

5.12 Moment of Inertia

The rotational inertia $I$ of an object may be determined by applying a torque $\tau$ and observing the resulting angular acceleration $\alpha$. For today’s lab, we will use a hanging mass and pulley to apply a constant torque, and an angular position sensor to measure the angular acceleration. Our goal is to experimentally determine the moment of inertia for various real masses.

Set up

In your lab report, make a large sketch of the apparatus. Next to the sketch draw free body diagrams and write the relations that follow from the diagrams. The apparatus is as shown schematically in figure 5.10, with the addition of a gyroscopic sensor (see section 4.3) attached to one end of the rotating bar to measure angular velocity. With the help of some free-body diagrams, you should be able to convince yourself that $Tr = I\alpha$ and $mg - T = ma$. From these equations it can be shown that

$$I = \frac{mr(g - ra)}{\alpha} \quad (5.6)$$

Please note: Do not mix up $r$ and $R$, or $m$ and $M$! Also note that for best results, your hanging mass should be at least 400 g. This helps to minimize the error due to friction.

1. Derive equation 5.6. Note that this equation is part of your measurement system: it converts easily measured values ($r, m, \alpha$) to rotational inertia, $I$. You will measure $m$ and $r$ only once, and $\alpha$ many times for different objects or object positions.
2. Measure $I$ as a function of mass placement radius $R$.

- Carefully measure the moment of inertia of the platform and sensor without any masses. This value must be subtracted from any subsequent measurements of rotational inertia, to obtain the rotational inertia of just the object in question.
- Use a square bolt-on mass, and record values of $I$ for at least 8 different radii. Use as wide a range of $R$ as possible. Record $R$ and $I$ in a data table.
- Plot $I$ vs. $R$, and use an appropriate curve fit to determine the dependence of $I$ on $R$. The parallel-axis theorem predicts that $I = I_{cm} + MR^2$, so you might try an equation of the form $y = A + Bx^C$. What is the meaning of $A$? Of $B$? Is $C$ actually 2 for your data?

3. Measure the rotational inertia of a solid disk rotating about its center. Attach the disc to the center of the rotating platform, using the adapter nut. Measure its moment of inertia. Measure the mass and radius of the disc and use these values to calculate the theoretical value of $I$. State the percent difference between your measurement and the theoretical value.

4. Measure the rotational inertia of solid disk rotating about a diametric axis. Follow the same procedure as before. Compare the results for the rotational inertia of the disk about these two axes, and explain why they are different.

5. Measure the rotational inertia of another object, and compare your results to the theoretical value. Possible objects may include (but are not limited to)

- ring
- solid disk rotating about an off-center axis
- solid disk not rotating about an off-center axis (!)

Conclusions:
Your lab report should include a summary of your measurements and a comparison of your measurements and theoretical values. You should also include some discussion of any potential sources of error and what steps you took to minimize them.