## Hit the Cut Off

## What is this about?

When outfielders chase down a ball hit in the gap, they rarely throw all the way to home plate. Instead, they are taught to throw the ball to an infielder positioned about halfway between them and home. This infielder, called the "cut-off," then throws on to home. You will learn why "hitting the cut-off" is the best play.

## What do I need?

This is a paper and pencil exercise that requires the ability to use the kinematic equations and a calculator.

What will I be doing?
First, you will calculate the time for a throw to get from the wall to home plate on the fly. Then you will calculate the time for the throw to get to the cut-off and for the cut-off to throw home.

## What do I think will happen?

Take a minute and write down a description of what you think will happen and why you think it. Which time will be shorter, the one throw from the wall or the two throws involving the cut-off?

## What really happened?

Let's assume an outfielder can throw a ball at $90 \mathrm{mph}(40 \mathrm{~m} / \mathrm{s})$ and the wall is $390 \mathrm{ft}(100 \mathrm{~m})$ away from home plate. First, we need to find the angle for the throw to be launched starting with the kinematic equations,

$$
x=x_{o}+v_{o x} t+\frac{1}{2} a_{x} t^{2} \quad \text { and } \quad y=y_{o}+v_{o y} t+\frac{1}{2} a_{y} t^{2} .
$$

Since the height the ball is released is about the height it will be caught, we'll set $y_{o}$ and y to zero. We can choose $x_{0}=0$. There is no acceleration along $x$ and along $y$ it is constant and downward so, $\mathrm{a}_{\mathrm{x}}=0$ and $\mathrm{a}_{\mathrm{y}}=-\mathrm{g}$. The initial speed along each direction must be written as a component of the velocity so, $\mathrm{v}_{\mathrm{ox}}=\mathrm{v}_{\mathrm{o}} \cos \theta$ and $\mathrm{v}_{\mathrm{oy}}=\mathrm{v}_{\mathrm{o}} \sin \theta$. Now we have,

$$
x=v_{o} t \cos \theta \quad \text { and } \quad v_{o} \sin \theta=\frac{1}{2} g t .
$$

Solving both equations for t ,

$$
t=\frac{x}{v_{o} \cos \theta} \quad \text { and } \quad t=\frac{2 v_{o}}{g} \sin \theta
$$

and setting them equal gives the so called "range equation,"

$$
x=\frac{v_{o}^{2}}{g} \sin 2 \theta .
$$

Finally, we can get an expression for the launch and you can calculate it,

$$
\theta=\frac{1}{2} \arcsin \frac{x g}{v_{o}^{2}}=
$$

$\qquad$ .
We can get the time from the x equation and you can calculate it.

$$
t=\frac{x}{v_{o} \cos \theta}=
$$

Now, let's suppose that the cut-off is halfway between the wall and home plate so that $\mathrm{x}=50 \mathrm{~m}$. Calculate the launch angle needed to get the ball from the outfielder to the cut-off,

$$
\theta_{o c}=\frac{1}{2} \arcsin \frac{x g}{v_{o}^{2}}=
$$

and the time for the ball to get there,

$$
t_{o c}=\frac{x}{v_{o} \cos \theta}=
$$

$\qquad$ .
You need to double this time for the ball to get from the cut-off to home plate.

$$
t_{c}=2 t_{o c}=
$$

$\qquad$
Compare this time to the time to throw home with a single throw from the outfield.

## What did I learn?

Did you get the time of flight for the two throws is slightly less than for a single throw? This is because more of the velocity given to the ball in the shorter throws is used to move it horizontally, so these throws cover the distance along the ground faster.

Of course, we haven't accounted for the short time required for the cut-off to catch the throw from the outfield and fire it home, which will make the two throws take slightly longer than the single throw. Hitting the cut-off has added advantages such as, stopping a throw that would be off line or too late, or redirecting the play to another base where an out is more likely. These advantages outweigh the slight disadvantage of the extra time.

What else should I think about?
You know that air drag and the Magnus force have a sizable effect on the flight of the ball, which we have ignored here. It turns out that the distance the ball will travel is more strongly affected than the time of flight when the air is taken into account. So the errors we have made are not as large as you might at first think.

