Using PITCHf/x to Teach Physics

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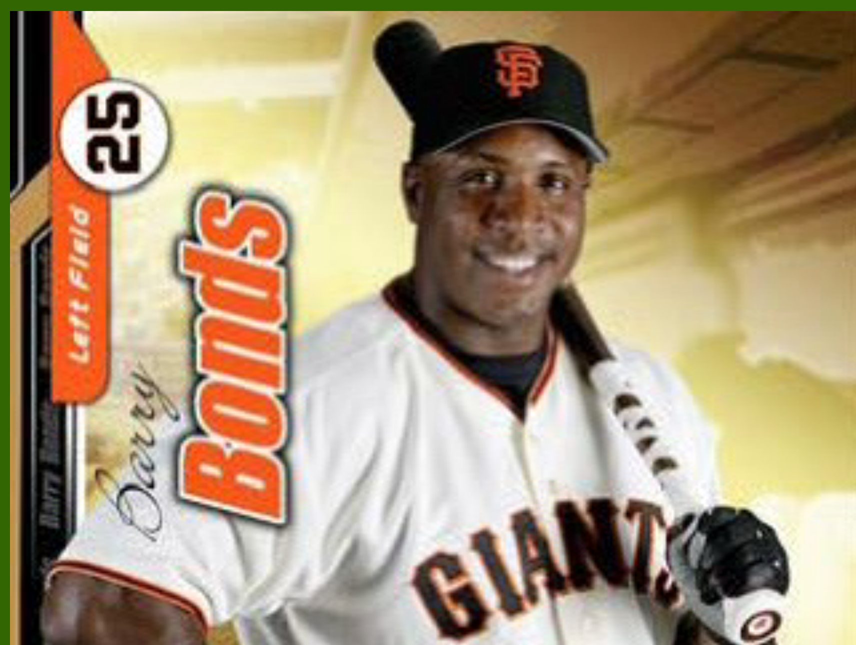
Typical Physics Problem

A football is kicked with an initial velocity of 25 m/s at an angle of 45-degrees with the horizontal. Determine the time of flight, the horizontal displacement, and the peak height of the football.

A Better Physics Problem

A Better Physics Problem

Not Just a Slugger…



A Better Physics Problem

Get the pitch data from mlb

Kinematic data

No.	Quantity	Value	Units	Description
1	des	In play, run(s)		A comment on the action resulting from the pitch.
2	type	X		B=ball, S=strike, X=in play
3	id	371		Code indicating pitch number
4	x=	112.45	pixels	x-pixel at home plate
5	y=	131.24	pixels	z-pixel at home plate (yes, it is z)
6	start_speed	84.1	mph	Speed at y0=50ft
7	end_speed	77.2	mph	Speed at the front of home plate y=1.417ft
8	sz_top	3.836	ft	The z-value of the top of the strike zone as estimated by a technician
9	sz_bot	1.79	ft	The z-value of the bottom of the strike zone as estimated by a technician
10	pfx_x	8.68	in	A measure of the "break" of the pitch in the x-direction.
11	pfx_z	9.55	in	A measure of the "break" of the pitch in the z-direction.
12	px	-0.012	ft	Measured x-value of position at the front of home plate (y=1.417ft)
13	pz	2.743	ft	Measured z-value of position at the front of home plate (y=1.417ft)
14	x0	1.664	ft	x-position at y=50ft
15	y0	50.0	ft	Arbitrary fixed initial y-value
16	z0	6.597	ft	z-position at y=50ft
17	vx0	-6.791	ft/s	x-velocity at y=50ft
18	vy0	-123.055	ft/s	y-velocity at y=50ft
19	vz0	-5.721	ft/s	z-velocity at y=50ft
20	ax	13.233	ft/s/s	x-acceleration at y=50ft assumed constant.
21	ay	25.802	ft/s/s	y-acceleration at y=50ft assumed constant.
22	az	-17.540	ft/s/s	z-acceleration at y=50ft assumed constant.
23	break_y	25.2	ft	Another measure of the "break."
24	break_angle	-32.1	deg	Another measure of the "break."
25	break_length	5.9	in	Another measure of the "break."

A Better Physics Problem

Problem 1: Find the initial speed of the ball (at y=50.0ft) in mph.

In 3-dimensions the initial speed is the magnitude of the initial velocity vector. Since the components are listed below we take the square root of the sum of their squares,

$$v_o = \sqrt{v_{ox}^2 + v_{oy}^2 + v_{oz}^2}$$

$$v_o = \sqrt{(-6.791)^2 + (-123.055)^2 + (-5.721)^2}$$

$$v_o = 123.375\, ft/s = 84.1 mph$$

5	y=	131.24	pixels	z-pixel at home plate (yes, it is z)
6	start_speed	84.1	mph	Speed at y0=50ft
7	end_speed	77.2	mph	Speed at the front of home plate y=1.417ft
8	sz_top	3.836	ft	The z-value of the top of the strike zone as

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

A Better Physics Problem

Problem 2: Find the components of the final velocity of the pitch when it reaches the front of home plate (y=1.417ft).

Since we know the initial and final y-values we can get the y-component of the velocity using the kinematic equation,

$$v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$$

$$v_y = -\sqrt{v_{oy}^2 + 2a_y(y - y_o)}$$

$$v_y = -\sqrt{(-123.055)^2 + 2(25.802)(1.417 - 50.00)}$$

$$v_y = -112.408\ ft/s$$

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

v_x = ?

v_y = -112.408ft/s

v_z = ?

A Better Physics Problem

Problem 2: Find the components of the final velocity of the pitch when it reaches the front of home plate (y=1.417ft).

The time of flight must be found to get the other velocity components. Using another kinematic equation,

$$v_y = v_{oy} + a_y t$$

$$t = \frac{v_y - v_{oy}}{a_y}$$

$$t = \frac{-112.408 - (-123.055)}{25.802}$$

$$t = 0.4127s$$

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

v_x = ?

v_y = -112.408ft/s

v_z = ?

A Better Physics Problem

Problem 2: Find the components of the final velocity of the pitch when it reaches the front of home plate (y=1.417ft).

Having the time of flight and using kinematic equations for the other two axes,

$$v_x = v_{ox} + a_x t = -6.791 + (13.233)(0.4127) = -1.330\, ft/s$$

$$v_z = v_{oz} + a_z t = -5.721 + (-17.540)(0.4127) = -12.960\, ft/s$$

$$t = 0.4127 s$$

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

v_x = ?1.330ft/s

v_y = -112.408ft/s

v_z = ?12.960ft/s

A Better Physics Problem

Problem 2: Find the components of the final velocity of the pitch when it reaches the front of home plate (y=1.417ft).

The final speed is the magnitude of the final velocity vector. Taking the square root of the sum of the squares,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$v = \sqrt{(-1.330)^2 + (-112.408)^2 + (-12.960)^2}$$

$$v = 113.160\,ft/s = 77.2mph$$

5	y=	131.24	pixels	z-pixel at home plate (yes, it is z)
6	start_speed	84.1	mph	Speed at y0=50ft
7	end_speed	77.2	mph	Speed at the front of home plate y=1.417ft
8	sz_top	3.836	ft	The z-value of the top of the strike zone as

t = 0.4127s

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

v_x = -1.330ft/s

v_y = -112.408ft/s

v_z = -12.960ft/s

A Better Physics Problem

Problem 3: Since a typical batter doesn't get a sense of the motion of the pitch until the ball is about 40ft away from home plate, find the time to get there and the x and z components of the position and velocity when it arrives.

The time can be found using the kinematic equation,

$$y = y_o + v_{oy}t_{40} + \tfrac{1}{2}a_y t_{40}^2$$

$$t_{40} = \frac{-v_{oy} \pm \sqrt{v_{oy}^2 - 2a_y(y_o - y)}}{a_y}$$

$$t_{40} = \frac{-(-123.055) - \sqrt{(-123.055)^2 - 2(25.802)(50 - 40)}}{(25.802)} = 0.08197s$$

$$t_{40} = 0.08197s$$

t = 0.4127s

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

t_{40} = 0.08197s

x_{40} = ?

v_{x40} = ?

z_{40} = ?

v_{z40} = ?

A Better Physics Problem

Problem 3:Since a typical batter doesn't get a sense of the motion of the pitch until the ball is about 40ft away from home plate, find the time to get there and the x and z components of the position and velocity when it arrives.

The x-position and velocity can now be found,

$$x_{40} = x_o + v_{ox}t_{40} + \tfrac{1}{2}a_x t_{40}^2 = 1.664 + (-6.791)(0.08197) + \tfrac{1}{2}(13.233)(0.08197)^2 = 1.152\,ft$$

$$v_{x40} = v_{ox} + a_x t_{40} = -6.791 + (13.233)(0.08197) = -5.706\,ft/s$$

as can the z-position and velocity,

$$z_{40} = z_o + v_{oz}t_{40} + \tfrac{1}{2}a_z t_{40}^2 = 6.597 + (-5.721)(0.08197) + \tfrac{1}{2}(-17.540)(0.08197)^2 = 6.069\,ft$$

$$v_{z40} = v_{oz} + a_z t_{40} = -5.721 + (-17.540)(0.08197) = -7.159\,ft/s$$

$t = 0.4127s$

$x_o = 1.664ft$	$v_{xo} = -6.791ft/s$	$a_x = 13.233ft/s^2$
$y_o = 50.00ft$	$v_{yo} = -123.055ft/s$	$a_y = 25.802ft/s^2$
$z_o = 6.597ft$	$v_{zo} = -5.721ft/s$	$a_z = -17.540ft/s^2$

$t_{40} = 0.08197s$

$x_{40} = 1.152ft$

$v_{x40} = -5.706ft/s$

$z_{40} = 6.069ft$

$v_{z40} = -7.159ft/s$

A Better Physics Problem

Problem 4: Now that the batter has a sense of the position and velocity of the ball, he can begin to plan his swing. If the ball only felt gravity in the z-direction and no force in the x-direction from this point on, where would it cross home plate.

The time of flight from y=40ft can be found from by subtracting the total time from the time to get to y=40ft,

$$t_h = t - t_{40} = 0.4127 - 0.08197 = 0.3307s$$

$t = 0.4127s$ $t_h = 0.3307s$

$x_o = 1.664ft$	$v_{xo} = -6.791ft/s$	$a_x = 13.233ft/s^2$
$y_o = 50.00ft$	$v_{yo} = -123.055ft/s$	$a_y = 25.802ft/s^2$
$z_o = 6.597ft$	$v_{zo} = -5.721ft/s$	$a_z = -17.540ft/s^2$

$t_{40} = 0.08197s$

$x_{40} = 1.152ft$

$v_{x40} = -5.706ft/s$

$z_{40} = 6.069ft$

$v_{z40} = -7.159ft/s$

A Better Physics Problem

Problem 4: Now that the batter has a sense of the position and velocity of the ball, he can begin to plan his swing. If the ball only felt gravity in the z-direction and no force in the x-direction from this point on, where would it cross home plate.

Along the x-direction there would be no acceleration,

$$x_{noair} = x_{40} + v_{x40}t_h + \tfrac{1}{2}a_x t_h^2 \Rightarrow x_{noair} = 1.152 + (-5.706)(0.3307) = -0.735\,ft$$

Along the z-axis there would only be gravitational acceleration,

$$z_{noair} = z_{40} + v_{z40}t_h + \tfrac{1}{2}a_z t_h^2$$

$$z_{noair} = 6.069 + (-7.159)(0.3307) + \tfrac{1}{2}(-32.174)(0.3307)^2 = 1.942\,ft$$

$t = 0.4127s$ $t_h = 0.3307s$ $x_{noair} = -0.735ft$ $z_{noair} = 1.942ft$

$x_o = 1.664ft$	$v_{xo} = -6.791ft/s$	$a_x = 13.233ft/s^2$
$y_o = 50.00ft$	$v_{yo} = -123.055ft/s$	$a_y = 25.802ft/s^2$
$z_o = 6.597ft$	$v_{zo} = -5.721ft/s$	$a_z = -17.540ft/s^2$

$t_{40} = 0.08197s$

$x_{40} = 1.152ft$

$v_{x40} = -5.706ft/s$

$z_{40} = 6.069ft$

$v_{z40} = -7.159ft/s$

A Better Physics Problem

Problem 5: Batters describe the effect of spin on the ball as the "break." One way to analytically define the break is the difference between where the ball actually arrives and where is would have arrived only feeling gravity. Find the break along the x and z directions.

The actual x and z positions are in the data table.

12	px	-0.012	ft	Measured x-value of position at the front of home plate (y=1.417ft)
13	pz	2.743	ft	Measured z-value of position at the front of home plate (y=1.417ft)

px = -0.012ft pz = 2.743ft

$t = 0.4127s$ $t_h = 0.3307s$ $x_{noair} = -0.735ft$ $z_{noair} = 1.942ft$

$x_o = 1.664ft$	$v_{xo} = -6.791ft/s$	$a_x = 13.233ft/s^2$
$y_o = 50.00ft$	$v_{yo} = -123.055ft/s$	$a_y = 25.802ft/s^2$
$z_o = 6.597ft$	$v_{zo} = -5.721ft/s$	$a_z = -17.540ft/s^2$

$t_{40} = 0.08197s$

$x_{40} = 1.152ft$

$v_{x40} = -5.706ft/s$

$z_{40} = 6.069ft$

$v_{z40} = -7.159ft/s$

A Better Physics Problem

Problem 5: Batters describe the effect of spin on the ball as the "break." One way to analytically define the break is the difference between where the ball actually arrives and where is would have arrived only feeling gravity. Find the break along the x and z directions.

This definition of break can now be calculated for the x and z directions.

$$x_{break} = px - x_{noair} = -0.012 - (-0.735) = 0.723\,ft = 8.68\,in$$

$$z_{break} = pz - z_{noair} = 2.743 - 1.942 = 0.801\,ft = 9.61\,in$$

10	pfx_x	8.68	in	A measure of the "break" of the pitch in the x-direction.
11	pfx_z	9.55	in	A measure of the "break" of the pitch in the y-direction.

px = -0.012ft pz = 2.743ft

t = 0.4127s t_h = 0.3307s x_{noair} = -0.735ft z_{noair} = 1.942ft

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

t_{40} = 0.08197s

x_{40} = 1.152ft

v_{x40} = -5.706ft/s

z_{40} = 6.069ft

v_{z40} = -7.159ft/s

A Problem on Forces

Problem 6: Given the weight of a baseball is 0.320lbs, find the x, y, and z components of the force exerted on the ball by the air during its flight.

Use Newton's Second Law along each direction. Along x and y the only force is due to the air,

$$F_x = ma_x = mg\left(\frac{a_x}{g}\right) = (0.320)\left(\frac{13.233}{32.174}\right) = 0.132lbs$$

$$F_y = ma_y = mg\left(\frac{a_y}{g}\right) = (0.320)\left(\frac{25.802}{32.174}\right) = 0.257lbs$$

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

F_x = 0.132lbs

F_y = 0.257lbs

F_z = ?

A Problem on Forces

Problem 6: Given the weight of a baseball is 0.320lbs, find the x, y, and z components of the force exerted on the ball by the air during its flight.

Along z gravity is also in play,

$$F_z - mg = ma_z \Rightarrow F_z = mg + mg\left(\frac{a_z}{g}\right) = mg\left(1 + \frac{a_z}{g}\right) = (0.320)\left(1 + \frac{-22.232}{32.174}\right) = 0.146lbs$$

The magnitude of the force caused by the air is,

$$F_{air} = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(0.132)^2 + (0.257)^2 + (0.146)^2} = 0.324\,lbs$$

The force exerted by the air is about equal to the weight!

x_o = 1.664ft	v_{xo} = -6.791ft/s	a_x = 13.233ft/s^2
y_o = 50.00ft	v_{yo} = -123.055ft/s	a_y = 25.802ft/s^2
z_o = 6.597ft	v_{zo} = -5.721ft/s	a_z = -17.540ft/s^2

F_x = 0.132lbs

F_y = 0.257lbs

F_z = 0.146lbs

Thanks to

For turning this…

Into this…