the air.



2. A runner sliding into second base.

3. A screen dragged over the infield during the 7th inning stretch.



4. A bat leaning against the dugout wall.

Problem 4.4:

In this commercial (http://www.youtube.com/watch?v=QZNR5mxQDwM&NR=1) a 90.0kg baseball player slides around the bases at a constant speed. (a)Explain why this violates Newton's Laws. Suppose that this ad was made by pulling the player with a rope along the ground and that the tension in the rope was 750N. (b)Find the magnitude of all four forces acting on the player.

Problem 4.5:

A baseball catcher is expected to catch a 100mph (44.7m/s) fastball. Find (a)the force his glove exerts on the 150g baseball if it is brought to rest in 0.200s and (b)the force the ball exerts on the catcher's glove.

Problem 4.6:

Charles "Gabby" Street was a catcher for the Washington Senators from 1909 to 1911. He reputedly caught a 145g baseball dropped from the top of the Washington Monument which is 152m tall. Modern wind tunnel measurements suggest that the terminal speed of a dropped baseball should be about 42.7m/s.

1. Ignoring air resistance, find (a)the time it takes for the ball to reach the ground and (b)the speed when it gets there.

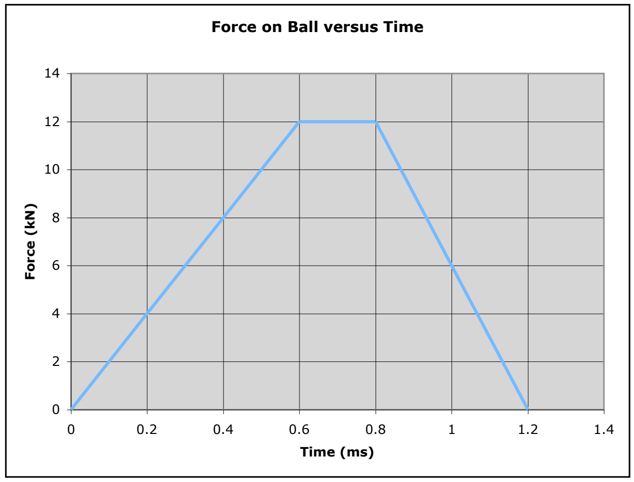
2. Find (a)the force of air resistance on the ball at terminal speed and (b)the drag coefficient for the ball.

3. Find the time it took the baseball to reach 90% of terminal speed. Compare this answer with the time to fall from part 1. Comment on the significance of air resistance for the fall of the baseball.

4. (a)Take the equation for the velocity versus time for an object falling with air resistance and derive the equation for the distance fallen as a function of time for the baseball. (b)Graph the distance fallen versus time to find the actual time to fall 152m. (c)Compare your answer with the answer from part 1.

Problem 4.7:

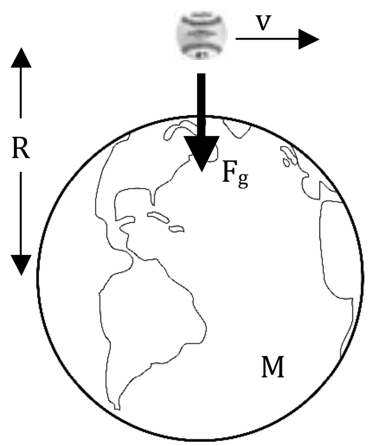
The force exerted on a baseball by a bat as a function of time is shown in the graph at the right. Find the impulse exerted during the collision (a)on the baseball and (b)on the bat. Find the change in momentum of (c)the baseball and (d)the bat.



5. Circular Motion and Gravitation (2)

Problem 5.1:

A baseball announcer describing a long home run states that the batter "put that one in orbit." Find the speed that the ball would have to leave the bat in order to go into orbit just above the surface of the earth.



Applying the Second Law to the ball, $\Sigma F = ma \Rightarrow F_g = ma$.

The acceleration is centripetal, $a = \frac{v^2}{r} \Rightarrow F_g = m\frac{v^2}{R}$.

Using the Law of Universal Gravitation, $F_g = G\frac{mM}{R^2}$,

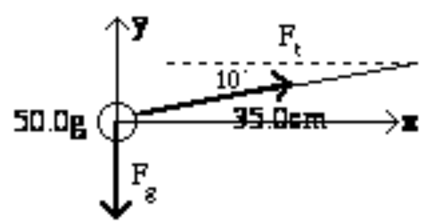
$$G\frac{mM}{R^2} = m\frac{v^2}{R} \Rightarrow G\frac{M}{R} = v^2 \Rightarrow v = \sqrt{G\frac{M}{R}}.$$

Plugging in the numbers,

$$v = \sqrt{(6.67x10^{-11})\frac{5.97x10^{24}}{6.38x10^{6}}} \Rightarrow \boxed{v = 7900m/s}.$$

Problem 5.2:

A 50.0g baseball is twirled overhead at the end of a 35.0cm string. The string makes a 10.0° angle with the horizontal. (a)Draw the forces that act on the ball is the sketch at the right. (b)Find the speed of the ball and (c)the number of revolutions per minute.



(b)Applying Newton's Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_t\cos 10° = ma_c \text{ and } \Sigma F_y = ma_y \Rightarrow F_t\sin 10° - F_g = 0 \Rightarrow F_t\sin 10° = F_g.$$

Using the mass/weight rule and the centripetal acceleration,

$$F_t\cos 10° = m\frac{v^2}{r} \text{ and } F_t\sin 10° = mg.$$

Dividing the vertical equation by the horizontal equation,

$$\frac{F_t\sin 10°}{F_t\cos 10°} = \frac{mgr}{mv^2} \Rightarrow \tan 10° = \frac{gr}{v^2}.$$

Noting that $r = \ell\cos 10°$ and solving for the speed,

$$\tan 10° = \frac{g\ell\cos 10°}{v^2} \Rightarrow v = \sqrt{\frac{g\ell\cos 10°}{\tan 10°}} = \sqrt{\frac{(9.80)(0.350)\cos 10°}{\tan 10°}} \Rightarrow \boxed{v = 4.38m/s}.$$

(b)Using the definition of speed,

$$v \equiv \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T} = 2\pi rf \Rightarrow f = \frac{v}{2\pi r} = \frac{v}{2\pi\ell\cos 10°} = \frac{4.38}{2\pi(0.350)\cos 10°} \Rightarrow$$

$$\boxed{f = \left(2.02\tfrac{rev}{s}\right)\left(\tfrac{60\,s}{min}\right) = 121rpm}.$$

6. Work and Energy (7)

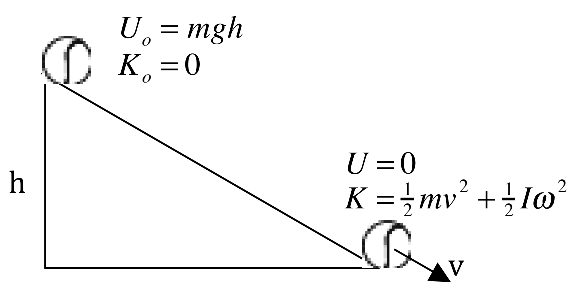
Problem 6.1:
Calculate the rotational kinetic energy of a curveball rotating at 2400rpm about the center. The mass of a baseball is 145g and it has a radius of 3.64cm. Compare that to the translational kinetic energy if the ball is moving at 40.0m/s.

Given: ω = 2400rpm = 251rad/s, m = 0.145kg, r = 0.0364m, v = 40.0m/s, and I = 7.68x10-5kg·m^2.
Find: K = ?

Using the rotational kinetic energy,
$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(7.68x10^{-5})(251)^2 \Rightarrow \boxed{K = 2.42J}$.

The translational kinetic energy of the ball is,
$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.145)(40)^2 \Rightarrow \boxed{K = 116J}$.

Problem 6.2:
A 0.145kg baseball starts near rest and rolls without slipping down a 55.0cm high pitchers mound. Find the speed of the ball at the bottom.



Using the Law of Conservation of Energy,
$\Delta U + \Delta K = 0 \Rightarrow (0 - mgh) + (\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2) = 0$

Since the ball rolls without slipping, $\omega = \frac{v}{r}$.

The rotational inertia of a sphere is, $I = \frac{2}{5}mr^2$.

$K_o = 0$
Substituting into the Conservation of Energy equation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mr^2)\frac{v^2}{r^2} \Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \Rightarrow mgh = \frac{7}{10}mv^2 .$$

Canceling the mass and solving for the speed,
$v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}(9.80)(0.550)} \Rightarrow \boxed{v = 2.77m/s}$.

Problem 6.3: A 0.150kg baseball thrown by a pitcher leaves his hand 2.00m above the ground at a speed of 102mph (45.6m/s). It is caught by the catcher 60.5ft (18.4m) away at a height of 1.00m and a speed of 97.0mph (43.4m/s). Find (a)the initial kinetic energy of the ball, (b)the final kinetic energy of the ball, (c)the net work done on the ball, (d)the net work done by gravity and (e)the work done by the resistive forces that act on the ball.

(a)Using the Definition of Kinetic Energy,
$K_o = \frac{1}{2}mv_o^2 = \frac{1}{2}(0.150)(45.6)^2 \Rightarrow \boxed{K_o = 156J}$.

(b)Similarly,
$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150)(43.4)^2 \Rightarrow \boxed{K_o = 141J}$.

(c)Using the Work-Energy Theorem,
$W_{net} = \Delta K = K - K_o = 141 - 156 \Rightarrow \boxed{W_{net} = -15J}$.

(d)Using the Definition of Work,
$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_g = F_g \Delta y = mg\Delta y = (0.150)(9.8)(1.00) \Rightarrow \boxed{W_g = 1.47J}$.

(e)The total work done is the sum of the work done by gravity and by the resistive forces.
$W_{net} = W_g + W_r \Rightarrow W_r = W_{net} - W_g = -15 - 1.47 \Rightarrow \boxed{W_r = -16J}$.

Problem 6.4:

Drop Zone

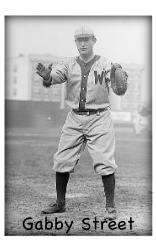
Gabby Street was a catcher for the Washington Senators from 1909 to 1911. He reputedly caught a baseball (m = 150g) dropped from the top of the Washington Monument known to be 555ft (170m) tall. Assume there is no air resistance and g = 10m/s². Fill in the blanks at the right.

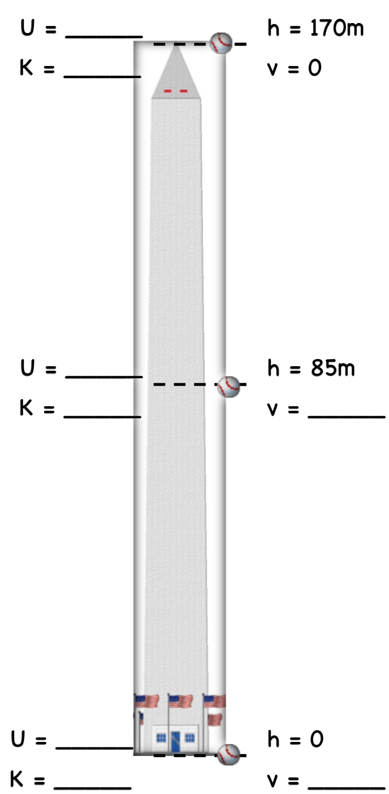
The principles of physics I used were:

1.

2.

3.





Problem 6.5:

Charles "Gabby" Street was a catcher for the Washington Senators from 1909 to 1911. He reputedly caught a 145g baseball dropped from the top of the Washington Monument which is 152m tall. Modern wind tunnel measurements suggest that the maximum speed of a dropped baseball should be about 42.7m/s. Find (a)the work done by gravity on the falling ball, (b)the net work done on the ball during its fall and (c)the work done by air resistance during the fall.

(a)Using the definition of work and the fact that the force of gravity is constant and in the direction of motion of the ball,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_g = F_g y = mgy = (0.145)(9.80)(152) \Rightarrow \boxed{W_g = 216J}.$$

(b)According to the Work-Energy Theorem,

$$W_{net} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_o^2 = \tfrac{1}{2}(0.145)(42.7)^2 - 0 \Rightarrow \boxed{W_{net} = 132J}.$$

(c)The total work done is the sum of the work done by each force. In this case,

$$W_{net} = W_g + W_{air} \Rightarrow W_{air} = W_{net} - W_g = 132 - 216 \Rightarrow \boxed{W_{air} = -83.8J}.$$

Note that the answer is negative because air resistance acts opposite the motion.

Problem 6.6:

Use energy methods to find the speed that the baseball in problem 6.5 would have struck the ground if there were no air resistance.

Applying the Law of Conservation of Energy and ignoring air resistance,

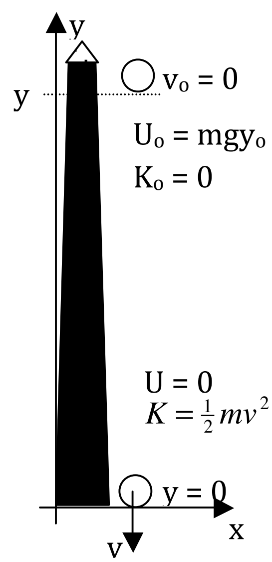
$$\Delta K + \Delta U = 0 \Rightarrow (K - K_o) + (U - U_o) = 0 \Rightarrow K - U_o = 0 \Rightarrow K = U_o$$

Using the kinetic and potential energies,

$$\tfrac{1}{2}mv^2 = mgy_o \Rightarrow v = \sqrt{2gy_o}$$

Plugging in the numbers,

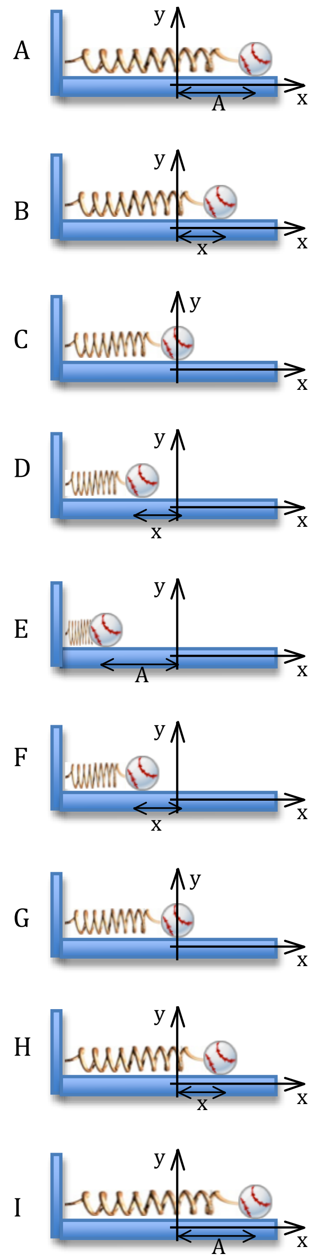
$$v = \sqrt{2(9.80)(152)} \Rightarrow \boxed{v = 54.6m/s}.$$



Problem 6.7:

Follow the Bouncing Ball

The nine images at the right were taken sequentially with a video camera as the ball oscillated back and forth on the end of the spring. Rank them from largest to smallest. If some are equal to others put an equal sign between them.



Rank the frequency of oscillation of the system:

___ ___ ___ ___ ___ ___ ___ ___ ___

Rank the acceleration vector of the ball:

___ ___ ___ ___ ___ ___ ___ ___ ___

Rank the speed of the ball:

___ ___ ___ ___ ___ ___ ___ ___ ___

Rank the velocity of the ball:

___ ___ ___ ___ ___ ___ ___ ___ ___

Rank the ball's kinetic energy:

___ ___ ___ ___ ___ ___ ___ ___ ___

Rank the spring's potential energy:

___ ___ ___ ___ ___ ___ ___ ___ ___

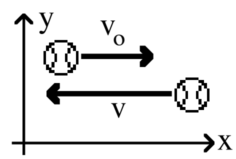
Rank the total energy of the system:

___ ___ ___ ___ ___ ___ ___ ___ ___

7. Impulse and Linear Momentum (5)

Problem 7.1:
A baseball is thrown at 42.0m/s. Miguel Caberra hits it back in the opposite direction at a speed of 52.0m/s. The ball is in contact with the bat for 1.20ms. Find (a)the initial momentum of the ball, (b)the final momentum of the ball, (c)the change in momentum of the ball, (d)the impulse imparted to the ball, and (e)the impulse imparted to the bat.

Problem 7.2:
A 150g baseball pitched at 30.0m/s is hit straight back at the pitcher at 50.0m/s. The ball is in contact with the bat for 1.00ms. Find (a)the initial momentum, (b)the final momentum and (c)the average force on the ball.



Given: m = 0.150kg, v_o = 30.0m/s, v = -50.0m/s, and Δt = 1.00x10-3s.
Find: p_o = ?, p = ?, and F = ?

(a)Using the definition of linear momentum,

$$\vec{p} \equiv m\vec{v} \Rightarrow p_o = mv_o = (0.150)(30.0) \Rightarrow \boxed{p_o = 4.50 kg \cdot {}^{m}\!/\!_{s}}.$$

(b)Using the definition of linear momentum again,

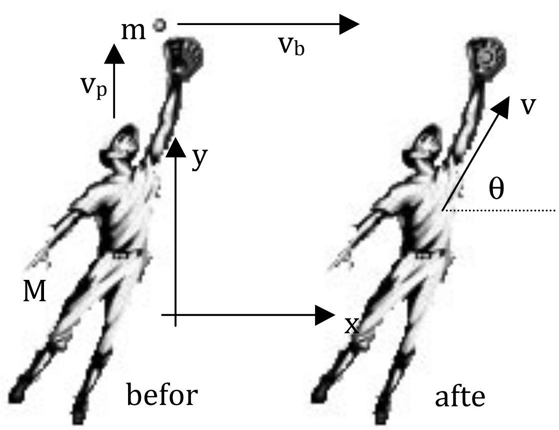
$$\vec{p} \equiv m\vec{v} \Rightarrow p = mv = (0.150)(-50.0) \Rightarrow \boxed{p = -7.50 kg \cdot {}^{m}\!/\!_{s}}.$$

Note that momentum is a vector quantity.
(c)Applying the Second Law,

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow F = \frac{\Delta p}{\Delta t} = \frac{p - p_o}{\Delta t} = \frac{-7.50 - (4.50)}{1.00m} \Rightarrow \boxed{F = -12.0kN}.$$

The force is to the left, in the negative x direction.

Problem 7.3:
A 75.0kg shortstop jumps upward to catch the 150g baseball traveling horizontally at 55.0m/s. At the instant before the catch the shortstop is traveling upward at 0.250m/s. Find the velocity (magnitude and direction) of the shortstop and ball just after the catch.



The initial components of the linear momentum are,

$$p_{ox} = mv_b \text{ and } p_{oy} = Mv_p.$$

Just after the catch the components can be written as,

$$p_x = (M+m)v\cos\theta \text{ and}$$
$$p_y = (M+m)v\sin\theta.$$

Applying the Law of Conservation of Linear Momentum,

$$p_{ox} = p_x \Rightarrow mv_b = (M+m)v\cos\theta \text{ and}$$
$$p_{oy} = p_y \Rightarrow Mv_p = (M+m)v\sin\theta.$$

Dividing the y-equation by the x-equation will eliminate the final speed,

$$\frac{Mv_p}{mv_b} = \frac{(M+m)v\sin\theta}{(M+m)v\cos\theta} = \tan\theta \Rightarrow \theta = \arctan\left[\frac{Mv_p}{mv_b}\right] = \arctan\left[\frac{(75.0)(0.250)}{(0.150)(55.0)}\right] \Rightarrow \boxed{\theta = 66.3^\circ}.$$

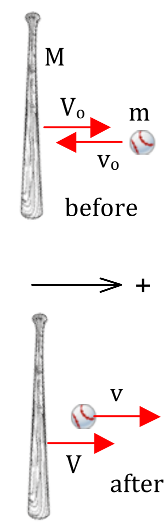
Squaring the y-equation and adding it to the square of the x-equation will eliminate the angle,

$$\left(Mv_p\right)^2 + \left(mv_b\right)^2 = (M+m)^2 v^2 \sin^2\theta + (M+m)^2 v^2 \cos^2\theta \Rightarrow \left(Mv_p\right)^2 + \left(mv_b\right)^2 = (M+m)^2 v^2$$

$$\Rightarrow v = \frac{\sqrt{\left(Mv_p\right)^2 + \left(mv_b\right)^2}}{M+m} = \frac{\sqrt{(75.0\cdot 0.250)^2 + (0.150\cdot 55.0)^2}}{75.0+0.150} \Rightarrow \boxed{v = 0.273 m/s}.$$

Problem 7.4:

A homerun can leave the bat at about 110mph (49m/s), while an average fastball heads toward the batter at about 92mph (41m/s). The mass of a baseball is 145g while a typical bat has a mass of 36oz (1.0kg). The (center-of-mass) speed of the bat when it strikes the ball is about 50mph (22m/s). Find the speed of the bat just after hitting the ball.



Given: v = 49m/s, v_o = 41m/s, V_o = 22m/s, m = 0.145kg, and M = 1.0kg.
Find: V = ?

The initial momentum is, $p_o = MV_o - mv_o$,
while the final momentum is, $p = MV + mv$.
Using the Law of Conservation of Linear Momentum,
$p_o = p \Rightarrow MV_o - mv_o = MV + mv$.
Solving for the final speed of the bat,
$V = \frac{1}{M}(MV_o - mv_o - mv) = V_o - \frac{m}{M}(v_o + v)$.
Plugging in the numbers,
$V = 22 - \frac{0.145}{1}(41+49) \Rightarrow \boxed{V = 9.0 \frac{m}{s}}$.

Problem 7.5:

Justin Verlander has a mass of 90kg. He is floating on a still lake in a 20kg canoe. He throws a 150g baseball at 45m/s. Find the speed the canoe moves after the baseball is thrown.

8. Rotational Motion (3)

Problem 8.1:
A sporting goods catalog lists the masses and lengths that are available in a given style of bat. Rank them according to their rotational inertia from smallest to largest. Be sure to explain your reasoning.

Label		Mass (kg)	Length (cm)
A		0.90	75
B		1.00	80
C		1.10	85
D		0.80	75
E		0.90	80
F		1.00	85

The rotational inertia of an object like a bat is some multiple of the mass and length squared. Since the multiple is the same for all bats of similar shape, the ranking goes as this product. The shortest lightest bat is the smallest (D) and the longest heaviest bat is the largest (C). The rest must be calculated.

A: $mL^2 = (0.900)(0.75)^2 = 0.51 kg \cdot m^2$
B: $mL^2 = (1.000)(0.80)^2 = 0.64 kg \cdot m^2$
C: $mL^2 = (1.100)(0.85)^2 = 0.79 kg \cdot m^2$
D: $mL^2 = (0.800)(0.75)^2 = 0.45 kg \cdot m^2$
E: $mL^2 = (0.900)(0.80)^2 = 0.58 kg \cdot m^2$
F: $mL^2 = (1.000)(0.85)^2 = 0.72 kg \cdot m^2$

The ranking is, D<A<E<B<F<C.

Problem 8.2:
Find the torque that a pitcher must exert on a 150g baseball with a radius of 3.50cm to get it to go from rest to a spin rate of 2400rpm in the 0.200s it takes to "snap" their wrist.

Using the definition of angular acceleration,

$$\alpha \equiv \frac{d\omega}{dt} \approx \frac{\omega - \omega_o}{\Delta t} = \frac{\omega}{\Delta t} = \frac{\left(2400 \frac{rev}{\min}\right)\left(\frac{\min}{60s}\right)\left(2\pi \frac{rad}{rev}\right)}{0.200s} \Rightarrow \alpha = 1260 \frac{rad}{s^2}.$$

Using the Second Law for Rotation,

$$\Sigma\tau = I\alpha \Rightarrow \tau = I\alpha.$$

The rotational inertia of a solid sphere is,

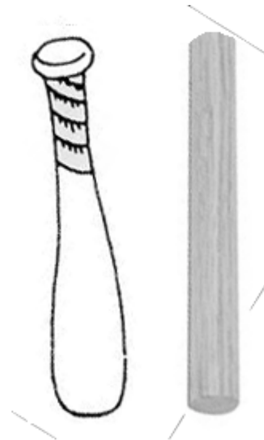
$$I = \tfrac{2}{5}mr^2.$$

Substituting,

$$\tau = \tfrac{2}{5} m r^2 \alpha = \tfrac{2}{5}(0.150)(0.0350)^2(1260) \Rightarrow \boxed{\tau = 0.0926 N \cdot m}.$$

Problem 8.3:

A baseball bat and a uniform stick of wood both have the same mass and length. Explain which one has the higher rotational inertia about the top end shown in the sketch at the right.



The rotational inertia of an object is defined to be,

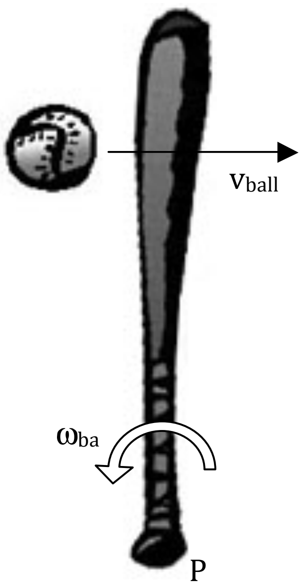
$$I \equiv \int r^2 dm.$$

This means that objects that have a greater fraction of their mass further away from the axis of rotation have a larger rotational inertia. Since a baseball bat is designed to get wider at the "business" end. It has more mass further away from the axis of rotation than a uniform stick. Therefore, the bat has a larger rotational inertia than the stick.

9. Torque and Angular Momentum (5)

Problem 9.1:

A 1.00kg bat is rotating about the knob (P) at 600rpm as the 150g ball approaches with a speed of 40.0m/s. The rotational inertia of the bat about the knob end is 0.350kg·m^2 and the ball strikes the bat 60.0cm from the knob. Find (a)the angular momentum of the ball about P, (b)the angular momentum of the bat about P, and (c)the velocity of the ball after the collision assuming the rotation rate of the bat is 300rpm.



(a)Using the definition of angular momentum,

$$\vec{L} \equiv \vec{r} \times \vec{p} \Rightarrow L = rmv = (0.600)(0.150)(40.0) \Rightarrow$$

$$\boxed{L = 3.60 kg \cdot m^2/s}.$$

The direction is into the paper by the right hand rule.

(b)First convert the units of the rotational rate,

$$\omega = (600\tfrac{rev}{\min})(\tfrac{\min}{60s})(\tfrac{2\pi rad}{rev}) = 62.8 rad/s.$$

The angular momentum of a rigid body is,

$$L = I\omega = (0.350)(62.8) \Rightarrow \boxed{L = 22.0 kg \cdot m^2/s}.$$

This is out of the paper by the right hand rule.

(c)The total angular momentum before the collision is the sum of the angular momentum ball and the angular momentum of the bat,

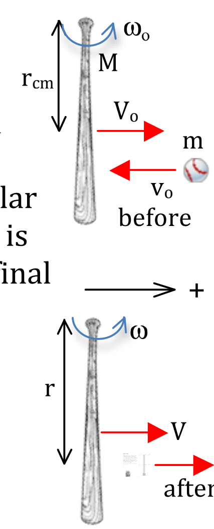
$$L_o = 22.0 - 3.60 = 18.4 kg \cdot m^2/s.$$

After the collision the total will be, $L = rmv + I\omega$.
Using the Law of Conservation of Angular Momentum,

$$L = L_o \Rightarrow L_o = rmv + I\omega \Rightarrow v = \frac{L_o - I\omega}{rm} = \frac{18.4 - (0.350)(31.4)}{(0.600)(0.150)} \Rightarrow \boxed{v = 82.3 m/s}.$$

Problem 9.2:

A homerun can leave the bat at about 110mph (49m/s), while an average fastball heads toward the batter at about 92mph (41m/s). The mass of a baseball is 145g while a typical bat has a mass of 36oz (1.0kg). The (center-o mass) speed of the bat when it strikes the ball is about 50mph (22m/s). Find (a)the initial angular momentum of the ball about the knob and the final angular momentum of the ball about the knob. Assuming the center of mass of the bat is 60cm from the knob, find (c)the initial rotational speed of the bat and (d)the final rotational speed of the bat. (e)Find the rotational inertia of the bat.



Given: v = 49m/s, vo = 41m/s, Vo = 22m/s, V = 9.0m/s,
m = 0.145kg, M = 1.0kg, rcm = 0.60m and r = 0.75m.
Find: Lo1 = ?, L1 = ?, wo = ?, w = ?, and I = ?

The angular momentum for a point object is $L = r_\perp p$.

(a)So, $L_{o1} = -rmv_o = -(0.75)(0.145)(41) \Rightarrow \boxed{L_{o1} = -4.5 \frac{kg \cdot m^2}{s}}$.

(b)Similarly, $L_1 = rmv = (0.75)(0.145)(49) \Rightarrow \boxed{L_1 = 5.3 \frac{kg \cdot m^2}{s}}$.

(c)The rotational speed is related to the linear speed by, $v = r\omega$. So,

$$\omega_o = \frac{V_o}{r_{cm}} = \frac{22}{0.6} \Rightarrow \boxed{\omega_o = 37 \tfrac{rad}{s}}.$$

(d)Similarly, $\omega = \frac{V}{r_{cm}} = \frac{9}{0.6} \Rightarrow \boxed{\omega = 15 \tfrac{rad}{s}}$.

(e)The total angular momentum before the collision is $L_o = L_{o1} + I\omega_o$.

The total angular momentum after the collision is $L = L_1 + I\omega$.

Using the Law of Conservation of Angular Momentum,

$$L_o = L \Rightarrow L_{o1} + I\omega_o = L_1 + I\omega$$

Solving for the rotational inertia of the bat,

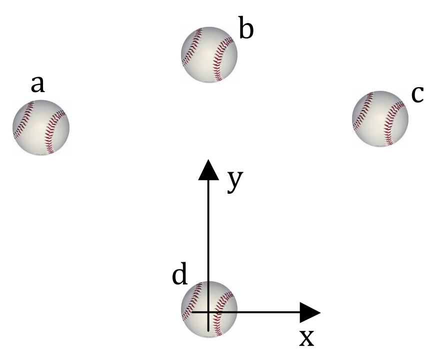
$$L_{o1} - L_1 = I\omega - I\omega_o = I(\omega - \omega_o) \Rightarrow I = \frac{L_{o1} - L_1}{\omega - \omega_o}$$

Plugging in the numbers,

$$I = \frac{-4.5 - 5.3}{15 - 37} \Rightarrow \boxed{I = 0.45 kg \cdot m^2}.$$

Problem 9.3:

For each of the situations at the right gravity acts along the negative y-axis. Find the direction of the torque on the baseball due to gravity about the origin in each case.



Problem 9.4:

A lazy fly ball is hit toward an outfielder 100m due north from home plate. Using a coordinate system with an origin at home, the x-axis toward the fielder, and the y-axis vertical, the position of the ball is given by $\vec{r} = 70.0\hat{i} + 8.00\hat{j}$ in meters. Find the torque exerted by gravity on the ball about (a)home plate and (b)the outfielder. The mass of a baseball is 150g.

Sixty Basebal

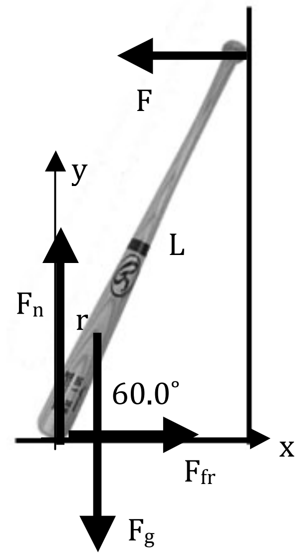
Problem 9.5:

A baseball catcher sits on a rotating stool. He reaches out 85.0cm to catch a 40.0m/s fastball. After catching the ball he spins at a rate of 60.0rpm. His mass is 80.0kg and the mass of the ball is 150g. Find the rotational inertia of the catcher and the stool.

10. Statics (3)

Problem 10.1:

A 1.00kg baseball bat 85.0cm long has a center of mass that is 20.0cm above the fat end. It leans against a wall smooth wall making a 60.0° angle with the ground. Find the magnitude of each force that acts on the bat and show its direction in the sketch below.



Applying the Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_w = 0 \Rightarrow F_{fr} = F_w$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g$$

Applying the Second Law for Rotation about the origin,

$$\Sigma\tau_o = I\alpha \Rightarrow LF_w \sin 60.0^\circ - rF_g \cos 60.0^\circ = 0 \Rightarrow F_w = \tfrac{r\cos 60.0^\circ}{L\sin 60.0^\circ} F_g$$

Using the mass/weight rule, $F_g = mg = (1.00)(9.80) \Rightarrow$ $\boxed{F_g = 9.80N}$.

Using the torque equation, $F_w = \frac{(20.0)\cos 60.0^\circ}{(85.0)\sin 60.0^\circ}(9.80) \Rightarrow \boxed{F_w = 1.33N}$.

Using the x-equation, $F_{fr} = F_w \Rightarrow \boxed{F_{fr} = 1.33N}$.

Using the y-equation, $F_n = F_g \Rightarrow \boxed{F_n = 9.80N}$.

Problem 10.2:

A baseball bat leans against a smooth wall making a 60° angle with the ground. The center of mass is two-thirds of the way down the bat. Find the minimum coefficient of static friction needed to keep the bat in place.

Applying Newton's Second Law,

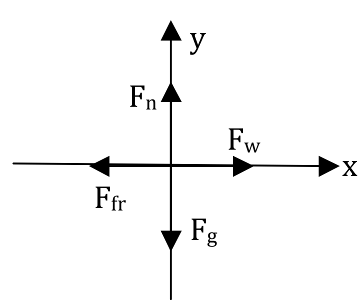
$$\Sigma F_x = ma_x \Rightarrow F_w - F_{fr} = 0 \Rightarrow F_{fr} = F_w$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g$$
$$\Sigma\tau_p = I\alpha \Rightarrow F_g \tfrac{\ell}{3}\cos 60^\circ - F_w \ell \sin 60^\circ = 0 \Rightarrow F_w \ell \sin 60^\circ = F_g \tfrac{\ell}{3}\cos 60^\circ$$

The definition of the COSF is, $\mu = \frac{F_{fr}}{F_n}$.

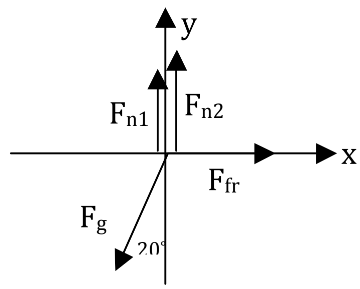
Using the force equations, $\mu = \frac{F_{fr}}{F_n} = \frac{F_w}{F_g}$

Using the torque equation,

$$\mu = \frac{F_w}{F_g} = \frac{\frac{\ell}{3}\cos 60^\circ}{\ell \sin 60^\circ} = \frac{1}{3\tan 60^\circ} \Rightarrow \boxed{\mu = 0.192}.$$



Problem 10.3:
The slope of a pitcher's mound makes a 20.0° angle with the horizontal. A 15.0N – 90.0cm baseball bat rests on the mound in such a way that only the ends are actually in contact with the mound. The center of mass of the bat is 60.0cm from the skinny end. Find the magnitudes of each of the normal forces and the total frictional force that the ground exerts on the bat.



Using the free body diagram to apply the 2nd Law along the axes,
$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_g \sin 20° = 0$ and
$\Sigma F_y = ma_y \Rightarrow F_{n1} + F_{n2} - F_g \cos 20° = 0$.
Finding the torques about the origin and applying the 2nd Law for Rotation,
$\Sigma \tau_o = I\alpha \Rightarrow \ell F_{n2} - \frac{2}{3}\ell F_g \cos 20° = 0$.
Solving the x-equation for the frictional force,
$F_{fr} = F_g \sin 20° = (15.0)\sin 20° \Rightarrow \boxed{F_{fr} = 5.13N}$.
Solving the torque equation for the second normal force,
$F_{n2} = \frac{2}{3}F_g \cos 20° = \frac{2}{3}(15.0)\cos 20° \Rightarrow \boxed{F_{n2} = 9.40N}$.
Solving the y-equation for the first normal force,
$F_{n1} = F_g \cos 20° - F_{n2} = (15.0)\cos 20° - 9.40 \Rightarrow \boxed{F_{n1} = 4.70N}$.

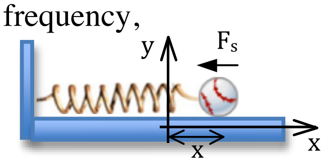
11. Oscillatory Motion (7)

<u>Problem 11.1:</u>
A 0.15kg baseball is on the end of a spring with spring constant 8.0N/m. The ball is pulled horizontally 20cm from equilibrium and released. When the ball is 10cm from equilibrium, find its speed.

Given: m = 0.15kg, k = 8.0N/m, A = 0.20m, and x = 0.10m.
Find: v = ?

Initially, there is no kinetic energy an all the energy is in the potential energy of the spring.
$K_o = 0$ and $U_o = \frac{1}{2}kA^2$.
Afterward, there is both kinetic and potential energy,
$K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$.
Using the Law of Conservation of Energy,
$E = E_o \Rightarrow K_o + U_o = K + U \Rightarrow 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$.
Solving for the final speed,
$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\frac{8}{0.15}\left[(0.2)^2 - (0.1)^2\right]} \Rightarrow \boxed{v = 1.6\frac{m}{s}}$.

<u>Problem 11.2:</u>
A 0.15kg baseball is on the end of a spring with spring constant 8.0N/m. The ball is pulled horizontally 20cm from equilibrium and released. Find (a)the angular frequency, (b)the period, and (c)the amplitude.

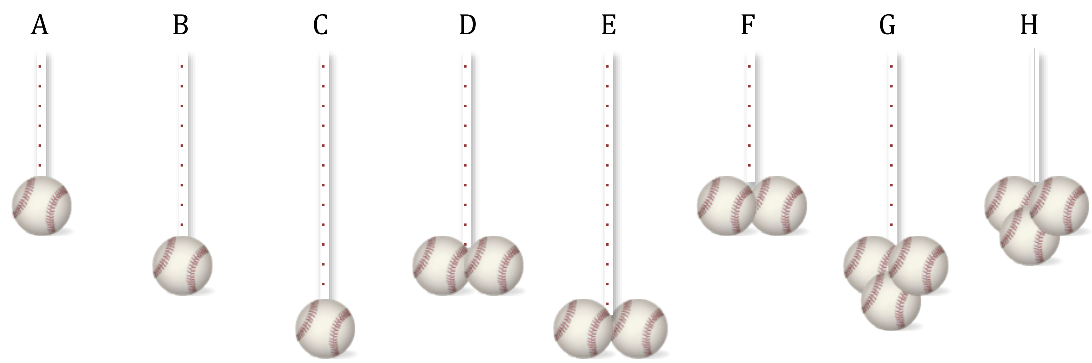


(a)The force acting on the ball is due to the spring. Applying the 2nd Law,
$\Sigma F = ma \Rightarrow -F_s = ma \Rightarrow -kx = ma \Rightarrow a = -\frac{k}{m}x$
The requirement for SHM is,
$a = -\omega^2 x \Rightarrow -\frac{k}{m}x = -\omega^2 x \Rightarrow \omega^2 = \frac{k}{m}$
The angular frequency is,
$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{0.15}} \Rightarrow \boxed{\omega = 7.3\frac{rad}{s}}$.
(b)The angular frequency is related to the frequency, which is the reciprocal of the period.
$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{7.3} \Rightarrow \boxed{T = 0.86s}$.
(c) The amplitude is the maximum displacement which will be $\boxed{A = 10.0cm}$.

Problem 11.3:

The Best Damn Pendulums Period!

The eight pendulums below are set oscillating. Rank them from largest to smallest based upon the period of oscillation. If some are equal to others put an equal sign between them.



Problem 11.4:

A 0.900±0.001kg baseball bat is 85.0±0.2cm long and its center of mass is 55.0 ±0.2cm from the knob end. When held at the knob end and allowed to oscillate, it is found to have a period of 1.60±0.05s. Find the rotational inertia of the bat about the knob end and find the uncertainty in this value.

The bat acts like a physical pendulum. The oscillation frequency is, $\omega = \sqrt{\frac{mgr}{I}}$.

The period is related to the frequency by, $\omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{mgr}{I}} \Rightarrow I = \frac{mgrT^2}{4\pi^2}$.

Plugging in the numbers, $I = \frac{(0.900)(9.80)(0.550)(1.60)^2}{4\pi^2} \Rightarrow \boxed{I = 0.315 kg \cdot m^2}$.

The uncertainty can be found using the multiplication rule, $\frac{\Delta I}{I} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + \left(2\frac{\Delta T}{T}\right)^2}$.

Plugging in the numbers, $\frac{\Delta I}{I} = \sqrt{\left(\frac{0.001}{0.900}\right)^2 + \left(\frac{0.2}{55.0}\right)^2 + \left(2\frac{0.05}{1.60}\right)^2} = 0.072$.

Therefore, $\Delta I = 0.072 I = (0.072)(0.315) = 0.02 kg \cdot m^2$.

Finally, $\boxed{I = 0.32 \pm 0.02 kg \cdot m^2}$.

Problem 11.5:

An 86.4cm long baseball bat has a mass of 0.820kg. It's center-of-mass is located 58.6cm from the handle end about which it oscillates with a period of 1.64s. (a)Find the rotational inertia of the bat about the handle. (b)Compare the result with the value for a uniform stick.

Given: l = 0.864m, m = 0.826kg, r = 0.586m, and T = 1.64s.

Find:

(a)The period is related to the frequency of the physical pendulum,

$$\omega = \sqrt{\frac{mgr}{I_p}} \Rightarrow T = 2\pi\sqrt{\frac{I_p}{mgr}}.$$

Solving for I_p,

$$I_p = \frac{mgrT^2}{4\pi^2} = \frac{(0.820)(9.80)(0.586)(1.64)^2}{4\pi^2} \Rightarrow \boxed{I_p = 0.321 kg\cdot m^2}.$$

(b)For a uniform stick pivoted about its end,

$$I = \tfrac{1}{3}m\ell^2 = \tfrac{1}{3}(0.820)(0.864)^2 \Rightarrow \boxed{I = 0.204 kg\cdot m^2}.$$

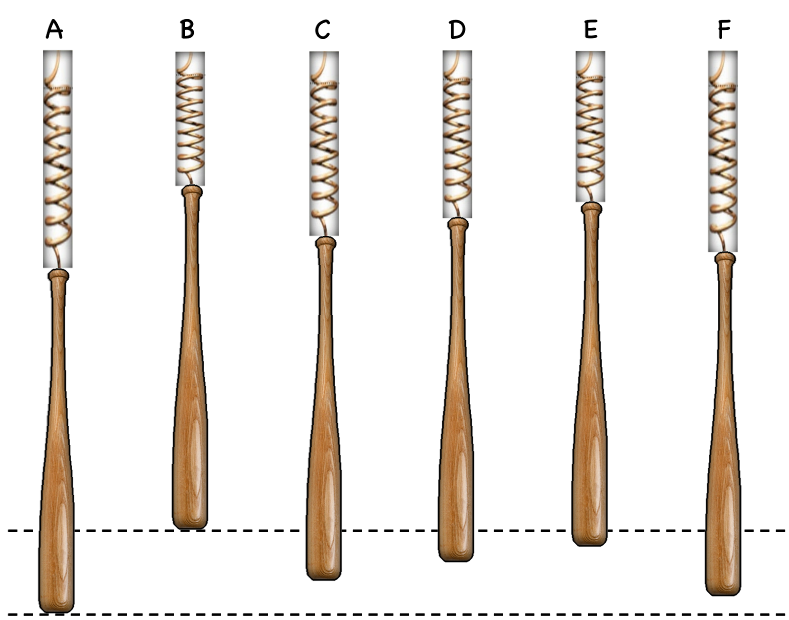
The uniform stick has a smaller rotational inertia because a baseball bat has a greater fraction of its mass at the far end, making it harder to accelerate. This is a disadvantage when your trying to speed up the bat, but a big advantage when the collision with the ball is trying to slow it down.

<u>Problem 11.6:</u>

Hanging Bats

Six different bats are hung from identical springs that stretch different amounts in equilibrium.

Rank these bats from greatest to least based upon their mass.

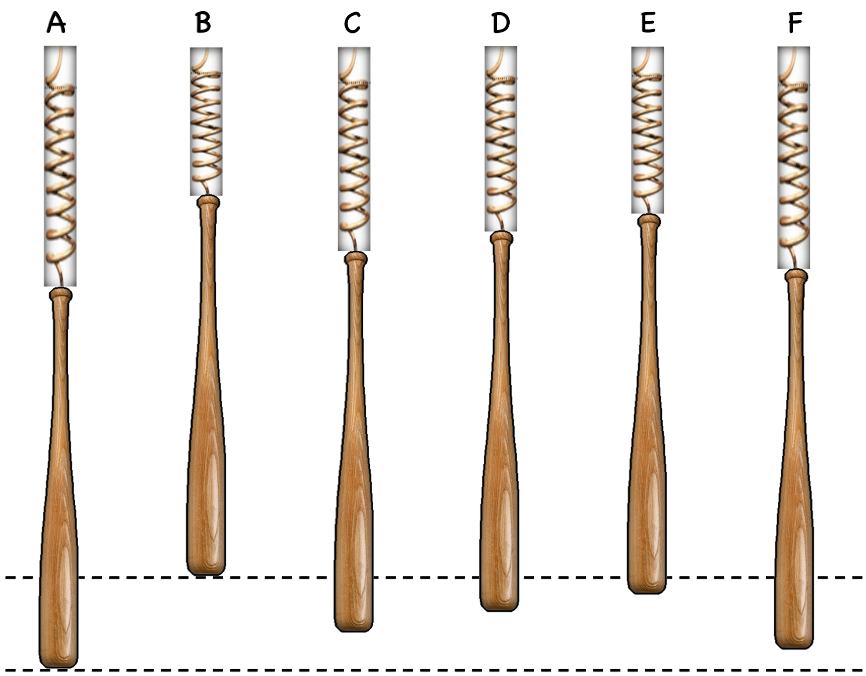


Problem 11.7:

Bouncing Bats

Six different bats are hung from identical springs that stretch different amounts in equilibrium. When the bats are pulled down and released. They oscillate up and down.

Rank these bats from greatest to least based upon the frequency of oscillation.



12. Wave Motion (1)

Problem 12.1:

The speed of sound in the wood of a baseball bats is about 4000m/s. Treat the bat 95cm long bat like a string free at both ends. (a)Sketch the first three harmonics. (b)Find their frequencies.

Given:
v = 4000m/s
L = 0.95m

Find: f1 = ?, f2 = ?, and f3 = ?

From the sketch, the wavelengths are,
$\lambda_1 = 2L = \frac{2L}{1}$, $\lambda_2 = L = \frac{2L}{2}$, and $\lambda_3 = \frac{2L}{3}$.

So, $\lambda_n = \frac{2L}{n}$

Since the speed of waves is given by,

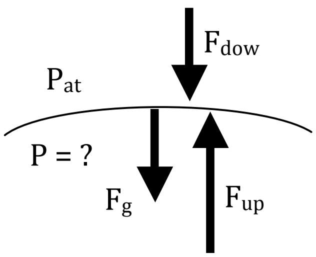
$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} \Rightarrow f_n = \frac{v}{\lambda_n} = \frac{v}{\left(\frac{2L}{n}\right)} \Rightarrow f_n = n\frac{v}{2L} = n\frac{4000}{0.95} = n(4210Hz)$$

Finally,
f1 = 4200Hz, f2 = 8400Hz, and f3 = 16,800Hz

13. Fluids (2)

Problem 13.1:

The Metrodome was a domed stadium in Minneapolis where the Minnesota Twins used to play. The air inside the stadium, which has a density of 1.29kg/m, almost entirely supports its roof. The mass of the roof is $2.640 x 10^5$kg and is it $7.44 x 10^4 m^2$ in area. Find (a)the pressure difference across the roof and (b)the speed that air would travel through an open door to the stadium. (c)Would the air move in or out?



(a)the pressure difference across the roof exerts the upward force to cancel the weight. Applying the Second Law,

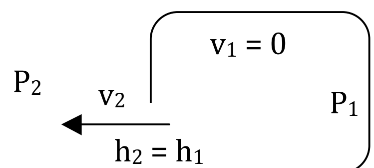
$$\Sigma F = ma \Rightarrow F_{up} - F_g - F_{down} = 0 \Rightarrow F_{up} - F_{down} = F_g \Rightarrow F_{up} - F_{down} = mg$$

Using the definition of pressure, $P \equiv \frac{F}{A} \Rightarrow F = PA$.

Substituting into the force equation,

$$PA - P_{atm}A = mg \Rightarrow P - P_{atm} = \frac{mg}{A} \Rightarrow \Delta P = \frac{mg}{A}.$$

Plugging in the numbers, $\Delta P = \frac{(2.640x10^5)(9.80)}{7.44x10^4} \Rightarrow \boxed{\Delta P = 34.8 N/m^2}$.



(b)Use Bernoulli's Principle,

$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$.

The height terms cancel and the v_1 term is zero,

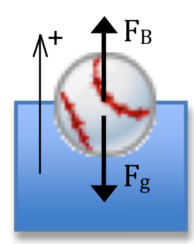
$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow \frac{1}{2}\rho v_2^2 = P_1 - P = \Delta P \Rightarrow v_2 = \sqrt{\frac{2\Delta P}{\rho}}.$$

Putting in the values, $v_2 = \sqrt{\frac{2(34.8)}{1.29}} \Rightarrow \boxed{v_2 = 7.34 m/s}$.

(c)Since the pressure inside is larger than outside $\boxed{\text{the air moves out}}$.

Problem 13.2:

A homerun lands in McCovey Cove. The mass of the ball is 0.145kg and its radius is 3.68cm. (a)Will the ball float or sink? (b)If it floats find the fraction of the ball that will be under water.



Given: m = 0.145kg and r = 0.0368m.

Find: rb = ? and $\frac{V_u}{V} = ?$

(a)Using the definition of density,

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{0.145}{\frac{4}{3}\pi(0.0368)^3} \Rightarrow \rho = 695 \frac{kg}{m^3}.$$

Since this is less than the density of water, the ball will float.

(b)Applying the Second Law,

$$\Sigma\vec{F} = m\vec{a} \Rightarrow F_B - F_g = 0 \Rightarrow F_B = F_g.$$

Using the mass/weight rule and Archimedes' Principle,

$$\rho_w g V_u = mg.$$

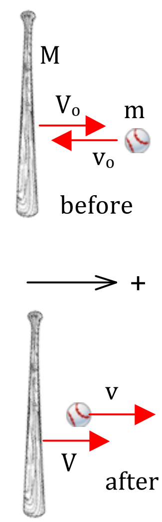
Using the definition of density for the ball,

$$\rho_w g V_u = \rho V g \Rightarrow \rho_w V_u = \rho V \Rightarrow \frac{V_u}{V} = \frac{\rho}{\rho_w} = \frac{695}{1000} \Rightarrow \boxed{\frac{V_u}{V} = 69.5\%}.$$

14. Thermal Physics (2)

Problem 14.1:

An average fastball heads toward the batter at about 92mph (41m/s). It is hit for a homerun leaving the bat at about 110mph (49m/s). The mass of the is 145g and the bat had a mass of 36oz (1.0kg). The (center-of-mass) speed of the bat when it strikes the ball is about 50mph (22m/s) and the speed of the bat just after hitting the ball was 9.0m/s. (a)Find the heat generated and (b)the amount of water that could be heated from 20˚C to 100˚C with that heat.



Given: v = 49m/s, vo = 41m/s, Vo = 22m/s, V = 9.0m/s,
m = 0.145kg, M = 1.0kg, To = 20˚C, and T = 100˚C.
Find: Q = ? and mw = ?

The initial kinetic energy is, $K_o = \frac{1}{2}MV_o^2 + \frac{1}{2}mv_o^2$,
while the final kinetic energy is, $K = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$.
By the Law of Conservation of Energy, the difference must be the heat produced,

$$Q = K_o - K = \frac{1}{2}MV_o^2 + \frac{1}{2}mv_o^2 - \frac{1}{2}MV_o^2 - \frac{1}{2}mv_o^2.$$

Plugging in the values,

$$Q = \frac{1}{2}(1)(22)^2 + \frac{1}{2}(0.145)(41)^2 - \frac{1}{2}(1)(9)^2 - \frac{1}{2}(0.145)(49)^2 \Rightarrow \boxed{Q = 150J}.$$

(b)The heat changes the temperature according to,

$$Q = m_w c\Delta T \Rightarrow m_w = \frac{Q}{c\Delta T} = \frac{150}{4180(100-20)} \Rightarrow \boxed{m_w = 0.00045kg = 0.45g}.$$

Problem 14.2:

It's A Long Fly Ball…

The distance a well hit ball can travel depends inversely upon the air density. The ball will go farther when the air is less dense. Below are three cities with their average summer temperature and atmospheric pressure.

Rank them based upon the distance traveled by a well hit ball.

City	Team	Average Summer Temperature (°C)	Average Atmospheric Pressure (kPa)
		28	101.4
		18	101.7
		28	101.6