LABORATORY EXPERIMENTS

FOR

PHYSICS 204B: Electricity and Magnetism

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LAB TIPS

There are some important issues to keep in mind as you do these lab experiments and record your observations. Following these suggestions is practically guaranteed to substantially increase both your lab scores and your enjoyment of the experience.

1. Read the write-up of the experiment before coming to lab and try to visualize yourself doing the experiment. Such preparation will help you and your group perform the measurements more efficiently than coming in “cold.” This will also allow you to pace yourself during the lab, so that you will finish, hopefully with time to spare. Be an asset to your team, not a liability.

2. Entries in the lab notebook should be in *pencil*; feel free to erase. Writing should be **large and dark** enough to be read easily. Each lab should have a title at the top of the first page of the entry. Write using complete sentences, so that someone who wasn’t there during the experiment would be able to read your notebook and have a good idea of what happened. In particular, avoid at all costs starting sentences (answers to questions) with the words “yes” and “no.” Remember that your work will be read by others, so ***NEATNESS COUNTS!***

3. UNITS: Don’t forget to indicate the *units* (where appropriate) on all numerical values. If these are recorded in a chart, you can indicate the units at the top of a column. By far the most common deficiency in lab entries is failure to give units the proper respect!

4. Show sample calculations to indicate how data is being combined to produce a calculated result, and how uncertainties are being estimated. Results should be reported in a form (i.e. with a number of significant figures) *consistent with its uncertainty*. Please note that it is almost *never* appropriate to report an uncertainty with *more than one significant figure*!

5. GRAPHS: Graphed data should take up at least half a page if feasible. If you are graphing a few data points (as opposed to the continuously recorded output of one of the sensors), make sure to plot the data points clearly and indicate the curve that best fits them. Don’t forget to label the axes with the quantity being plotted (e.g., “time,” “voltage,” etc.) along with the units. Always give the graph a title (nothing fancy—just say what’s being plotted). When comparing predictions and observations, you can use the same set of axes for both graphs—just be sure to distinguish the two plots (with, e.g., solid and dashed lines, or with different colors). Please do not put arrowheads on the ends of best-fit lines!

6. If your observations or calculations give results that are so far from those that you expect that something must be wrong and you and your group can’t figure it out, *ask the instructor for help*.

7. Always write a conclusion in which you answer the indicated questions at the end of the write-up. Try to base your statements on what you and your partners actually observed.

8. Always review graded labs for comments intended as feedback. *Don’t make the same mistake twice*!

9. Laptop Usage: The laptops are intended as *read only* devices: do not leave files on the disk or alter the desktop in any way. Deliberate and malicious changes to the laptops will be considered vandalism and as such will be subject to disciplinary action.

10. Courtesy: As a courtesy to your group, to the class, and to the instructor, don't come in late to lab. Do not write on the tabletop. Backpacks must NOT be placed in the aisles (stow them under the table or by the entrance). Before you leave: Clean up any cups, wrappers, etc. from your work area as a courtesy to the next group and to the lab technician. Put everything back the way it was when you started. *Report broken apparatus immediately*. The cell phone policy stated in the syllabus applies in lab as well.

Electrostatic Forces and Coulomb's Law

Introduction. In this set of experiments, you will investigate the forces between charges. In part I, you will do this qualitatively by producing excess charges on objects and observing the attraction or repulsion they exhibit for each other. In part II you will do a more careful quantitative assessment of the way the force between two charged objects behaves as a function of the distance between them. You can then compare your result to the prediction of Coulomb's law.

Procedure. You will need the following:

- Scotch tape in dispenser
- aluminum foil pieces
- 1 ea. hard plastic/Lucite/glass rod
- Styrofoam packing peanut on a string
- 2 metal rods
- laptop
- Coulomb balance apparatus with two 5kV high voltage supplies (back bench)
- Van de Graaff generator with insulated conducting sphere (front bench)
- 2 small rod stands
- long thin plastic strip
- 1 fur piece
- 1 silk cloth
- 2 right angle clamps
- string and scissors
- 1 ea. conducting/insulating pith ball (5 mm)
- French curve

I. Qualitative Observations. The nature of electrical interactions is not obvious without careful experimentation and reasoning. We will first state two hypotheses about electrical interactions. You will then observe some of these interactions and determine whether your observations are consistent with these hypotheses.

> *Hypothesis One:* The interaction between two objects that have been rubbed is due to a *property* of matter that we will call *charge*. There are *two* types of electrical charge that we will call, for the sake of convenience, positive charge and negative charge.
>
> *Hypothesis Two:* Excess charge moves readily on certain materials, known as conductors, and not on others, known as insulators. In general, metals are good conductors, while glass, rubber, and plastic tend to be insulators.

Note: In completing the following activities, you should not report outcomes based on assumptions or previous experience. You must report your actual observations and devise a sound and logical set of reasons to support (or refute) the hypotheses.

Hypothesis One: Testing for Different Types of Charge

1. Interactions of Scotch Tape Strips

(a) Stick two 15 cm (or so) strips of Scotch tape side by side on the back of a chair; the end of each tape should be curled to make a non-stick handle. Peel the strips off the chair and bring the non-sticky side of the tapes toward each other. What happens? How does the distance between tapes affect the interaction between them?

(b) Stick two strips of tape on the chair and label them "B" for bottom. Press another strip of tape on top of each of the B pieces; label these strips "T" for top. Pull each pair of strips off the chair. Then pull the T and B strips apart.
i.) Describe the interaction of the two T strips when they are brought toward one another.
ii.) Describe the interaction between the two B strips.
iii.) Describe the interaction between a T strip and a B strip.

(c) Are your observations of the tape strip interactions consistent with the hypothesis that there are two types of charge? Please explain your answer carefully, in complete sentences, and cite the outcomes of *all* your observations.

2. Charging Insulators with Rods

(a) Use tape to secure a doubled-over strip of thin plastic[1] to a horizontal rod so that the two approximately equal lengths hang facing each other. Try rubbing the black plastic rod with fur and then use the rod to rub the facing surfaces of the plastic strips near the bottom. What happens to the hanging pieces? What happens when you rub the rod with the fur again and bring it near the pieces?

(b) What happens if you now immediately rub a glass rod with silk and then bring it into the vicinity of the strips where they were rubbed by the plastic rod?

(c) Recalling the interactions between like and unlike charged objects that you observed before, can you explain your observations?

(d) Touch the entire surface of the plastic strip with your hands. Now what happens when you let them hang again? Is there an interaction between them?

Note: Benjamin Franklin *arbitrarily* assigned the term "negative" to the nature of the charge acquired by the hard plastic (or rubber) rod when it is rubbed with fur. Conversely, the nature of the charge found on the glass rod after it is rubbed with silk is defined as "positive."

Hypothesis Two: Using Induction to Distinguish Conductors from Nonconductors

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1 If the strip appears to be already charged, you may need to discharge it with your hand or by moistening and then drying it.

A neutral object with no external influences has equal numbers of both kinds of charges that, in general, are distributed uniformly. In the presence of external charges, however, the positive and negative charges in the body, since they feel different forces, can be induced to separate so that the body as a whole feels a net force. This phenomenon is known as *electrostatic induction*.

3. Insulators, Conductors and Induction

Using pieces of Scotch tape, hang two small pith balls--one coated (conducting) and one uncoated (insulating)--by their threads from a horizontal bar. Space them as far apart as possible, and touch each one to make sure it is uncharged.

(a) What happens when you rub the Lucite rod with the fur and bring it near the uncharged *insulating* pith ball--
i.) Before they touch?
ii.) Now immediately charge the rod again and bring it near the area that was just touched as well as near an area that was *not* touched. What happens?

(b) Repeat observation (a) using the hanging uncharged *conducting* pith ball.. What happens--
i.) Before they touch?
ii.) After they touch (bring the rod near both touched and untouched areas)?

(c) Use Hypothesis Two, which claims that charges move readily on conductors, to explain why the conducting pith ball is *attracted* to the rod *before* touching and *repelled after* touching it.

(d) Use Hypothesis Two, which claims that if an object is an insulator, its electrons will stay in the vicinity of their atoms, to explain the behavior of the insulating pith ball after being touched by the rod.

(e) Watch the video AlPan&FoamBoard.mov, which should be available on your desktop in the folder **Electrostatics Videos**. Explain the behavior of the charged strips using what you've learned about electrostatic forces.

II. Quantitative Investigation: Coulomb's Law

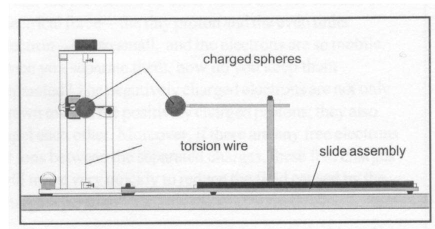
Coulomb's law is the basis for the calculation of the electrical forces that act between charged objects. The law applies specifically to the simplest case of two "point" charges (q_1 and q_2) separated by a distance *r*. These charges experience a mutual force given as

$$F_{Coul} = k\frac{|q_1 q_2|}{r^2}, \qquad (1)$$

where k_e is the Coulomb constant, and has a value of 8.99×10^9 N·m^2/C^2. The force is attractive if $q_1 q_2 < 0$ and repulsive if $q_1 q_2 > 0$. Application of the principle of superposition shows that

Eq. (1) also applies to two *uniformly charged* spheres. (This is a good thing, since point charges are hard to find.)

In this experiment you will use the Coulomb balance pictured at the right to determine how the force between two identical spheres with equal positive charges depends on the distance r between their centers. For each value of r (varied by moving the sliding sphere) the electrostatic force on the sphere suspended on the torsion thread is balanced by twisting the thread, using the knob at the top, so that the sphere returns to its initial angular position. The angle scale then indicates the amount of twist (and hence the torque) exerted by the thread, which is now equal to the torque due to the charges. Since the lever arm is constant, the angle reading for each distance is proportional to the mutual force of repulsion between the charged spheres.



Procedure. The Coulomb balance should be already adjusted for the experiment. Make sure that the torsion balance is zeroed by setting the dial at the top to zero and rotating the bottom torsion wire retainer until the pendulum assembly is at its zero displacement position as indicated by the index marks. When both spheres are discharged, they should just touch when the position marker for the sliding sphere reads 3.8 cm, which is the sphere diameter; the thumbscrew on the top of the rod supporting the sliding sphere can be loosened and the position of this sphere can be adjusted so that this latter condition is met. The high voltage supply should be set at about 8 kV and held constant so that the same charge is always delivered to the spheres by the charging probe.

For each separation r for which the force is to be measured:

1. Move the spheres to their maximum separation and charge each one by briefly touching it with the charging probe connected to the high voltage supply.

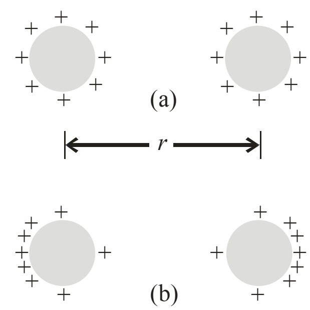
2. Quickly move the sliding sphere until the scale indicator reads the desired separation r.

3. Turn the knob until the pendulum is brought back to zero displacement as indicated by the alignment of the index marks. Record the distance r and the angle reading θ of the knob.

4. Repeat this measurement a few (two or three) times and obtain an average value of the angle reading, $\overline{\theta}$, corresponding to this value of r.

5. Each group should repeat this procedure for two different values of r between 20 cm and 6 cm. The data ($\overline{\theta}$ vs. r) from all the groups for different r values should be combined for analysis.

Analysis. Open **LoggerProFile.cmbl** and produce a plot of $\overline{\theta}$ (along the y-axis) vs. the separation *r* (along the x-axis). Try fitting the data[2] (Analyze→Curve Fit) with a function of the form Ax^B. Does your result agree with Coulomb's law, Eq. (1)? Make a careful sketch of this graph (data points with best-fit curve) in your notebook.



The "Dipole" Correction: It should be noted that Coulomb's law actually applies directly only to point charges and, by extension, to uniformly charged spheres, as depicted at the right in (a). Our spheres are conducting, so the positive charges on one sphere are free to move and respond to the repulsive force from positive charges on the other sphere. As the spheres are brought into close proximity, the + charge distribution becomes more like that in (b), so we would expect a deviation from Coulomb's law behavior for the force at distances *r* that are not large compared with the sphere radius *a*.

Conclusion. Briefly summarize how your qualitative observations in part I support or contradict the two hypotheses given concerning the behavior of charge.

Do the results of your quantitative measurements of the interaction between two charged spheres support Coulomb's law? Explain.

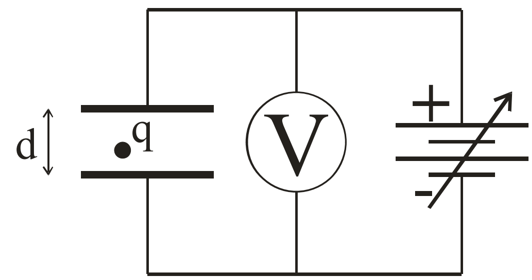
Question: Considering the "dipole" correction discussed above, how does your observed θ vs. r curve differ from the curve you would have obtained if your spheres represented actual "point" charges (or if the charge were uniformly distributed over their surfaces)? Illustrate your answer by roughly sketching both the observed and "point charge" curves on the same axes for comparison.

[2] You will first need to sort the data (use the Data menu).

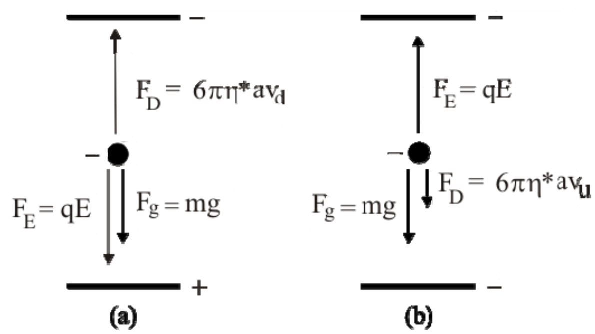
The Millikan Oil Drop Experiment

Introduction. The American physicist R. A. Millikan, in a famous series of measurements performed around 1910, studied the motion of charged oil droplets between two charged plates. These experiments (a) showed that charge is *quantized*, i.e., appears only as multiples of an elementary unit of charge *e*; (b) produced a value for this charge *e*, and (c) helped Millikan win the Nobel prize in 1923. We will perform a version of this historic experiment.

The oil drop apparatus is pictured schematically at the right. By applying a potential difference (voltage) V between the two parallel plates separated by a distance d, a region of uniform electric field $E = \frac{V}{d}$ is created between the plates. Oil droplets are introduced into this region and their motion is observed after they reach terminal velocity under the influence of their weight, the electric field, and the resistive drag force due to the viscosity of the air. By carefully measuring the terminal speeds, the charge on each drop can be determined, as described below.



The force diagrams for an oil droplet with a *negative* charge[3] q are shown at the right (a) with the top plate negative and (b) with the top plate positive. The size of the voltage V (and hence the magnitude E of the field) between the plates is the same for both polarities.



For the situation depicted in (a), the droplet (of radius a) is pulled down under the influence of the electric field ($\mathbf{F_E = qE}$) and its weight $F_g = mg$; the viscous drag force F_D acts upward and is given by Stokes' law as $F_D = 6\pi\eta^* a v_d$, where v_d is the terminal downward speed of the droplet and η^* is the effective viscosity (see below) of the air. Since the droplet is not accelerating, Newton's second law gives $\sum \mathbf{F} = \mathbf{F_D} + \mathbf{F_g} + \mathbf{F_E} = 0$, from which we get

$$6\pi\eta^* a v_d - mg - |q| E = 0 . \tag{1}$$

Note that the buoyant force on the droplet is neglected because it is small compared to its weight.

If the plates are charged so that the sphere moves *upward* with constant speed v_u, as in

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[3] Although the charge on the droplet may be of either sign, we assume that $q < 0$ for this calculation. The sign of q on any droplet can always be determined by observing its direction of motion relative to the polarity of the voltage on the plates.

(b), the force balance again reads $\sum \mathbf{F} = \mathbf{F_D} + \mathbf{F_g} + \mathbf{F_E} = 0$, which gives us

$$|q|E - mg - 6\pi\eta^* a v_u = 0. \tag{2}$$

Eliminating η^* between Eqs. (1) and (2) gives the charge on the droplet in terms of the measured terminal up and down speeds[4] v_u and v_d:

$$\boxed{|q| = \frac{mg}{E}\left(\frac{v_d + v_u}{v_d - v_u}\right)}. \tag{3}$$

We could calculate the charge q on any droplet from Eq. (3) if we knew its mass, but this must be calculated for each droplet from its radius: mass = (volume)(density) $\Rightarrow m = \frac{4}{3}\pi a^3 \rho$, where ρ is the density of the oil. Plugging this expression for the mass into Eqs. (1) and (2), then eliminating the charge, would give for the radius a:

$$a = \sqrt{\frac{9\eta^*}{4g\rho}(v_d - v_u)}. \tag{4}$$

But the value of the effective viscosity η^* is different from the "ordinary" viscosity η because the droplets are so small that their radii are comparable to the mean free path of air molecules. In terms of the "ordinary" viscosity η, this effective viscosity η^* is given as

$$\eta^* = \eta\left(1 + \frac{b}{pa}\right)^{-1}, \tag{5}$$

where p is the atmospheric pressure and b is a constant (given below). Substituting Eq. (5) for η^* in Eq. (4) and solving for a again gives the corrected radius as

$$a = -\frac{b}{2p} + \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta}{4g\rho}(v_d - v_u)}. \tag{6}$$

This value of a can be used to compute the mass of the droplet, and thus the charge directly from Eq. (3). By observing several droplets and measuring v_d and v_u, you will compute their charges and look for evidence of a fundamental charge unit.

Procedure. You will need the following (FB = front bench only):

- Millikan oil drop apparatus (FB)
- 0-500 V voltage supply w/readout (FB)
- digital multimeter (FB)
- Stopwatches (2)
- VideoFlex camera with adapters (FB)
- Video projector (FB)
- Computer with video analysis software

[4] Note that here, v_u and v_d are speeds, and are hence positive quantities.

Each group should take data on droplets from the apparatus and from the videos on the laptops:

I. Taking data from the apparatus: (Each group takes data on one droplet.)

The oil drop apparatus should already be set up and optically aligned, so there is no need to make any adjustments, except as indicated by the instructor.

1. Determine the temperature T of the droplet viewing chamber (for the calculation of η) from the resistance of the thermistor and the thermistor resistance table at the very end of this write-up. You will need to linearly interpolate since the measured resistance is unlikely to be one of the table entries. Alternatively, you can input this resistance in the worksheet (see below under "Analysis") and let it calculate T.

2. Measure and record the voltage V at the input terminals of the plates.

3. Move the ionization source lever to the "Spray Droplet" position and eject a spray of oil droplets from the atomizer into the chamber with one quick squeeze followed by a slow squeeze. When the droplets appear in the viewing field, switch the source lever to "OFF."

4. Select a droplet that can be made to rise and fall depending on the polarity of the plate voltage, as controlled by the 3-position plate voltage switch. Noting that *the vertical distance between major reticle lines is 0.5 mm*, time the motion of this droplet with the polarity such that the droplet moves downward and calculate v_d. Switch the voltage polarity so that the droplet rises, and measure v_u by timing this upward motion. Make a note of which polarities cause the drop to rise/fall and so deduce the sign of its charge.

Repeat these measurements two more times for this droplet, if possible, to produce a total of at least three pairs of speeds v_d and v_u. For each pair of speeds you can produce a value of the charge q on the droplet (see below under "Analysis"). Averaging these values for a single droplet gives the best value $\overline{q}$ of its charge; calculating the standard deviation (the "spread") of these values gives an estimate of the uncertainty associated with q. Recall that the uncertainty in $\overline{q}$ is smaller than that in q (see "Uncertainties" below). You can also estimate the uncertainties from the worksheet.

You can attempt to change the charge on this droplet by moving the ionization source lever to the ON position (with the E-field off) for a few seconds; if the velocity with E back on has changed, make a new set of measurements to determine the new charge. As time permits, measure the terminal speeds for additional droplets in various charge states.

Post your group's average values for the down and up speeds ($\overline{v}_d$ and $\overline{v}_u$) for your drop along with the chamber temperature (T) and plate voltage (V) in a table on the board (like the one below) to share with the class:

Group #	$\overline{v}_f$ (m/s)	$\overline{v}_r$ (m/s)	T (°C)	V (V)
1				
2				
3				
4				
5				
6				

II. Taking data from the videos: (Each group analyzes three droplet videos.)

In the folder "Millikan Oil Drop Videos" on your desktop you will find a series of QuickTime videos shot using the Oil Drop Apparatus that can be analyzed in the same way as the observations you made at the front bench. Make measurements on three different charges, obtaining the up and down speeds for each one.

Analysis. Compute a value of the charge $\overline{q}$, as described below, for each of the droplets observed by the lab groups using the apparatus, as well as for any three droplets featured on the videos. A Mathcad file entitled "Millikan Oil Drop Worksheet.mcd" can be found in the Dietz's 204B LABS\Millikan Oil Drop folder accessible from the desktop. This is a worksheet in which you can fill in the data for the computation of the droplet charge, which appears at the bottom of the sheet[5].

Note: It is recommended that $q \pm \Delta q$ for each droplet be calculated and recorded *before* moving on to take data for the next droplet. That way you'll have some charge values at the end even if you don't have time to finish calculations on all of the selected droplets.

5 If you find the speeds using the VideoPoint or Logger Pro graphical analysis tools, you can enter these results directly into the worksheet (instead of times and distances) by overriding the definitions of the speeds.

For each charge state of each of the droplets you observed, you can use Eq. (3) to determine its charge q (don't forget the sign). To do this, however, you will need to know (in addition to the average terminal speeds):

(a) The *radius a* of the droplet (to get the mass): This can be calculated from Eq. (6), where

1. The atmospheric pressure p is in Pascal (Pa) and 1 Pa = 1 N/m^2. Note that 1 atmosphere (atm) is 760 mm Hg = 1.013×10^5 N/m^2, so 1 atm is 1.013×10^5 Pa. The pressure in the room should be around 1 atm, depending on atmospheric conditions. Read p from the barometer mounted on the west wall of the lab room and convert to Pa.

2. The viscosity η of the air inside the viewing chamber. This is a function of the temperature, given as $\eta(T) = \left[1.729 + \left(4.765 \times 10^{-3}\right) T\right]\left(\times 10^{-5}\ \mathrm{Nsm^{-2}}\right)$, where T is in $^\circ$C as determined from the thermistor reading using the chart (with interpolation) at the very end of this write-up (or with the numerical fit from the worksheet).

3. The constant b, used to correct the viscosity for the smallness of our droplets, is given as $b = 8.20 \times 10^{-3}\,\mathrm{Pa \cdot m}$.

4. The density of the mineral oil, given as ρ = 886 kg/m^3.

(b) The *mass m* of the droplet, calculated from $m = \frac{4}{3}\pi a^3 \rho$.

(c) The *electric field E* between the plates (in N/C or V/m). This is calculated from the plate voltage V using $E = \frac{V}{d}$, where d is the plate separation, measured as 7.60×10^{-3} m for our apparatus.

Note that the above calculations can be done manually or by filling in values on the worksheet.

Finally:

(1) Tabulate the charges you calculated for the droplets (with estimated uncertainties) and look for regularities, i.e., are the charges multiples of some unit, say the smallest *difference* between charges?

(2) Assuming these charges *are* in fact integer multiples of the elementary charge e, determine the number of elementary charges on each drop, along with the value of e derived from this assessment (include uncertainties). What is the average value of e from all the drops that you observed?

Uncertainties. One way to estimate the uncertainty in any charge measurement is to look at the "spread" (the standard deviation Δq) of several measurements of the same charge q. Since you are averaging these values, the uncertainty in an average charge $\overline{q}$ can be estimated as $\Delta\overline{q} \simeq \frac{\Delta q}{\sqrt{n}}$, where n is the number of charges averaged. To get an idea of the fractional uncertainty $\left(\frac{\Delta\overline{q}}{\overline{q}}\right)$ for these measurements, each group should compute $\Delta\overline{q}$ for the charges calculated from the sets of (v_d, v_u) data for the droplet they observed on the apparatus.

Note that the worksheet will also compute the uncertainty in a single charge from the estimated uncertainties in the experimental parameters, which you must provide. Uncertainties in parameters given to you as data in the movie (e.g. pressure p), you can make estimates based on the number of significant figures presented.

Conclusion. Report your best estimate (with uncertainties) of the value of the elementary charge. Is this within uncertainties of the accepted value of e?

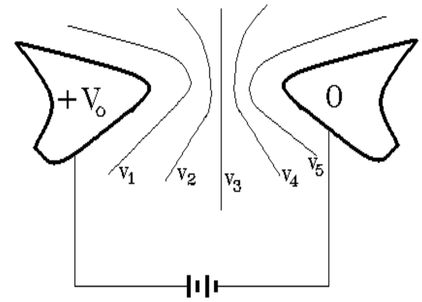
Appendix:

Millikan Oil Drop Apparatus Thermistor Resistance at Various Temperatures

THERMISTOR RESISTANCE TABLE

°C	X 10^6 Ω	°C	X 10^6 Ω	°C	X 10^6 Ω
10	3.239	20	2.300	30	1.774
11	3.118	21	2.233	31	1.736
12	3.004	22	2.169	32	1.700
13	2.897	23	2.110	33	1.666
14	2.795	24	2.053	34	1.634
15	2.700	25	2.000	35	1.603
16	2.610	26	1.950	36	1.574
17	2.526	27	1.902	37	1.547
18	2.446	28	1.857	38	1.521
19	2.371	29	1.815	39	1.496

Electric Potential and Electric Field

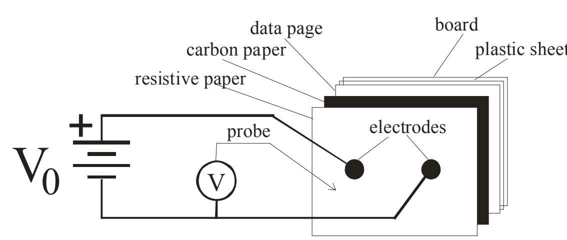


Introduction. If a set of conductors is maintained at a fixed potential difference by a battery (or power supply), an electric field is created in the surrounding region by the charge that the battery deposits on them. In the configuration shown at the right, for example, the battery (which provides a voltage V_0) maintains the conductors at potentials V_0 and 0.

The value of the potential V for each point in the region of the conductors depends on the details of how the charge is distributed on them. This in turn depends on the size, shape and positioning of the conductors, along with V_0. One way to represent the resulting pattern is to draw *equipotential lines*--lines of constant potential--and label them with the corresponding voltages (V_i) as shown above. The lines representing the electric field may then be drawn perpendicular to these equipotentials. It is sometimes important to know the equipotential and field patterns associated with various electrode configurations (for example, in the design of ion optics). Unfortunately, patterns for all but the simplest of electrode arrangements are quite difficult to compute. In this exercise you will learn a technique which utilizes resistive (Teledeltos) paper to construct equipotential lines. In addition you will see how well the measured pattern of potentials for concentric cylinders agrees with the prediction of Gauss's law.

Procedure. You will need the following:

- Teledeltos apparatus
- low voltage power supply
- digital multimeter
- Vernier calipers and cm ruler
- Laptop with Logger *Pro*
- dipole and cylinder electrodes
- cables/alligator clips/pointed electrodes
- French curve

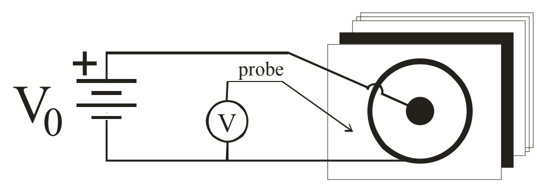


1. Dipole Pattern (Qualitative): Arrange the board, plastic sheet, data page, carbon paper and resistive paper as shown at the right. Tightly screw the dipole electrodes to the assembly so that the grooved sides make contact with the resistive paper.

Connect the leads from the DC supply to each electrode and connect the common side ("COM") of the digital multimeter (DMM) to the negative terminal of the supply. The other side of the DMM is the "probe," (plugged into the "V" socket). If connected correctly, the DMM (configured to read voltage) should read zero volts when the probe is touched to one of the electrodes and the full supply voltage V_0 when in contact with the other. Mark the location, polarity and voltage of the electrodes on the data page by pressing down firmly on the resistive paper with the probe.

With V_0 adjusted to some convenient value, which you should record, gently slide the probe (without the alligator clip) over the surface of the resistive paper and observe the output of the voltmeter. Locate the point at a voltage $V = V_0/2$ between the dipole electrodes, and continue to find points at this potential on your sheet. Repeat this for four more values of V, so that you have points for five equipotentials corresponding to five uniformly spaced values of V between 0 and V_0 (be sure to distinguish points corresponding to different values of V as you mark them). Remove the data page and sketch your equipotential lines for the dipole and label them with voltage values. Noting that the lines of electric field are everywhere perpendicular to the equipotential lines, sketch the lines of the E-field (include directions). Sketch and label both sets of lines in your notebook.

2. Concentric Cylinders (Quantitative): Arrange the apparatus as shown at the right with the ring and the small disk attached concentrically. Tighten the electrodes securely with screws and wing nuts through *each hole*. Measure the outer diameter of the inner electrode and the inner diameter of the outer electrode. Record the respective radii a and b.



Set the power supply to $V_0 = 15.0$ V. Move the probe along three different radii (about equally spaced in angle) until you locate the point where V = 10.0 V on each radius; pressing down with the probe will record these positions on the data sheet. Repeat this for V = 7.50 V, 5.00 V, 3.00 V, 2.00 V, and 1.00 V. Make sure you keep track of the potential for each point.

Use a pencil to outline the positions of the inner and outer electrodes, and remove the data page. Tabulate the *average* values of r, the distance from the center, corresponding to each of the potentials.

Theory: Due to the constraints imposed by the resistive paper, this pattern of potentials corresponds to that of two very long concentric cylinders with radii a and b, rather than that of two circles. For the cylinders, the electric field in the region between them points radially outward; Gauss's law predicts that the field strength diminishes with r, the distance from the axis, and is given by

$$E = \frac{\lambda}{2\pi r \varepsilon_0}, \qquad (1)$$

where λ is the linear charge density on the inner cylinder. The difference between the potential of the center conductor (V_0) and the potential V(r) at a distance r from the axis is found by integrating this expression for **E** along a radius:

$$\Delta V_{0\to r} \equiv V(r) - V_0 = -\int_a^r \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{a}{r}. \quad (2)$$

If we put ground (V = 0) at the outer electrode (r = b), then setting V(b) = 0 in Eq. (2) gives us λ for this situation, which can the be substituted back into the same equation to give V(r):

$$\boxed{V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}}. \quad (3)$$

(Be certain you know how to obtain this result for V(r)!)

<u>Analysis</u>. In Logger *Pro*, tabulate and graph V/V_0 (on the y-axis) vs. $\ln(b/r)$ (on the x-axis). Find the best fit line and determine if the slope is within uncertainties[6] of what you would expect from Eq.(3), which was derived from Gauss's law. Sketch this graph in your notebook.

<u>Conclusion.</u> Are the equipotential and E-field lines that you sketched for the dipole arrangement approximately consistent with what you expected from your knowledge of Coulomb's law applied to + and − point charges? To what three-dimensional configuration does this resistive paper pattern *actually* correspond?

[6] In Logger *Pro*, you can obtain the uncertainty in the slope of the best-fit line by double-clicking the fit box and checking the box for the standard deviation in the slope.

Capacitors and Charge

Introduction. A device that acquires a charge in response to the application of a potential difference across its terminals has a property we call *capacitance*. Capacitors are important components in electronic devices that perform functions such as timing, tuning, filtering, energy storage and sensing. In this set of experiments you will determine how the capacitance of a set of parallel plates depends on their separation and on the material in the space between them. You will also use capacitors to see if charge is conserved.

For two conductors, we define the capacitance C as the number of Coulombs of charge Q that it "holds" (i.e., its capacity) per unit of applied potential (V):

$$Q \equiv CV \tag{1}$$

There are only a few configurations of conductors for which this property is readily computed from the geometry. As has been (or soon will be) discussed in class, the parallel plate arrangement has a capacitance given by

$$C = K\varepsilon_0 \frac{A}{d}. \tag{2}$$

Procedure. You will need the following:

- laptop
- low voltage DC power supply
- cables and alligator clips
- Sencore LC103 Capacitor & Inductor Analyzer
- Fluke 179 digital multimeter (DMM)
- 2 (good) electrolytic capacitors
- Vernier calipers and cm ruler
- PASCO variable capacitor
- Sencore Analyzer
- cold pack

I. Dependence of capacitance on plate separation:

Set up the LC103 "ReZolver" to measure capacitance[7]: Connect and zero the test leads (see p. 1 in the pull-out instructions) and press "all other caps" under "Component Type." Connect your capacitor (the set of circular parallel plates on the plastic track) to the test leads (use alligator clips); press and hold the "Capacitor Value" button to read C (p. 4, part 1).

Slide one of the plates along the track to get different plate separations[8] d (indicated on the scale) and measure the capacitance C with the analyzer for each value of d. Take 10 data points (values of C and d) and tabulate the data in your notebook. Open the **Dependence of C** file in the "Dietz's 204B LABS\Capacitors and Charge" folder and plot C vs. d. What function

[7] Never connect a battery or power supply to a device while it's connected to the Sencore analyzer! The Sencore has its own internal power supply. ***Always discharge the capacitor before connecting it to the analyzer!***

[8] Keep the value of d small compared to the linear dimension (diameter) of the plates.

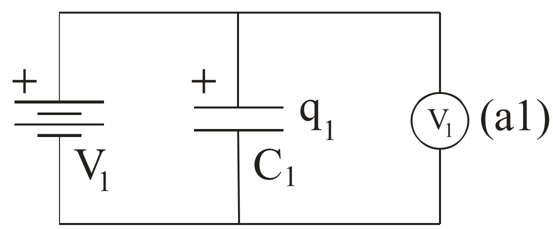
gives the best fit for the data? Is this consistent with the prediction of Eq. (2)? Sketch the best-fit curve in your notebook.

Set the plate separation at a value equal to the thickness of the "cold pack" provided. Measure and record the capacitance both without and with the cold pack inserted between the plates. What can you conclude about the effective dielectric constant of the material inside the pack?

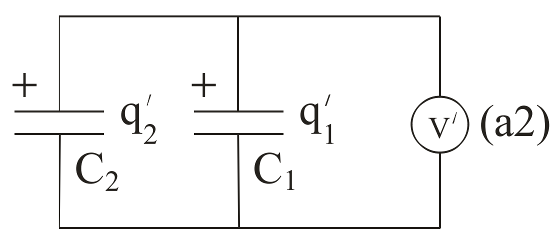
II. <u>Capacitors and Charge Conservation</u>: For each of two electrolytic capacitors (C_1 and C_2), measure the capacitance and the leakage resistance as follows:

1. Select the component type: "Aluminum Lytics."
2. Enter the operating voltage, i.e., the maximum voltage[4] you will apply during the measurements described below (p. 2, part 4 in pull chart instructions near the bottom of the unit.).
3. Read and record C as in part I above (red test clip to +). Note the uncertainty ΔC from Appendix B.
4. Perform the leakage test (p. 4, part 4) and record the leakage resistance R_L--this is the effective resistance in parallel with the capacitor; the smaller R_L, the faster it looses its charge. If you calculate the time $\tau_C = R_L C$, this will give you an idea of the time scale on which the capacitor will spontaneously discharge--and how fast you must work in performing the measurements that follow[9].

(a) Discharge each capacitor by shorting the terminals with a cable. Using the circuit at the right in (a1), charge C_1 and record the voltage[10] $V_1 \pm \Delta V_1$ read on the DMM (see Appendix B for uncertainties). Be sure to observe the polarities! Disconnect C_1 from the supply[11] and connect C_1 and C_2 as shown in (a2) and measure the common voltage $V' \pm \Delta V'$.



Predict: How will the charges q_1, q_1', and q_2' be related?



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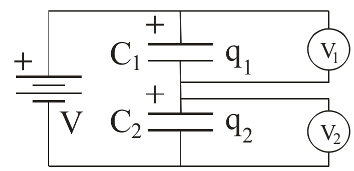
[9] Circuit Tips: 1. Before measuring component values with a meter, always remove the component from the circuit. 2. Lay out the components physically as they appear in the circuit diagram, then interconnect with cables. 3. Connect the "main" circuit first and the meters and/or scope last.

[10] Do not exceed the voltage rating on the side of the can.

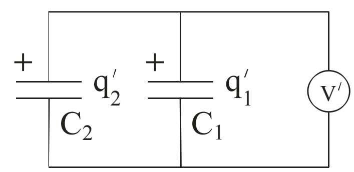
[11] For both parts (a) and (b), it is important to disconnect the capacitor(s) from the supply before turning it off (to prevent charge leakage through the supply).

Use Eq. (1) to calculate the initial charge q_1 on C_1 and the final charges q_1' and q_2' on C_1 and C_2 respectively, along with the uncertainties (see below).

(b) Short out the two capacitors again and connect them with the power supply as shown in (b1) at the right. Measure the voltage $V_1 \pm \Delta V_1$ and then $V_2 \pm \Delta V_2$. Disconnect the capacitors from the power supply and then reconnect them as in (b2) (observe polarities!). Measure the new voltage $V' \pm \Delta V'$.



(b1)



(b2)

Predict: What relationship do you expect to discover between the charges q_1, q_2, q_1' and q_2'? Are q_1 and q_2 related? How?
Compute all the charges q_1, q_2, q_1' and q_2', along with their uncertainties.

<u>Uncertainties.</u> The uncertainties for the capacitances measured by the LC103 ReZolver can be determined from Appendix B, as can the uncertainties for all the voltages measured with the Fluke 179.

For the charges, note that Eq. (1) implies, by way of Eq. (8b) in Appendix A, that their uncertainties are approximated as

$$\frac{\Delta q}{q} = \sqrt{\left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta V}{V}\right)^2} \,. \tag{3}$$

Also note that every time you add two quantities that each have an uncertainty, you compound the uncertainty for the sum as per Eq. (8a) of Appendix A. Thus the uncertainty in the *sum* (q_{SUM}) of two charges q_1 and q_2 would be given as

$$\Delta q_{SUM} = \sqrt{(\Delta q_1)^2 + (\Delta q_2)^2} \,. \tag{4}$$

It is seldom necessary or desirable to keep more than one significant figure for the uncertainty in a final result.

<u>Analysis</u>. From your measurements in part II, determine whether your predictions for the relationships among the charges in each of the two experiments were confirmed, *within uncertainties*.

Conclusion. What did you learn about capacitors in Part I? In particular, how does the capacitance of the parallel plate arrangement depend on its dimensions and on what's in between the plates, according to your investigation?

What do you conclude about charge conservation from your work in Part II? Are there uncertainties or systematic errors for which you have not accounted?

Ohm's Law

<u>Introduction</u>. The application of a potential difference (voltage) between two points on a conductor will cause a current to flow. If this current (I) is directly proportional to the voltage (V), then the conductor is said to obey "Ohm's Law:"

$$V = IR . \tag{1}$$

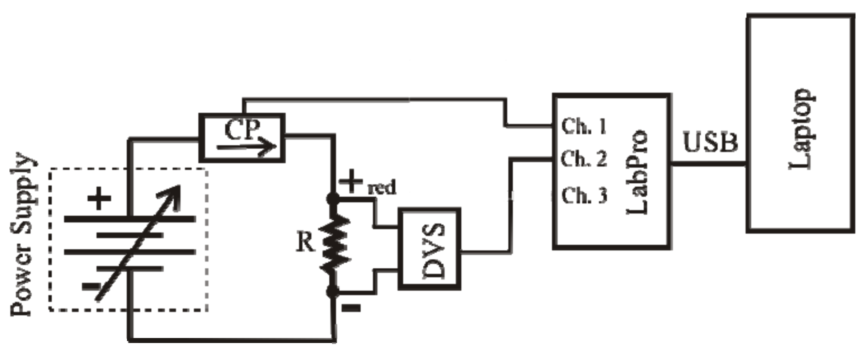
The constant of proportionality, R, between V and I is the *resistance* of the object and has units of *Ohms* (Ω). Conducting objects for which this law holds for over a range of conditions (e.g., temperature) are referred to as "Ohmic devices[12]" (e.g., resistors, wires).

In this lab, you will investigate the relationship between V and I for a resistor and for a light bulb and see if Ohm's law is obeyed.

<u>Procedure</u>. You will need the following:

- Laptop with LabPro interface
- current probe
- French curve
- differential voltage sensor (for LabPro)
- DC power supply
- 10 Ω and 50 Ω resistor
- light bulb (6.3 V) and holder
- multimeter
- wires and clips

Boot up the laptop and connect the LabPro to DC power with the adapter and to the laptop with the USB cable. Connect the Current Probe (CP) and the Differential Voltage Sensor (DVS) to Chs. 1 and 2 of the LabPro, respectively, as shown below.



I. <u>Voltage vs. Current for a resistor</u>: Set up the circuit with the power supply (OFF initially!) and the resistor with the Current Probe (CP) in the circuit as indicated above (note the

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[12] The resistance can be *defined* as $R \equiv \frac{V}{I}$ even for *non*-Ohmic devices, but for these, R is not constant as V is changed.

+ direction of the current flow as indicated by the arrow on the CP). Clip the DVS leads across the resistor. Turn the voltage knob on the power supply fully counterclockwise and the current knob (current limit control) fully clockwise.

Run Logger *Pro* and open the **Ohm's Law** file (File→Open) in the "Dietz's 204B LABS" folder. With the power supply off and disconnected and the leads to the DVS shorted, use the zero (Ø) button (3rd from the right on the toolbar) to set the zero for V and I. Turn the power supply on and click on the "collect" button (2nd from the right on the toolbar) to begin taking data. *Slowly* begin increasing the voltage (***do not exceed 6.0 V!***) and note the increase in V and I values on the "meter" at the lower left. *After the values of voltage and current have stabilized*, click on the rightmost button on the toolbar to "keep (the) current value" as a data point. Take several data points for the resistor and when finished click "stop" (in same position as "collect").

NOTE: Turn off the power supply when not in use.

Analysis. Determine the best-fit line for your V vs. I data: sort the I values in ascending order using the data menu, the use Analyze→Automatic Curve Fit. Sketch the graph in your notebook. Record the slope with its uncertainty and write an equation for V as a function of I (include units).

Take the resistor out of the circuit and measure its resistance with the digital multimeter (DMM), noting the uncertainty (Appendix B). Compare this value with the slope of your V vs. I line. Finally, compare these two resistance values with the value indicated on the resistor body[13]. Are these three values of the resistance consistent with each other, within uncertainties?

II. Voltage vs. Current for the light bulb: Clear the resistor data (Data→Clear All Data) and replace the resistor in the circuit with the light bulb. Again, use Logger *Pro* to collect and analyze the V vs. I data for V between 0 and 6 V (try to get several data points in the range between 0 and about 0.4 V).

Predict: What will be the resistance of the bulb as measured by the DMM?

Test your prediction by removing the bulb from the circuit and measuring its resistance with the DMM. Comment on the result in light of your prediction.

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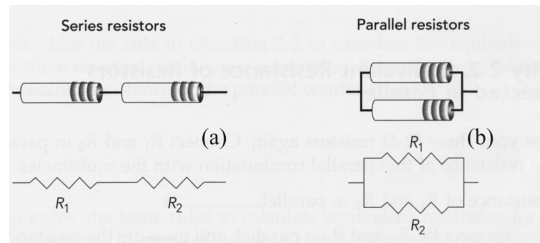
[13] The uncertainty in the resistor value printed on the body is given by the tolerance code. If there are colored bands on the body, then the value of R and its uncertainty are given in Appendix C. The letters "J" and "K" indicate 5% and 10% uncertainties respectively.

Analysis. Is the relationship between V and I for the bulb a linear one? Use Analyze→Curve Fit to find a function that gives a relationship V(I) that fits your data. Sketch the graph.

Resistor Circuits

Introduction. In this set of exercises, you will investigate several tools (or rules) that can be applied to the analysis of even very complex circuits. Rules for dealing with resistor combinations are often quite useful, as are the more generally applicable Kirchhoff's circuit rules.



Several resistors can be wired in *series* to increase their effective length and in *parallel* to increase their effective cross-sectional area, as shown at the right. For resistors in series, the total equivalent resistance (ratio of V to I) is given as

$$R_{s,eq} = \sum_i R_i \,. \tag{1}$$

For resistors wired in parallel, the equivalent resistance is found from

$$\frac{1}{R_{p,eq}} = \sum_i \frac{1}{R_i} \,. \tag{2}$$

It is often the case that even many simple circuits with only batteries and resistors cannot be analyzed with just the series and parallel combination rules. For these we need the power and generality of Kirchhoff's circuit rules:

1. Voltage Rule (KVL): The sum of the voltage changes (drops or rises) taken around any loop in the circuit is zero, i.e.,

$$\sum_i \Delta V_i = 0 \,. \tag{3}$$

2. Current Rule (KCL): The sum of all currents into (or out of) any junction is zero, i.e.,

$$\sum_i I_i = 0 \,. \tag{4}$$

Procedure. You will need the following:

- Resistors, 10 Ω and 50 Ω (3 ea.)
- Cables (of various lengths) and clips
- batteries and holders to make 3V and 6V sources
- Fluke DMM
- light bulb (6.3V) and holder

I. Resistor Combinations: Measure the resistances of all of your resistors using the DMM configured as an ohmmeter; record the uncertainties (Appendix B) as well. Make sure to keep track of which values go with which resistors.

(a) Series combination: Connect two resistances in series as in Fig. (a) above.

Predict: What will be the series resistance, predicted from the combination rules above? Include an uncertainty (Appendix A, Eq. 8(a)).

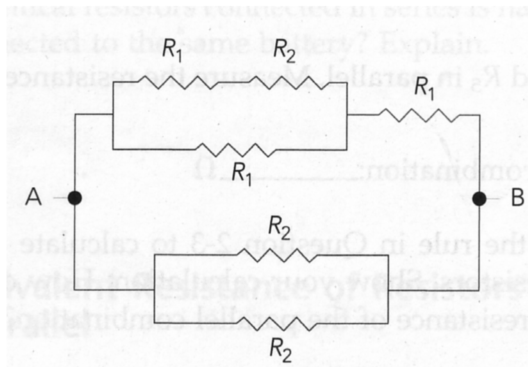
Now use the DMM to measure the series resistance and note the uncertainty. Is your result consistent with the combination rule for series resistances?

(b) Parallel combination: Connect two resistances in parallel (Fig. (b) above).

Predict: What will be the equivalent resistance of this combination?

Again, test your prediction with the DMM and compare.

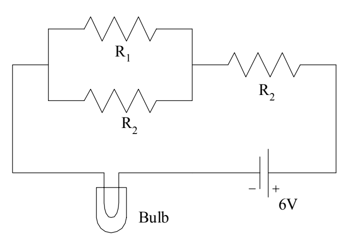
(c) Construct the resistor combination shown below, using your two different values for R_1 and R_2.



Predict: What will be the equivalent resistance between points A and B?

Use the DMM to measure this resistance, and compare with your prediction.

(d) Construct the circuit shown at the right with $R_1 = 10\ \Omega$ and $R_2 = 50\ \Omega$. The bulb should glow.

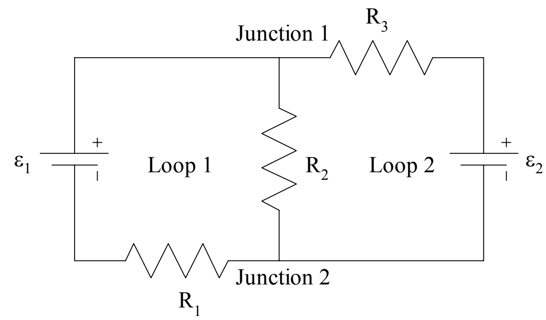


Predict: What will happen to the brightness of the bulb when you
(i) short out R_2 on the right by connecting a cable across its ends?
(ii) disconnect R_1 from the parallel combination?
(iii) short out the parallel combination?

Try each of these actions, returning the circuit to the original condition after each one. Did any of the results surprise you?

<u>NOTE</u>: For all experiments using batteries: Please disconnect them when not in use!

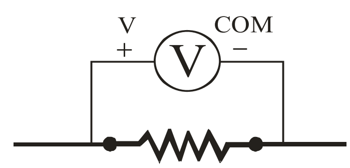
II. Kirchhoff's Laws: Construct the circuit below using $\varepsilon_1 = 3$ V, $\varepsilon_2 = 6$ V, and any combination of values for R_1, R_2, and R_3. Sketch the circuit with the values. Note that the combination rules for series and parallel resistances cannot be applied directly here to give the currents flowing in the wires.



Note: Remember to disconnect the batteries from the circuit when not making measurements!

1. Voltage Rule: Set up the DMM to measure DC voltage (potential difference). Go around Loop 1, measuring and recording the potential difference across each element, carefully noting the polarity and the uncertainty for each. Write the values and indicate the polarities on your sketch.

Note: Differences in potential are measured in parallel as shown at the right—*across* the element (a resistor in this case). The reading will represent the change in potential from the "COM" input to the "V" (+) input. In the case shown, the reading will be *positive* if current flows to the *right* through the resistor.

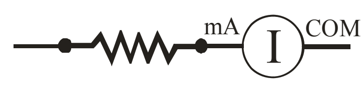


Repeat these voltage measurements for Loop 2.

Are your measurements consistent with the Voltage Rule (KVL), within uncertainties?

2. Current Rule: Set up the DMM to measure DC current[14]. Measure all the currents flowing *into* Junction 1, carefully noting directions (see note below) and uncertainties. Record the values and directions on your sketch.

Note: Currents are measured in series as shown at the right; you will need to "break" the circuit to insert the ammeter. The reading represents the current flowing *into* the "mA" input (towards the "COM" connection point). For the connection shown, the meter will display a *positive* number if the current flows to the *right* (*into* the mA socket). **Caution**: Never connect the meter ***across*** an element (as in the voltage measurement above) while it is set up to measure ***current***!



[14] If necessary, press the yellow button until the "DC" indication appears on the display.

Repeat these current measurements for Junction 2.

Are your measurements consistent with the Current Rule (KCL) to within uncertainties?

Uncertainties. In Appendix A, Eq. (8) and, in particular, Eq. 8(a) for sums, offer some guidance in estimating uncertainties for predicted values of resistances derived from formulas for their combination, if the individual ΔR's are estimated from Appendix B.

For the series equivalent of two resistances: $\Delta R_s = \sqrt{(\Delta R_1)^2 + (\Delta R_2)^2}$.
For the parallel equivalent, the uncertainty in the *reciprocal* of R_p, is given by
$\Delta\left(\frac{1}{R_p}\right) = \sqrt{\left[\Delta\left(\frac{1}{R_1}\right)\right]^2 + \left[\Delta\left(\frac{1}{R_2}\right)\right]^2}$. The uncertainty in each of these reciprocals is
$\Delta\left(\frac{1}{R}\right) \simeq \frac{\Delta R}{R^2}$, which can be substituted for each of the three terms above to find ΔR_p. (The alternative is to grind through the derivatives in Eq. (8).)

Eq. 8(a) will also be useful in estimating the uncertainties in the sums involved in KVL and KCL.

Remember to resist the temptation to keep more than one significant figure for the final uncertainty in any reported quantity.

Conclusion. Do the rules for series and parallel combinations of resistors appear to be valid? Did the light bulb exercise serve to make them clearer conceptually?

RC Circuits

Introduction. In this set of investigations we will explore the behavior with time of the charge on the plates of a capacitor as it charges and discharges through a resistance. As an interesting timing application for capacitors, you will also construct and investigate a relaxation oscillator (flasher).

When Kirchhoff's voltage rule was applied to a capacitance (C) and a resistance (R) in series, we found that the voltage across the capacitor as it *discharges* through the resistor is given by

$$V = V_0\, e^{-t/\tau_C}\,, \tag{1}$$

where V_0 is the voltage at t = 0 and $\tau_C \equiv RC$ is called the capacitive time constant for the RC combination; this constant sets the time scale on which the discharge occurs.

A similar analysis for the *charging* process, in which a battery or other source of emf (V_0) is placed in series with R and C, shows that the voltage is given by

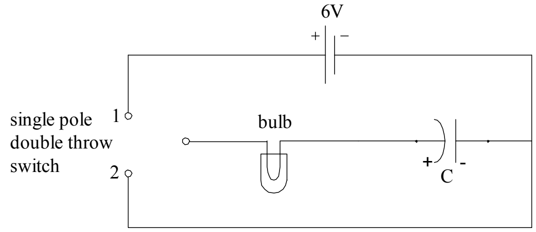
$$V = V_0\left(1 - e^{-t/\tau_C}\right) \tag{2}$$

if it is zero at t = 0. In this case the voltage approaches that of the battery asymptotically as the charging proceeds.

Procedure. You will need the following:

- Laptop with LabPro interface
- current probe
- neon lamp
- differential voltage sensor
- SPDT switch
- Digital multimeter
- stopwatch
- two 1 μF capacitors (250 V)
- Battery holder and batteries for 6V source
- bulb (6.3V) and socket
- 18000 μF capacitance
- wires, alligator clips, spade lugs
- 100–Ω resistor
- 90 V battery source
- two 1 MΩ resistors
- Sencore LC103 Capacitor & Inductor Analyzer

I. Charging and Discharging (qualitative investigation): Connect the switch, the 6.3 V light bulb, the large capacitor, and the 6 V source as in the circuit at the below:



Predict: What will be the behavior of the brightness of the bulb vs. time when the switch is moved to position 1 (charge) and then after a while is moved to position 2 (discharge)?

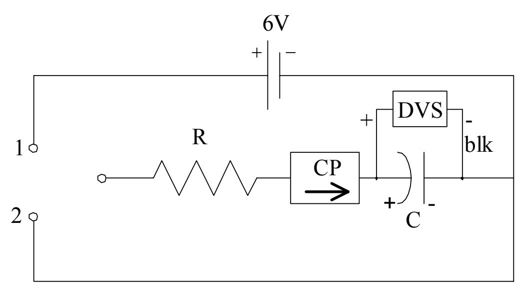
1. Move the switch to the discharge position (2) to drain any residual charge from the capacitor. After at least 30 s, switch it to the charging position (1) and describe what happens to the brightness of the bulb. Make a sketch of *approximate* brightness vs. time in your notebook.

2. After the switch has been in the charging position for a while, move it back to discharge C. Again, describe what happens to the brightness of the bulb and sketch a brightness vs. time curve for this process.

Try to explain your observation in terms of the charge on the capacitor and its motion (i.e., the current) in the circuit.

II. Charging and Discharging (quantitative investigation): Since (as you discovered in the Ohm's law investigation) the resistance of the bulb does not remain constant with varying current, we will replace it with a 100–Ω resistor to do a careful quantitative investigation of how the charge and current vary with time in your capacitor circuit.

Use the DMM to measure R ± ΔR for a 100–Ω resistor and the Sencore analyzer to measure C ± ΔC for the capacitor[15] you used in the bulb observation above. Use these elements to construct the circuit below:



Connect the current probe in series with R and C with the arrow as indicated to measure the capacitor current (dQ/dt); clip the differential voltage probe (DVS) across the capacitor terminals to measure capacitor voltage (proportional to the charge Q on the plates).

Connect the current probe (CP) and the Differential Voltage Sensor (DVS) to Chs. 1 and 2, respectively, of the LabPro. Bring up Logger *Pro* on the laptop and open up the file **Charging and Discharging**. Zero both probes and move the switch to position 2 to discharge the capacitor. Begin collecting data and move the switch to position 1. When the current and voltage stop changing, move the switch back to position 2. Sketch the current vs. time and the voltage vs. time curves in your notebook for both the charging and discharging processes.

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15 ***Always discharge the capacitor before connecting it to the analyzer!***

Analysis:

1. Discharge: Use the Analyze→Examine cursor to determine the time interval between the closing of the switch to position 2 and the reduction of the charge (or the voltage) on the capacitor to 37% of its maximum value obtained when the switch was in position 1. (Another way to look at it: how long does it take for 63% of the change to occur when the switch is moved?) Estimate an uncertainty based on how closely you can read the graph.

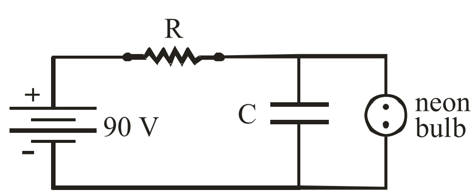
Select the portion of the V vs. t discharge curve where the voltage is changing and try to fit a curve with the form of Eq. (1) above to the data (Analyze→Curve Fit). Note: you will most likely need to "define function" and enter "$t - t_0$" instead of just "t", since the discharge doesn't start at t = 0 on your graph, as assumed by Eq. (1). Comment on the quality of the fit and whether this indicates that the process is exponential. Note the value of τ_C for your fit, along with the uncertainty.

Compare the three values of your time constant τ_C: (a) from the component values, $\tau_C = RC$; (b) from the value of "t" at 63% of the way down on the discharge curve; and (c) from the exponential curve fit. Are all these values consistent, considering uncertainties?

2. Charge: Repeat the analysis for the charging (beginning) portion of the V vs. t curve, comparing the three values of τ_C from the component values, the 63% point and the fit to Eq. (2). If this value is different from that obtained for the charging process, can you think of a reason?

III: Flasher (Relaxation Oscillator): Construct the circuit below with C = 1 μF and R = 1 MΩ. Measure C and R and record the values.

Caution: When building this circuit, make connections to the battery **last**. Disconnect the battery **first** when changing or dismantling the circuit. Avoid contact with both battery terminals simultaneously.



The neon bulb has the property that when the voltage applied between its terminals exceeds a certain value (the *threshold* voltage), the gas in the bulb breaks down and conducts current.

When the connections are completed and the button pressed, the neon bulb should flash periodically[16]. From your knowledge of RC circuits and the behavior of the bulb, sketch a curve

[16] If the bulb doesn't flash, try reversing the polarity of its leads.

that you think represents the voltage across C vs. time. With your stopwatch, measure the time interval T between flashes (average over several intervals!).

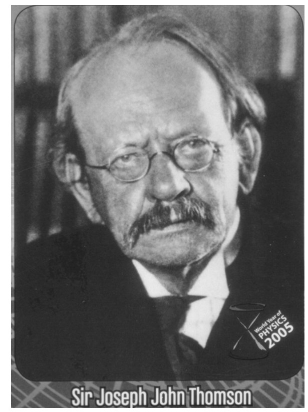
Predict: What will be the new value of T when C is replaced by *two* 1-μF capacitors in series? What assumptions are you making with this prediction?

Make this change (again, *disconnect the battery* when working on this circuit!). Measure the new time interval between flashes. How was your prediction?

Uncertainties. Meter uncertainties for R and C are in Appendix B. Eq. 8(b) will help you deal with uncertainties in the RC time constant.

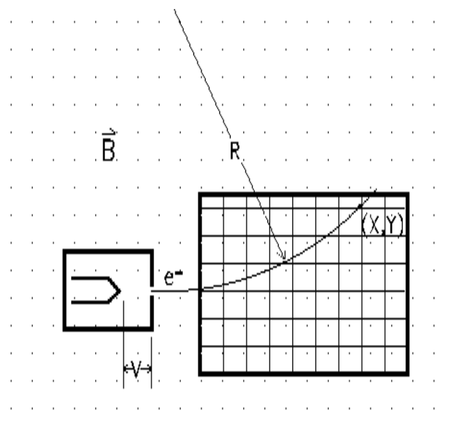
Conclusion. Do your observations indicate that charging and discharging a capacitor through a resistance is an exponential process? Are your results in quantitative agreement with Eqs. (1) and (2) above?

The Charge-to-Mass Ratio of the Electron



Introduction. In 1897, J. J. Thomson, renowned physicist, performed an experiment which established the electron as a fundamental particle and measured the ratio of its charge (e) to its mass (m). Using a variation of his technique, you will do a measurement of e/m by observing the trajectory of speeding electrons as they are deflected by a magnetic field.

The apparatus for the observation of electron trajectories is pictured schematically below:



Electrons are boiled off a heated cathode and accelerated through a potential difference V. They emerge from the exit aperture of the electron gun housing (the anode) in the presence of a magnetic field **B** and are thus deflected into a circular arc of radius R; this arc is made visible by electron's impact against a fluorescent grid. The radius R can be related to the electron mass m, its charge e, the electron speed v, and the field strength B by writing $F_M = ma = mv^2/R$, where F_M is the magnetic deflecting force of magnitude evB. The ratio e/m is then given as

$$\frac{e}{m} = \frac{v}{RB} \tag{1}$$

We can therefore calculate e/m from the measured value of R if we know v. By conservation of energy, the speed with which the electrons emerge from the gun is related to the accelerating voltage V: $\frac{1}{2}mv^2 = Ve$, so that v is given by

$$v = \sqrt{2V\left(\frac{e}{m}\right)} \tag{2}$$

Combining Eqs. (1) and (2) allows us to solve for e/m in terms of measurable quantities:

$$\boxed{\frac{e}{m} = \frac{2V}{B^2R^2}} \tag{3}$$

The quantities V, R, and B will be measured using your apparatus:

1. The *radius R* is measured by recording the coordinates (x, y) of some point on the arc as far as possible from the exit aperture. Since the electron beam emerges horizontally from the anode, located at the origin of the coordinate grid, the radius is given in terms of x and y as

$$R = \frac{x^2 + y^2}{2y} \tag{4}$$

2. The *magnetic field **B*** is created by a pair of "Helmholtz coils," and is nearly uniform in the region of the electron beam. Its magnitude in Teslas can be calculated from the measured current I in the coils:

$$B = (4.23 \times 10^{-3} \left[\tfrac{T}{A}\right])\, I \tag{5}$$

3. The *accelerating voltage V* is read directly from the meter (top scale) on the high voltage supply.

Procedure. You will need the following:

- e/m apparatus
- coil current power supply
- desk lamp
- laptop
- compass
- HV power supply
- DMM (for coil current measurement)

The apparatus at your bench should already be completely wired; do not alter the wiring unless asked specifically to do so by your instructor.

With the coil current off, align the apparatus using the compass so that the earth's magnetic field is parallel to the plane of the coils; this prevents it from contributing to the beam deflection.

1. Turn on the high voltage supply and the current supply for the coils; observe the trajectory of the electron beam and notice that the radius of curvature depends on both the accelerating voltage V and the current I through the coils.

2. Choose a convenient point (x,y) on the grid as far as possible away from the origin (e.g. (10, 2.6)) so that, with V set at 5000 V, the beam can be made to go through this point with the coil current available[17]. **Do not exceed 400 mA of current through the coils!** Compute the radius R for the point you have chosen, using Eq. 4. Note that since the grid coordinates are in cm, so is your computed value of R.

3. For 5 values of V between 2500 V and 5000 V, record V and the value of the coil current I that causes the beam to pass through the point (x, y) that you chose. Make sure, in each case, that reversing the direction of I (by reversing the leads) causes the beam to go through (x, −y). If the deflection is *not* symmetrical, then you will have to average the current required to produce deflections through (x, ±y). Make a chart in your notebook, like the one that follows, to record your data:

$V \pm \Delta V$	$I \pm \Delta I$	$B \pm \Delta B$	$e/m \pm \Delta(e/m)$
1.			
2.			
3.			
4.			
5.			
			$\overline{e/m} \pm \Delta\left(\overline{e/m}\right) =$

Fill in the first two columns with your measurements of V and I together with their respective uncertainties (see below).

Analysis.

1. Calculate the magnetic field B produced by each current I by using Eq. 5.

[17] To read the current, the DMM must be in DC mode (yellow button).

2. Calculate a value of e/m for the electron for each of the 5 combinations of V and I, using Eq. 3, and record these values in the rightmost column of your chart. Compute an average value of e/m, along with an uncertainty see below).

Uncertainties.

The uncertainty in the measurements of V can be taken as the uncertainty in the meter reading.

The uncertainty in the current I can be considered as arising from the width of the beam. Take ΔI as half of the current range for which some portion of the beam passes through the chosen grid point.

The propagated uncertainty in B is given in terms of ΔI, according to Eq. 5 and Appendix A, by ΔB/B = ΔI/I.

Assume no uncertainty in the radius R of the circular arc.

The uncertainty in the single measurement of e/m calculated from each combination of V and I is given by Eq. 3 and Appendix A as

$$\frac{\Delta\left(\frac{e}{m}\right)}{\left(\frac{e}{m}\right)} = \sqrt{\left[\frac{\Delta V}{V}\right]^2 + \left[2\frac{\Delta B}{B}\right]^2} \tag{6}$$

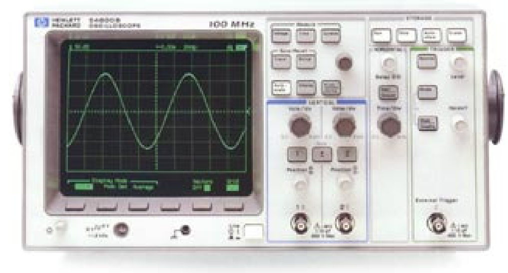
but recall that the uncertainty in the *average* value of e/m will be less than that for a *single* measurement. If the uncertainties in the single measurements (Δ(e/m)) are all the same to one sig fig, then the uncertainty in the *average* of n such values would be $\Delta(\overline{e/m}) \simeq \frac{\Delta(e/m)}{\sqrt{n}}$, according to Eqs. 6 & 7 of Appendix A.

Conclusion.

Report your experimental value of e/m, together with your estimated uncertainty.

Calculate the "accepted" value of e/m using accepted values for e and m. Determine whether this value and your experimental value are equal to within your experimental uncertainty.

The Oscilloscope



Introduction. So far you have used the LabPro/Logger *Pro* combination to observe and record voltages as a function of time. This method of observation, however, is limited in terms of the range of voltages and frequency (or time scale of the variations) that may be explored. In science and engineering laboratories, an *oscilloscope* is used to observe graphs of voltage vs. time (i.e., the *waveform*) for periodic signals with a much wider range of voltage and frequency. In this lab exercise you will have an opportunity to practice using the HP 54600B Digital Oscilloscope (pictured at the right) to study signals from a *function generator*—a device that outputs a variety of voltage waveforms with adjustable frequency and amplitude. The goal of this exercise is to prepare you to use these instruments for measurements to be performed in the coming weeks.

Procedure and Analysis. You will need the following:

- HP 54600B Oscilloscope
- HP200AB signal generator
- cables

Connect the ground ("G") terminal of the "600 Ω" output of the function generator to the GND terminal below the screen on the scope; the other terminal at the far right should be connected to the input to channel 1 on the scope. Turn on the scope and the generator and adjust the Volts/div, Time/div and TRIGGER knobs until a stable trace is obtained. If you have trouble getting a stable trace, push the white "Autoscale" button to the right of the screen and press the "2" button to turn off that unused input if two traces appear. If your trace is still jittery, press the "Source" button in the trigger area and select "1" as the trigger source[18].

Voltage: The *peak-to-peak voltage* (V_{pp}) of the wave may be measured in three different ways:

(a) Analog. The vertical voltage scale can be adjusted by the Volts/div knob for the active channel and is indicated in the upper left hand corner of the screen. This number is multiplied by the number of big (1 cm) divisions that span the waveform vertically from peak to peak to give V_{pp}.

---

[18] The scope takes its cue to begin a sweep from the "trigger" signal, which in many cases is best taken from the input itself. Note also that pressing "Autoscale" generally brings up signals from both channels, in which case it will be necessary to press "2" until the channel 2 signal (which is just noise in this case) goes away.

(b) Digital (cursors). Push the gray "Cursors" button to the right of the screen and select V1 and V2 as active cursors with the "softkeys" at the bottom of the screen, respectively aligning the cursors (by adjusting the small gray knob) with the upper and lower peaks of the wave. V_{pp} is then read directly as ΔV below the trace.

(c) Digital (direct readout). Pushing the gray "Voltage" button and selecting V_{p-p} with the softkeys gives a direct V_{pp} reading below the trace.

Time (period and frequency): The *period* of the wave is the amount of time elapsed between peaks (or between any two corresponding points on successive cycles). The frequency is given as the reciprocal of the period: $f = \frac{1}{T}$. As with the voltage, there are three ways to make time measurements:

(a) Analog. The horizontal time scale can be adjusted by the Time/Div knob and is indicated at the top of the screen (right of center). The period is the product of the time scale and the number of divisions spanning one period of the wave horizontally.

(b) Digital (cursors). Press the "Cursors" button and measure the time interval between peaks with a procedure analogous to (b) under Voltage.

(c) Digital (direct readout). Pushing the gray Time button and making menu selections can get you a direct readout of both period and frequency.

Measurements: Fill in the chart below for some different frequency settings of the function generator, with the amplitude dial on the highest setting that allows you to see the entire signal on the scope screen. The frequency setting of the generator is read as the product of the number on the dial and the power of ten selected by the "range" button. For columns with a "*" at the top, determine the quantity using *all* of the techniques discussed; unstarred columns refer to analog measurements.

NOTE: Every student at the lab table should operate and read the scope for the completion of at least one row of the chart.

Generator Setting (Hz)	**# of boxes for a period**	**Time Scale**	**Period***	**Frequency***	**# of boxes for V_{pp}**	**Voltage Scale**	**V_{pp}***
200							
800							
2000							
8000							
20,000							

Uncertainties. For the analog readings, the suggested uncertainty is one-half the smallest division on the scope. (Note that each 1-cm box is divided both horizontally and vertically into smaller divisions) This means that the relative uncertainties for scope measurements can be minimized by choosing voltage and time scales that *utilize as much of the screen as possible.*

For the direct digital readouts, an estimate of the uncertainties can be gleaned from the size of the fluctuations in the value of the quantity being measured. In the absence of fluctuations, the number of significant figures in the readout can be your guide. For cursor measurements, the uncertainty has to do with the "play" in the cursor position—how precisely can you position the cursor? The manufacturer's claims regarding uncertainties are listed in Appendix B. These should be used if they are larger than those determined by other means.

Conclusion. Are the results for period, frequency and peak-to-peak voltage, as derived from the various methods, consistent? How reliable is the dial on the function generator for setting the frequency of its output?

Magnetic Field Mapping

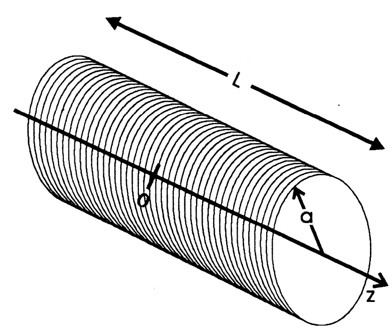
Introduction. The Biot-Savart law allows us to calculate the magnetic field **B** at any point due to any distribution of steady or slowly varying currents. In this experiment we will test the predictions of the law by measuring and plotting the magnetic field due to three common current configurations: the long straight wire, the solenoid, and a pair of Helmholtz coils. In each case you will observe how the magnetic field varies with position near the current and compare it with the theoretical prediction.

The probe we will use to measure the magnetic field consists of a tiny coil wound around a magnetic core. The currents that produce the magnetic fields you will study are rapidly varying and sinusoidal, so that they produce a rapidly changing magnetic flux through the probe coil. Faraday's law predicts that, for a fixed frequency, the peak-to-peak voltage produced between the ends of the coil leads will be proportional to the peak-to-peak magnetic field being produced by the current at the location of the probe coil. As the probe is moved, this voltage can be read on an oscilloscope as a *relative* measure of the field strength at different locations. A full understanding of the physics of the probe will have to await our discussion of Faraday's law, AC tuned LRC circuits, and magnetic materials.

1. Long Straight Wire. The magnetic field from a long straight wire carrying a current can be calculated either from the Biot-Savart law or from Ampère's law. The result for the magnitude of the **B** field at a point located a distance r from the wire is

$$B = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{r}. \tag{1}$$

The field vectors are in a plane perpendicular to the wire and point tangent to circles concentric with the wire.



2. The Solenoid. The solenoid consists of a wire wound in a tight helix around a cylindrical form. This is closely approximated by the arrangement of contiguous circular current loops shown at the right. The z-component of the magnetic field for points along the axis of a solenoid of length L, radius *a,* consisting of N turns each carrying a current I is given by the Biot-Savart law as

$$B_z(z) = \frac{\mu_0 N I}{2L}\left(\frac{\frac{L}{2} - z}{\sqrt{a^2 + \left(\frac{L}{2} - z\right)^2}} + \frac{\frac{L}{2} + z}{\sqrt{a^2 + \left(\frac{L}{2} + z\right)^2}} \right) \tag{2}$$

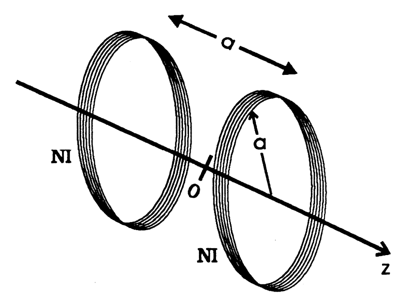
with z = 0 at the center of the solenoid. Since our probe only measures the *relative* strength of the **B** field for different locations, we can convert Eq. (2) into an expression for the relative field strength B_{rel} by dividing both sides by $B_z(0)$, the field strength at the center. It is also convenient to express the result in terms of the dimensionless coordinate $Z \equiv \frac{z}{L}$.

The result for B_{rel} is

$$B_{rel}(Z) \equiv \frac{B_z(Z)}{B_z(0)} = \sqrt{4\rho^2+1} \cdot \left(\frac{1-2Z}{\sqrt{(4\rho)^2+(2-4Z)^2}} + \frac{1+2Z}{\sqrt{(4\rho)^2+(2+4Z)^2}} \right), \qquad (3)$$

where the ratio of the radius of the solenoid to its length, represented above as $\rho \equiv \frac{a}{L}$, determines how B_{rel} varies with distance from the center. The smaller ρ is for a solenoid (i.e., the more "infinite" it is), the more uniform the field is along the length inside the windings and the more rapidly it diminishes outside. (So for values of ρ approaching zero, B_{rel} in the above expression becomes 1 for all values of Z.)

3. Helmholtz Coils. When a uniform magnetic field is required over an accessible region of space in the laboratory, *Helmholtz coils* are often used, as they were in the e/m experiment. This arrangement, shown at the right, consists of two thin circular coils mounted so that they are parallel and separated by a distance equal to their common radius *a.* Each coil has N turns that each carries a current I. For this configuration, the axial (z-) component of the magnetic field is given by the Biot-Savart law as



$$B_z(z) = \frac{\mu_0 N I}{2a} \left[\frac{1}{\left(1+\left(\frac{z}{a}+\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(1+\left(\frac{z}{a}-\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} \right], \qquad (4)$$

where z = 0 halfway between the centers of the two coils.

As for the solenoid, it is more convenient to express the B-field strength relative to its magnitude midway between the coils in terms of the dimensionless position $Z \equiv \frac{z}{a}$:

$$B_{rel} \equiv \frac{B_z(Z)}{B_z(0)} = \left(\frac{5^{3/2}}{16}\right) \cdot \left[\frac{1}{\left(1+\left(Z+\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(1+\left(Z-\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} \right]. \qquad (5)$$

Procedure. You will need the following:

- B-field mapping apparatus
- Long straight wire
- Helmholtz coils with probe guide tube
- Vernier calipers and straight edge
- HP model 200AB audio oscillator
- oscilloscope
- laptop/Logger Pro
- 500:4 impedance matching transformer

1. Long Straight Wire. Plug the signal generator output into the "500" terminals of the transformer. Attach the probe to the oscilloscope input and attach the long straight wire to the "4" terminals of the transformer. Try to keep the transformer and other magnetic materials away from the probe. With the sine wave frequency setting of the generator (audio oscillator) around 32 kHz, "tune" your apparatus by adjusting the frequency carefully so that your signal is maximized while the probe is held in a fixed position near the wire. Keep the generator set to this frequency for the entire experiment. Note that the amplitude of this signal is proportional to the amplitude of the magnetic field at the probe.

Examine the signal from the long wire with your probe by pointing the probe axis in various directions (keeping the distance from the tip of the probe to the wire constant!). Note the probe alignment that gives the strongest signal. What does this tell you about the direction of the B-field from the wire (given that the signal is maximum when the direction of the field is parallel to the probe axis)?

With the probe aligned for maximum signal[19], record the voltage peak-to-peak amplitude as given by the scope for at least ten different distances[20] r between the probe axis and the wire (e.g., from 2 cm to 10 cm in steps of 0.5 cm). You can explore as wide a range of distances as your apparatus will permit, being careful to keep the distance of your probe from other parts of the wire circuit large compared to its distance from the straight portion of the wire. Tabulate your data.

2. Solenoid. Measure and record the diameter (2*a*) and length (L) of the solenoid windings. Mount the probe axially with respect to the solenoid on the field mapping apparatus and connect the solenoid windings to the "4" terminals of the transformer, which is connected to the audio oscillator. Working backward from the midpoint[21] of the windings at z = 0, tabulate the peak-to-peak signal strength vs. distance (z) from the center for the range of distances allowed by the probe mount. Take data every half cm or so at least for $0 \le z \le L$.

3. Helmholtz Coils. Measure the distance *a* (which is both the separation between coils and their common radius); connect the coils to the "4" side of the transformer. Mount the probe in the transparent tube

---

[19] The probe should already be mounted in the clamp so that a horizontal line drawn perpendicular to the vertical wire that extends through the center of the small probe coil is also perpendicular to the probe axis. No adjustments of the probe seating should be necessary.

[20] Measure the first value of r directly from the wire to the center of the probe and note the reading on the reference scale; you can use the reference scale for the rest of the measurements.

[21] The position of the center of the solenoid windings may be determined as follows: Note the V_{pp} reading near the center and advance the probe until the reading decreases by, e.g., 20% and record the V_{pp} reading and the probe location at this point. Back the probe off to the other side of the center until the *same* V_{pp} *reading* is obtained and again note the position. The average of these two positions is the center of the solenoid.

along the axis of the coils and, again starting at the center (z = 0), tabulate the peak-to-peak signal strength vs. distance from the center for the range of distances allowed by the axial mounting tube.

Analysis:

1. Long Straight Wire: Use **LoggerProFile.cmbl** to plot the peak-to-peak signal strength (on the y-axis) vs. the distance (r) between the probe and the wire (x-axis). Use Analyze→Curve Fit to try to fit the data to a function of the form[22] A*r^B. Is the result consistent with Eq. (1)? Explain. Sketch the plotted data and best fit curve in your notebook.

2. Solenoid: Use your data to tabulate $B_{rel} \equiv \frac{B(Z)}{B(0)} = \frac{V_{p-p}(Z)}{V_{p-p}(0)}$ for your values of Z ≡ z/L in Logger *Pro*. Try to fit your data (Analyze→Curve Fit→Define Function) to the Biot-Savart expression for B_{rel} in Eq. (3) to determine the best fit value of the parameter $\rho \equiv \frac{a}{L}$ for your solenoid.[23] (NOTE: You will have to make sure that the variable in your curve fit expression for the horizontal coordinate is the same as the name—both "Name" and "Short Name"—of the corresponding data column!) Calculate ρ from the measurements on your solenoid and compare to the fit value. Sketch the plotted data and best fit curve in your notebook.

3. Helmholtz Coils: Tabulate your *experimentally determined* B_{rel} vs. Z ≡ z/a and use Logger *Pro* to examine the agreement between your data and the *theoretical* B_{rel} of Eq. (5). As you did for the solenoid, you can use Analyze→Curve Fit→Define Function, but select "Manual" fit (since there are no adjustable parameters in the theory) and enter the expression in Eq. (5). Sketch the data and theoretical curve in your notebook and comment on the agreement.

---

[22] Note that if there is a substantial amount of noise that does not depend on distance from the wire, it may be advantageous to try and fit the V_{pp} vs. r curve to an expression like A*r^B + C; the fitted value of C then represents the constant value of the "pickup" noise. Try it both with and without the "C."

[23] For the solenoid and Helmholtz coil data, you may find it very advantageous to compose your expression in a word processor like Notepad (Start→Accessories) and paste it into the Define Function dialog box. To start you off, the form for the expression of Eq. (3) is:

```
sqrt(4*rho^2+1)*((1-2*Z)/(sqrt((4*rho)^2+(2-4*Z)^2))+(1+2*Z)/(sqrt((4*rho)^2+(2+4*Z)^2)))
```

Conclusion.

Does your data for the long straight wire support the behavior of the field with distance predicted by theory? Be explicit as to the basis for your answer.

Are the data for the solenoid and the Helmholtz coils consistent with the Biot-Savart law?

What did you learn in this lab about magnetic fields produced by current distributions?

The Earth's Magnetic Field

Introduction. The direction and magnitude of the earth's magnetic field varies from place to place, as can be seen from Fig. 1. In this experiment you will use Faraday's law of induction and a spinning coil apparatus to measure the horizontal and vertical components of the earth's B-field at your location.

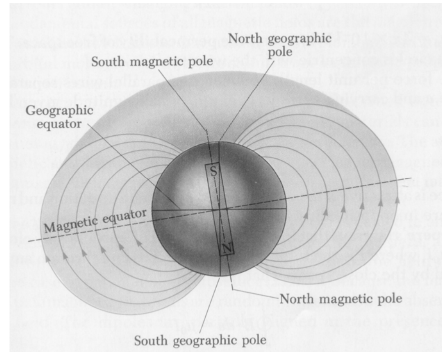


Figure 1

If a coil of cross-sectional area A with N turns of wire is set spinning about a vertical axis with angular speed ω as shown in Fig. 2, then the horizontal component of the earth's magnetic field (B_{eh}) will produce a time-changing flux Φ_m through the coil:

$$\Phi_m = N B_{eh} A \cos\theta , \qquad (1)$$

where θ is the angle that the compass needle makes with the normal to the plane of the coil; the vertical component B_{ev} of the earth's field produces no flux (why?). Since the instantaneous rate at which θ changes is ω, Faraday's law gives the induced emf ε(t):

$$\varepsilon(t) = -\frac{d\Phi_m}{dt} = N B_{eh} A\omega \, \sin\omega t , \qquad (2)$$

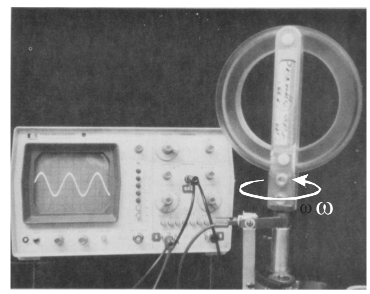


Figure 2

if we assume that θ = 0 at t = 0. We can express ε(t) in terms of the quantities you will measure by noting:

(i) The peak-to-peak emf, ε_{pp}, that you will measure on the oscilloscope, is just twice the peak value of ε(t) in Eq. (2) (achieved when $\sin\omega t = 1$): $\varepsilon_{pp} = 2 N B_{eh} A\omega$.

(ii) The angular speed ω is given from the period read on the scope as the time between successive peaks of the sine wave: $\omega = \frac{2\pi}{T}$.

(iii) The effective area A of the coil is given in terms of the *average* diameter $\bar{D}$ of the windings: $A = \frac{\pi \bar{D}^2}{4}$.

Substituting all of this back into Eq. (2) and solving for B_{eh} gives

$$\boxed{B_{eh} = \frac{\varepsilon_{pp} T}{N \pi^2 \bar{D}^2}} . \qquad (3)$$

The vertical component of the field, B_{ev}, can be measured in similar fashion with the rotation axis aligned parallel to the N-S direction (the direction of $\mathbf{B}_e$ in the horizontal plane); in this orientation it is the horizontal component B_{eh} that produces no flux through the coil. The expression for the measured B_{ev} is then identical to that for B_{eh} in Eq. (3).

Procedure. You will need the following:

- Spinning coil apparatus (coil+rotator)
- oscilloscope
- straight edge with cm scale
- bubble level
- compass
- shielded cable

Measure the average turn diameter $\bar{D} \pm \Delta\bar{D}$ of the coil windings; the value of N should be marked on the coil (no need to count). Align the rotation axis vertically as well as possible using the bubble level. Connect the cables from the ground and the coil output to the corresponding oscilloscope inputs.

NOTE: The spinning coil is dangerous. Exercise caution!

Set the coil spinning, adjust the rotator period to around 50 ms, and observe the voltage signal on the scope. Make sure you can distinguish the signal from the noise: the signal goes away when you turn off the rotator or short the leads at the coil! Also note that this noise can be minimized by increasing the separation between the scope and the rotator. If you wish, put the scope in single sweep mode by pressing "MODE" and then the "single" softkey. This freezes a single sweep on the screen for measurements.

Sketch the $\varepsilon(t)$ vs. t curve displayed on the scope. Measure and record ε_{pp} and T for the sinusoidal emf produced by the spinning coil in this orientation. Also estimate the uncertainties in these quantities (see below). Note that you will need to make these measurements by eye using the grid on the scope screen; the signal is much too noisy to expect good results from the digital values the scope prints on the bottom of the screen.

Use the compass and the bubble level to realign the coil axis so that it is horizontal and points north-south. Repeat the measurements of the emf and the period for the displayed signal and estimate their uncertainties.

Analysis. Use Eq. (3) to calculate B_{eh} and B_{ev} at your lab station. Also calculate the magnitude and *inclination* or *dip angle* θ_{dip} (angle below the horizontal) for $\mathbf{B_e}$.

Uncertainties. The noise on top of the signal should help you estimate uncertainties for both the peak-to-peak emf and the period. Estimate the uncertainty in the turn diameter from the precision of the cm scale.

For the components of $\mathbf{B_e}$, the form of Eq. (3) suggests (using Eq. 8(b) in Appendix A):

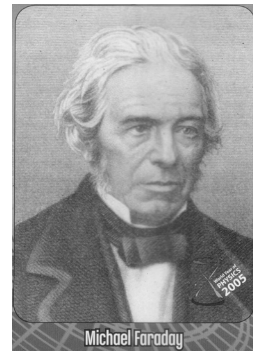
$$\frac{\Delta B_{eh,v}}{B_{eh,v}} = \sqrt{\left(\frac{\Delta\varepsilon_{pp}}{\varepsilon_{pp}}\right)^2 + \left(\frac{\Delta T}{T}\right)^2 + \left(2\frac{\Delta\bar{D}}{\bar{D}}\right)^2} \quad (4)$$

You can calculate the uncertainties in the magnitude and dip angle for $\mathbf{B_e}$ by using Eq. (8) in Appendix A along with ΔB_{eh} and ΔB_{ev} given by Eq. (4), or simply use ΔB_{eh} and ΔB_{ev} to estimate the range in B_e and θ_{dip}.

Conclusion. The National Geophysical Data Center gives values for the earth's magnetic field at different locations (http://www.ngdc.noaa.gov/seg/geomag/magfield.shtml). For Chico on 7/12/2004, the magnitude of B_e was 50.79 μT and changing at the rate of −68.58 nT/year. The inclination was 63.12° and changing by −0.026°/year.

Do your measured values of θ_{dip} and the magnitude for $\mathbf{B_e}$ agree with this data to within your uncertainties? Are there factors, other than underestimated uncertainties, that would account for a disagreement?

Electromagnetic Induction Laboratory



Introduction. Faraday's law states that there is an induced emf whenever there is a changing flux: $\mathcal{E} = -\frac{d\Phi_m}{dt}$; Lenz's law helps to predict the sense (direction) of the emf. This apparently simple statement has a multitude of ramifications that are crucial to our understanding of the current electromechanical technology. The purpose of this lab is to present to you several working examples of devices and situations that are tied together with the common themes of changing flux and Lenz's law.

Procedure. You will need the following:

- 9 lab stations, as described below

Lab groups will rotate together among the 9 lab stations, as directed by the instructor. For each station, write a description of your observations and a careful *explanation*, using fundamental principles and clear sketches correctly showing polarities, currents, fields, forces, etc.

NOTES:
1. Please make sure that all power supplies and batteries are disconnected or turned off when not in use!
2. Some of the meters require a button to be pressed in order to read a voltage/current.

Station #1 includes a permanent bar magnet that can ride a toy car down a ramp through a coil. The emf induced across the ends of the coil can be read with a differential voltage sensor (DVS) connected to a LabPro. The file C:\Dietz's 204B Labs\Electromag Ind Lab\Induced EMF can be used in Logger Pro to collect emf vs. time data. (You can use Experiment→Store Latest Run to compare data curves.) Don't let the car hit the floor.

Predict: Sketch a graph of the emf vs. time as the magnet speeds through the coil. Collect data and test your prediction.

Predict: What will be the effect of changing the height of release for the car on the emf vs. time curve? Test.

Predict: What happens if the car is turned around so the magnet's poles are reversed? Test.

Station #2 is a permanent magnet with a wire or a 1000-turn coil connected to a galvanometer. The wire should be moved rapidly through the magnetic field in various ways. What happens if the wire is wrapped to form a coil with a few turns? How do the effects produced by the wire and the 1000-turn coil compare?

Station #3 has two aluminum pendulums, identical except that one is slotted, to swing between the poles of a large permanent magnet. Note the marked difference in damping, and try pushing them rapidly through the field. Notice an identical device on the triple beam balance.

Station #4 is the ring toss apparatus with a coil and light. Note the different heights to which the various rings jump. Note that one is slotted. Try holding one ring slightly above another floating one, and note the interaction. Try the coil and light at various positions. You need only explain the behavior of the light.

Station #5 has a dry cell which can be connected briefly and then disconnected from the primary windings of two concentric coils. The secondary coil is connected to a galvanometer. Note when you do and do not get deflections on the galvanometer, and the directions and magnitudes of these deflections. Try it with the iron core in and out of the tube.

Station #6 is a dissectible transformer with a coil connected to a neon bulb. The other coil is to be connected briefly to a dry cell. Observe when the bulb flashes and note that you can determine which side of the bulb glows and change it by changing the polarity. Try removing the top bar of the rectangular iron core both with and without the dry cell connected to the coil.

Station #7 has a coil of wire which can be rotated between the poles of permanent horseshoe magnets. Wires are connected to a neon bulb and also to a galvanometer. At which positions do the moving wires produce the largest currents?

Station #8 has a coil that can be rotated between iron poles which are magnetized by a stationary coil. The stationary coil is connected to a power supply when in use. Brushes can be changed to allow for alternating or direct current situations. The movable coil can be rotated by hand with connections to a galvanometer for use as a generator, or the coil can be connected to a power supply to be run as a motor. (Note: Connect the momentary key switch in one of the lines to the power supply to avoid continuous use of power.)

Station #9 is an aluminum tube and a plastic tube. A strong magnet can be dropped either tube. Notice the differences in falling times. (Caution: Keep the magnet away from other magnets and magnetic materials!)

RL Circuits

Introduction. In this lab you will investigate the way the current through a coil changes when a sudden (step) change is made in the applied voltage across its terminals. You will also observe the AC behavior of the current through the coil and observe its action as a frequency filter.

Inductors, by means of the emf induced across their terminals, act to oppose sudden changes in the current they carry; they thus tend to slow down changes in current brought about by sudden changes in a circuit. In particular, if the emf across an RL series combination carrying a steady current I_0 is suddenly brought to zero, the current decays gradually (not suddenly) due to the presence of the inductance L:

$$I(t) = I_0\, e^{-t/\tau_L} \qquad (1)$$

where t = 0 when the change is made and $\tau_L \equiv \dfrac{L}{R_T}$ is the *inductive time constant* of the circuit. The resistance R_T includes any resistor in series with L as well as the resistance of the inductor coil itself. If an emf is applied suddenly to the same circuit, the buildup of current is again gradual, rather than sudden, and is characterized by the same time constant:

$$I(t) = I_{max}\left(1 - e^{-t/\tau_L}\right). \qquad (2)$$

If, instead of a step change, a sinusoidal (AC) signal voltage with amplitude ε_m is applied across the series resistor-inductor combination, the amplitude of the current (I_m) through both elements is given as

$$I_m = \left(\frac{\varepsilon_m}{R_T}\right)\left(\frac{1}{\sqrt{1+\omega^2\tau_L^2}}\right), \qquad (3)$$

where R_T is again the *total* resistance in the series combination. Additionally, the current and signal voltage are not in phase, but rather exhibit a phase difference ϕ that is a function of frequency:

$$\phi = \tan^{-1}\left(\omega\tau_L\right). \qquad (4)$$

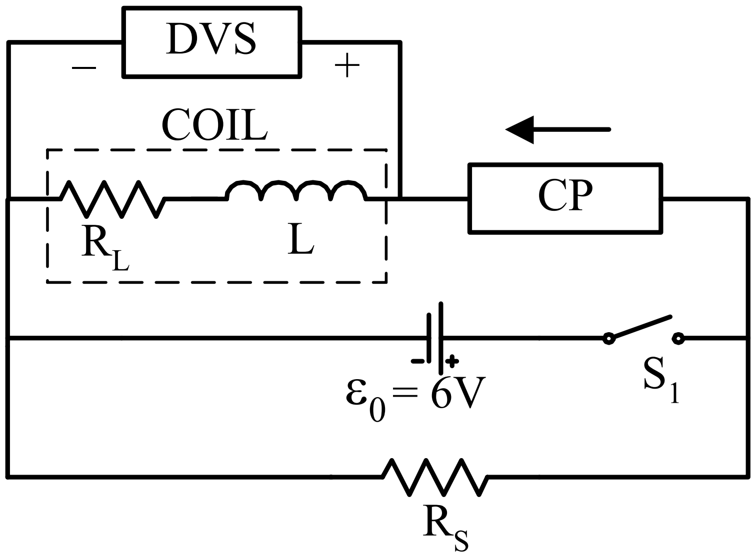
Procedure. You will need the following:

- laptop/LabPro/Logger *Pro*
- differential voltage sensor (DVS)
- current probe (CP)
- large inductor (about 700 mH)
- cables and alligator clips
- HP200AB audio oscillator
- oscilloscope
- 6V battery supply
- Sencore "ReZolver"
- digital multimeter
- momentary switch (S_1)
- impedance matching transformer
- 100 Ω resistor (R_S)

I. Step Changes: Use the Sencore "ReZolver" to measure the inductance L of the coil[24] and the DMM to measure the coil's resistance R_L and the resistance R_S of the 100-Ω resistor. Record these values along with their uncertainties (Appendix B). If values are written on the coil itself, don't believe them!

[24] Zero the test leads as described on p. 1 of the rollout instructions and then perform the out-

Construct the RL circuit shown below; observe the marked polarities and the direction of the current probe (CP) arrow. Connect the current probe (CP) and the Differential Voltage Sensor (DVS) to Chs. 1 and 2, respectively, of the LabPro. Plug in the LabPro power supply and plug the LabPro into the laptop's USB port.



Bring up Logger *Pro* and open the C\Dietz's 204B Labs\RL Circuits**RL Circuit Current Decay** file from the desktop. With S_1 open, zero both probes.

Note that after S_1 has been closed for a while, a constant current ($I_0 = \varepsilon_0/R_L$) flows through L and R_L. When it is opened, the emf is taken out of the circuit and this current starts to decay through R_S.

Close S_1 by pressing firmly on the key for a couple of seconds so that a steady current flows through the RL combination; the live readout for the voltage probe should read close to the battery voltage. Start the data collection and then quickly release the switch after the program indicates it is waiting. (Note that, for this file, data collection is delayed until the program detects a decrease in voltage sensed by the DVS). You may wish to use the Autoscale tool to make the features of the I vs. t curve more visible. Make sketches of the voltage and current vs. time in your notebook.

Analysis. Does Eq. (1) adequately describe the decay of the current through the inductor when the emf is removed? Try a curve fit of the decay data to the function $I(t) = Ae^{-t/\tau_L}$ with A and τ_L as free parameters to be determined for the best fit. (Be sure to select only the decaying portion of the curve.)

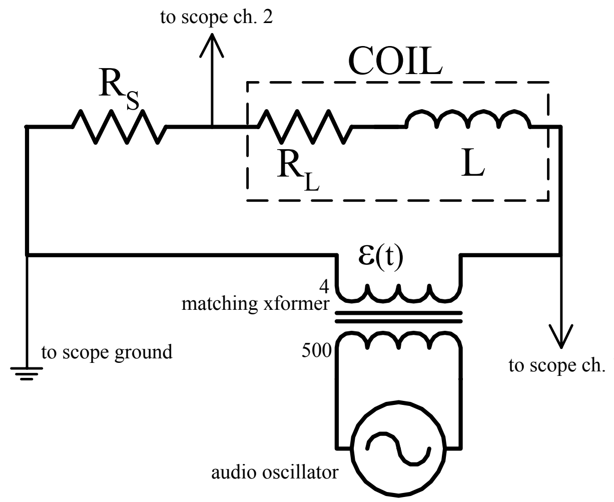
---

of-circuit inductance test described on p. 7, #1. When measuring inductance with this meter, make sure the scope and oscillator are OFF.

Compute the inductive time constant for this circuit from the values of L, R_L and R_S measured with the meters (see below for the uncertainty in this "theoretical" value of τ_L). Compare this value to the time constant obtained from the curve fit to I(t).

Close Logger *Pro* and disconnect the LabPro from the USB port of your laptop.

II. <u>Response to AC Applied Voltage</u>: To study the current that flows when a source of sinusoidal emf is applied to an RL series combination, construct the circuit shown below. The signal on Ch. 2 is the voltage across R_s, which by Ohm's law is proportional to the current I.



(a) Current Amplitude: Turn on the scope and audio oscillator (it needs a few seconds to warm up) and adjust the scope for a stable trace using ch. 1 as the trigger source. Make sure both inputs are DC coupled[25]. Adjust the time base and vertical scale so a few cycles of both signals are visible on the screen. Ch. 1 displays the oscillating emf ε(t) presented to the series RL combination and ch. 2 displays the voltage across R_S, which is directly proportional to the current I(t) through the inductor.

<u>Predict</u>: What will happen to the amplitude V_{mR} ($\frac{1}{2}V_{pp}(2)$ on the scope) of the voltage across R_S if the oscillator is adjusted slowly from lower frequency settings toward higher frequencies? (Look at Eq. (3).) Note that the amplitude ε_m of the emf ε(t) is supposed to be constant as the frequency is changed.

Set the oscillator[26] at about 500 Hz and set up the scope to measure frequency and peak-to-peak voltage for both channels using the "measure" buttons and the softkeys below the screen. Adjust the amplitude setting

[25] To DC-couple an input, press its numbered button on the "VERTICAL" section of the panel and use the softkeys below the screen to select DC coupling. This reduces the amount of extraneous capacitance introduced at the scope input.

[26] Try to maximize the distance between the inductor and the scope, oscillator and ReZolver, as the lengths of your cables permit.

on the oscillator to the smallest amplitude that gives a readable signal. (Check to see that you will be able to maintain ε_m at this level for all frequencies from 50 Hz to 500 Hz.)

Tabulate (1) the frequency, (2) $V_{p\text{-}p}$ for the ch. 2 voltage (this is $2V_{mR}$—proportional to the inductor current amplitude) and (3) $V_{p\text{-}p}$ for the ch.1 voltage (the oscillator signal $2\varepsilon_m$) as the oscillator is adjusted from about 500 Hz to 50 Hz in steps of 50 Hz. If necessary, slightly adjust the amplitude of the oscillator output to maintain a constant ε_m each time the frequency is adjusted.

Analysis: Test the prediction of Eq. (3) opening **LoggerProFile.cmbl** and entering the $V_{p\text{-}p}$ (ch.2) (y-axis) vs. frequency[27] data and trying to fit a curve of the form $\frac{A}{\sqrt{1+\omega^2\tau_L^2}}$ to determine the best-fit values for A and τ_L (remember $\omega = 2\pi f$). How does this value of τ_L compare with the one you computed from the component values? Make a sketch in your notebook of the best-fit curve with your data points.

(b) Phase of the Current: You may have noticed by now that the peaks of the current through the inductor do not occur at the same times as the peaks of the applied emf $\varepsilon(t)$. The relative *phase angle* (in radians) between the current and applied emf is just $2\pi \cdot \frac{\delta t}{T}$, where δt is just the displacement ("shift") in time between the two waves, which have period T. Do the current peaks lead or lag the voltage peaks phase-wise?

Slowly tune the oscillator from the lowest frequency available to the higher ones and observe how the phase angle changes as you change the frequency of the applied emf. Are your qualitative observations consistent with Eq. (4)?

Uncertainties. The uncertainty in the "theoretical" value of the inductive time constant calculated from the measured values of the inductance L and the total series resistance R_T is given from Eq. 8(b) in Appendix A as

$$\frac{\Delta\tau_L}{\tau_L} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta R_T}{R_T}\right)^2}, \qquad (5)$$

where the uncertainty in R_T is given by Appendix A, Eq. 8(a) as

$$\Delta R_T = \sqrt{\Delta R_S^2 + \Delta R_L^2}. \qquad (6)$$

Conclusion:

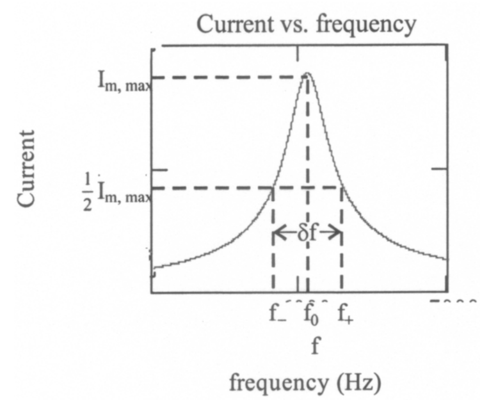
1. How does an inductor "react" to sudden changes in a circuit? Are your measurements consistent with the theoretical behavior of an RL combination?

---

[27] Engineers frequently plot quantities like $\log(V_{mR}/\varepsilon_m)$ vs. $\log(\omega)$ instead and work with "octaves" and "decibels." The quantity V_{mR}/ε_m is referred to as a "transfer function."

2. For AC circuits, in what sense does an inductor-resistor combination act as a “filter?” If the circuit you studied were powered by a stereo amplifier (instead of the oscillator) and the resistor R_S replaced by a speaker (with the same resistance and an inductance small compared to L), should this speaker be a woofer (for low frequencies) or a tweeter (for high frequencies)?

The Driven RLC Tuned Circuit



Introduction. If a sinusoidal emf signal $\varepsilon(t) = \varepsilon_m \sin(\omega t)$ is applied to a series combination of a resistance R_T, an inductance L, and a capacitance C, a sinusoidal current $I(t) = I_m \sin(\omega t - \phi)$ flows through all three elements. The amplitude I_m of the current is given as a function of frequency f ($\equiv \omega/2\pi$) from KVL and phasor analysis as

$$I_m = \frac{\varepsilon_m}{\sqrt{R_T^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}. \qquad (1)$$

As you can see from a plot of I_m vs. f at the right, this current amplitude (the "response" to the applied emf) is large for only a small range of frequencies. It is a maximum at the *resonance frequency* f_0, given from Eq. (1) as

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}. \qquad (2)$$

Outside of a small range of frequencies $\delta f \equiv f_+ - f_-$, the amplitude of the current is less than half of what it is at resonance. In terms of component values, Eq. (1) predicts that δf is given by

$$\boxed{\delta f = \frac{\sqrt{3}R_T}{2\pi L}}, \qquad (3)$$

where R_T is the total series resistance. This frequency range δf (labeled in the plot) is sometimes referred to as the *full width at half maximum* (FWHM) of the response curve.

As with the RL circuit you studied in the last lab, the current I(t) and the emf ε(t) will not be in phase (except, in this case, for emf's at the resonant frequency f_0). The phase angle φ by which the emf *leads* the current is given from phasor analysis and KVL as

$$\boxed{\tan\phi = 2\pi f\tau_L - \frac{1}{2\pi f\tau_C}}, \qquad (4)$$

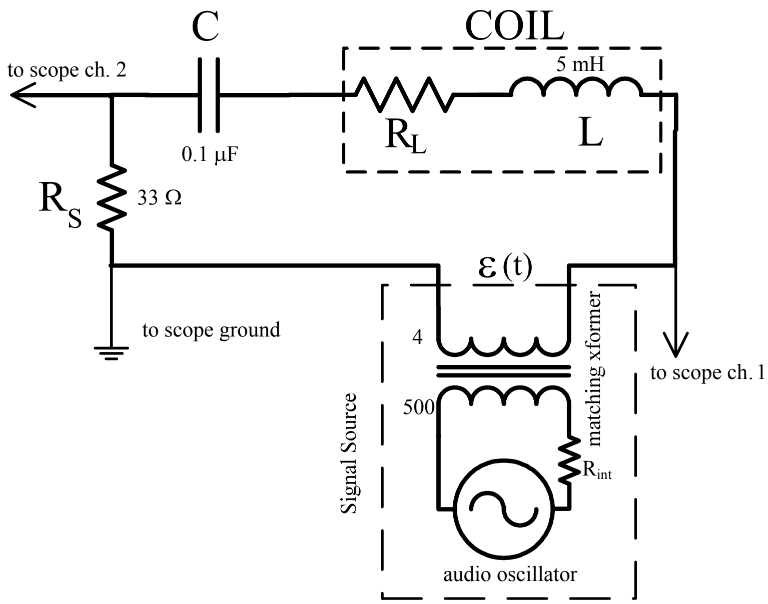
where $\tau_L = L/R_T$ and $\tau_C = R_T C$ are, respectively, the inductive and capacitive time constants studied in previous labs.

In this lab you will have the opportunity to study the current response I_m as a function of frequency and the phase difference between the emf and the current in an RLC circuit.

Procedure. You will need the following:

- Sencore "ReZolver"
- HP200AB audio oscillator
- 5 mH inductor
- two 0.1-μF capacitors
- French curves
- digital multimeter
- HP 54600B oscilloscope
- 5.6-Ω and 33-Ω resistors, one each
- laptop
- impedance matching transformer

With the oscillator and the scope off, use the ReZolver to measure and record the component values for the capacitors[28] and inductors, along with the uncertainties. Use the digital multimeter to measure the resistances R_L and R_S. Construct the circuit shown below:



Turn on the scope and signal generator and make sure the scope is DC-coupled (see footnote #2 on the RL Circuits write-up). One of the plugs on the "500" side of the transformer should be connected to the terminal on the oscillator marked "G," which should be attached by a jumper bracket to one of the other two terminals; the remaining third terminal is connected to the other transformer lead. The signal on Ch. 1 ($V_{PP}(1)$) is then the emf ε(t) (across the "4" side of the transformer) being presented to the series combination[29]; The Ch. 2 voltage $V_{PP}(2)$ is the voltage across R_S, which is proportional (by Ohm's law) to the instantaneous current response I(t) of the series circuit.

[28] ***Always discharge the capacitor before connecting it to the analyzer!*** Also, make sure that all components are disconnected before attempting to measure their values.

[29] Note that the oscillator is modeled here (within the dashed rectangle) as an "ideal" emf in series with an internal resistance R_{int} amounting to about 75 Ω for the HP200AB. The "step-down" transformer impedance-matches the oscillator to the circuit so that ε_m will be somewhat independent of the frequency setting.

With everything connected, you should see a sine wave on each channel. Display the values of $V_{PP}(1)$, $V_{PP}(2)$, and the common frequency f of the waves.

(a) Current Amplitude Response:

Predict: At what frequency f_0 will the current amplitude I_m be a maximum?

Use your prediction to search for the resonance by slowly adjusting the oscillator frequency in the range near your predicted f_0. Note that at resonance the current amplitude I_m will be maximum *and* the instantaneous current I(t) will be in phase with the emf ε(t). Record the observed value of f_0 along with the uncertainties (see below). Also note the value of $V_{PP}(1)$.

Find and record the frequencies f_+ and f_- at which the amplitude I_m ($\propto V_{pp}$ across R) is diminished to half of its value at f_0; estimate your observed FWHM $\delta f \equiv f_+ - f_-$.

Produce a graph that shows the behavior of the current I_m vs. frequency f. Note that the amplitude ε_m (indicated as $V_{PP}(1)$) of the signal output from the signal source does not stay strictly constant[2] as the frequency is changed; to correct for this you will need to adjust it back to the value it had at resonance for each new frequency f.

Measure and tabulate the frequency along with $V_{PP}(2)$ (keeping $V_{PP}(1)$ constant) at intervals of approximately 200 Hz between f_- and f_+. Make five measurements on each side of this range (i.e., below f_- and above f_+) every 400 Hz or so.

Use a French curve to sketch a careful graph of $V_{PP}(2)$ ($\propto I_m$) vs. frequency f in your notebook (use **LoggerProFile.cmbl** if desired). Compare the appearance of your graph with that in Figure 1. Use your graph to produce another estimate the resonant frequency f_0 along with δf.

Predict: How will f_0 (the peak frequency) and δf (the FWHM) of your current amplitude response curve change if

(1) A second 0.1 μF capacitor is connected in parallel with the one in your circuit?

(2) Instead of replacing the capacitor, you replaced the 33-Ω resistor with a 5.1-Ω one?

Try these changes (one at a time) and see whether your predictions were *qualitatively* correct.

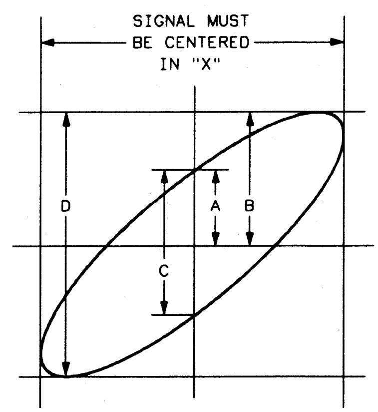
Analysis: Compare your predicted values of f_0 and δf (from Eqs. (2) and (3)) to those observed for your circuit. Are they consistent? Does your plotted curve resemble the theoretical one?

(b) Phase of the Current:

Qualitatively observe and record what happens to the phase of the current relative to the applied emf as you sweep the oscillator frequency from far below resonance to substantially above it. For what range of frequencies does the current lead the emf? For what range does it lag? Is this consistent with the prediction of Eq. (4)?

The Lissajous method: Placing the oscilloscope in "XY mode" allows us to accurately measure the phase difference between the signals on the two input channels. In this mode, the horizontal and vertical deflections each represent one of the two voltages applied at the inputs; the result is a Lissajous figure from which the phase angle can be calculated. For two signals of the same frequency, the figure will be an ellipse.

To use the XY mode press "Autoscale," then "Main/Delayed," then the "XY" softkey below the screen. If you are *near* but not *at* resonance, you will see an ellipse like the one at the right. The phase angle can be measured with the help of the Y-cursors and is given as $\sin\phi = \frac{C}{D}$.



Measure f_0 from phase observation:

Put the scope in XY mode and search for the resonant frequency, at which $\phi = 0$. At this frequency, the figure should degenerate from an ellipse to a straight line segment. Go back to the main display (press the "Main" softkey) to read and record the value of $f_0 \pm \Delta f_0$ using this method. How does f_0 found from this "phase method" compare with the value found from the current amplitude response investigation above? How do these compare with the value predicted from Eq. (2)?

Uncertainties: Uncertainties in quantities like $V_{p\text{-}p}$ and frequency read on the scope screen can be estimated by noting the fluctuations in these displayed numbers.

For the value of f_0 calculated from component values, we have

$$\frac{\Delta f_0}{f_0} = \sqrt{\frac{1}{2}\left(\frac{\Delta L}{L}\right)^2 + \frac{1}{2}\left(\frac{\Delta C}{C}\right)^2}\,.$$

Conclusion.

In words, describe the response of the RLC tuned circuit to emf's of different frequencies.

Do the width and the position of the peak of your observed current vs. frequency curve coincide with those predicted from the values of the components? Can you think of some reasons why your measured δf might be larger than the one you calculated from Eq. (3)? Which components affect the peak position? Which one(s) affect the width?

Appendix A: Measurements and Uncertainties

The establishment or verification of a physical law or the experimental determination of a physical quantity usually involves measurement. A reading taken from the scale on a stopwatch, a meter stick or a voltmeter, for example, may be directly related by a chain of analysis to the quantity or law under study; any uncertainty in these readings would thus be directly reflected in an uncertainty in the final result. A measurement by itself, without at least a rough quantitative statement as to the uncertainty involved, produces a result of limited usefulness. It is therefore essential that any introductory physics laboratory course include a discussion of the nature of experimental uncertainties and the manner in which they should be assigned to experimental results.

1. UNCERTAINTY vs. DISCREPANCY

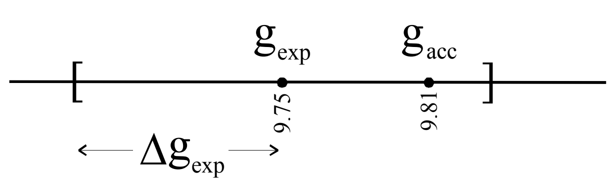
When you report the result of a measurement of a quantity x, you should also give the uncertainty Δx, e.g.,

5.0 m ± 0.1 m
⇗ ⇗
x Δx

The uncertainty tells us how precise you think your measurement is. It is also often useful to compare your result with a "true" or accepted value; the difference between these is the discrepancy and is a reflection of the overall accuracy of the measurement.

An estimate of the uncertainty of a measurement should always be made; a calculation of the discrepancy can be made only if an accepted value or result happens to be available for comparison. The conclusion for an experiment should, whenever possible, address the question: ***Do the uncertainties involved in my measurements account for the discrepancies between my result and the accepted one?***

As an example, suppose you do an experiment to measure the acceleration of gravity and you report the experimental value ($g_{exp} \pm \Delta g_{exp}$) to be $9.75 \pm 0.08 \frac{m}{s^2}$ where the "accepted" value is $g_{acc} = 9.81 \frac{m}{s^2}$. As you can see from the graphic representation below, the *uncertainty* Δg_{exp} in the measurement accounts nicely for the *discrepancy* between g_{exp} and g_{acc}.



2. ORIGINS OF UNCERTAINTIES

Problems which lead to discrepancies between experimental values and "true" values can be placed in two general categories:

I. Systematic Errors are inaccuracies due to identifiable causes and can, at least in principle, be eliminated. Errors of this kind result in values for the measured quantity which are **consistently** either too high or too low. Such errors can be

a) Theoretical - due to simplifications of the model system or approximations in the equations describing it.

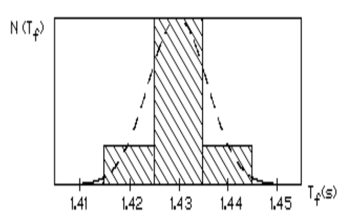
b) Instrumental - e.g., a poorly calibrated instrument.

c) Environmental - e.g., factors such as inadequately controlled temperature and pressure.

d) Observational - e.g., parallax in reading a meter scale.

II. Random Uncertainties are the result of small fluctuating disturbances which cause about half the measurements of any quantity to be too high and half to be too low. It is often not possible in practice to identify all the sources of such errors, to evaluate their effects individually or to completely eliminate them. They can, however, often be well characterized mathematically.

To illustrate the difference between systematic errors and random uncertainties, we consider the measurement of the length of time T_f taken for a ball to fall some fixed distance, say 10 m. Suppose we drop the ball about 1000 times and obtain as many values of T_f, rounding each time to the nearest .01 s. If $N(T_f)$ is a function of T_f that represents the number of times a particular measured value of T_f occurs, then a histogram of N versus T_f might look like



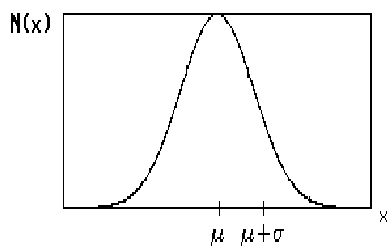
The "best" value for T_f is just the average (mean) value $\overline{T_f}$:

$$\overline{T_f} = \frac{\sum_{i=1}^{n} N_i T_{fi}}{n} \qquad \textbf{(1)}$$

where N_i is the number of times the value T_{fi} appears as a measurement and n is the total number of times the ball is dropped. In this example we see that $\overline{T_f}$ = 1.43 s; the discrepancy between this number and the "true" value of T_f, if known, would provide a measure of the accuracy of the measurement. Systematic errors (e.g. the reaction time involved in starting and stopping the clock) will affect $\overline{T_f}$ and hence the accuracy. The spread of T_f values, indicated by the width of the curve, is a reflection of the precision with which T_f is being measured. Random uncertainties contribute to this width and may be attributable, to e.g., small fluctuations of the height from which the ball was dropped or to the difficulty in determining the exact moment of impact with the ground. In what follows we discuss the mathematical treatment of such random uncertainties and the role they play in reporting laboratory measurements.

3. CHARACTERIZING A SET OF DATA: THE NORMAL DISTRIBUTION

It is most often the case that repeated measurements of the same quantity will, as in the timing experiment example described above, exhibit a spread about the average value related to the random errors associated with the measurement process. If we make "many" (say 10^6) measurements of a quantity x and plot the frequency of occurrence N(x), we quite often obtain a curve that approximates a Gaussian, or normal distribution, as pictured below.



This curve N(x) represents the relative probability with which values of x are obtained as the result of any single measurement of this quantity, which may be, for example, the reading of a meter or stopwatch. The analytical expression for such a curve is

$$N(x) = \frac{N_0}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \textbf{(2)}$$

where the parameters μ and σ determine the position and width of the peak, respectively. The "normalization" parameter N_O would correspond, in our timing example, to the total number of readings taken from the stopwatch.

The curve representing Eq. (2) is of importance for the analysis of experimental data because in many cases this data is distributed normally and thus has a frequency distribution that can be "fit" to this curve. The more data taken, the better the fit will be. But for any set of data, regardless of the number of data points or the nature of the distribution, we can define quantities which characterize the "best value" and "spread" of the data.

For a set of data points x_i, the quantity $\overline{x}$ is the mean of all values obtained for x, and is defined by

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \textbf{(3)}$$

where n is the number of measurements. For data in which each value of x_i generally occurs more than once, Eq. (1) may provide a more convenient way to calculate the mean value of x. The "best value" that one finally reports for the quantity x is generally its mean, as defined above. If a large number of normally distributed data points are taken, then $\overline{x}$ should be close to the value of μ for which the curve representing Eq. (2) best fits the data.

We can characterize the spread of any finite set of n data points by its standard deviation, symbolized by s.d., which we will define as

$$\text{s.d.} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}} \qquad \textbf{(4)}$$

The s.d. for a set of data thus represents the square root of the average square of the deviation (i.e., the "rms" deviation) that the data points exhibit with respect to their average value. For a large number of normally distributed data points, s.d. ≈ σ, where σ is the value of the parameter of Eq. (2) which produces the best description of the data.

4. REPORTING THE VALUE OF A MEASURED QUANTITY

Because repeated measurements of the same quantity give us, as discussed above, a range of values reflecting the random fluctuations inherent in the measurement process, we need to report the result of these measurements in a way which reveals not only the best value of the quantity, but also the precision to which we are stating it: we report **both** the best value **and** the uncertainty. The distributions discussed above refer to a collection of single measurements. So if we make a single measurement of a quantity x and had some knowledge of the single measurement distribution curve, then we could report this value of x as

$$x \pm \text{s.d.} \qquad \textbf{(5)}$$

where x is the single measured value and s.d. is the standard deviation of the single measurement distribution. To determine an appropriate value for s.d. without actually making several measurements, we would have to know something, a priori, about the single measurement curve; we would either have to be given a value of s.d. or we would have to guess.

Suppose, as is more common, we take n measurements instead of just one. Intuitively, it seems reasonable that the mean value should be reported with a smaller uncertainty than the single measurement discussed above. In fact, if one took **many** sets of n measurements (all with the same s.d.) and then calculated the mean for **each** set, the distribution of these **means** would have a smaller standard deviation (denoted S.D.) than that for a single measurement (s.d.). It is not hard to show (from Eq. (8a) below) that

$$\text{S.D.} \approx \frac{\text{s.d.}}{\sqrt{n}} \qquad \textbf{(6)}$$

What this means is that one could report the result of a set of n measurements as

$$\bar{x} \pm \text{S.D.} \approx \bar{x} \pm \frac{\text{s.d.}}{\sqrt{n}} \qquad \textbf{(7)}$$

so that the uncertainty associated with the average of 5 measurements, e.g., could be reduced by about a factor of 2 from the single measurement s.d. **NOTE**: The expression above for reporting an average is only meant to serve as a guideline in reporting results; it assumes that all uncertainties involved are purely random in nature. Systematic errors must always, at some level, be considered, so that in most cases we **cannot** create an arbitrarily small uncertainty simply by taking more and more measurements!

Relative vs. Absolute Uncertainty. One often reports the relative (or percentage) uncertainty associated with the best value in lieu of the absolute uncertainty Δx = s.d. or S.D. No units are associated with these relative uncertainties, so that these may be used to directly compare the precision of two different measurements. On the other hand, the absolute uncertainty is more convenient for comparing two measured values.

Example. Suppose we make 10 measurements of the period T of a pendulum, and that the results are tabulated to the nearest millisecond (ms) as

TRIAL #	T (s)	TRIAL #	T (s)
1	1.411	6	1.468
2	1.452	7	1.437
3	1.403	8	1.446
4	1.414	9	1.425
5	1.459	10	1.434

We need to report the best value for T along with the uncertainty, which we assume is due to random fluctuations. For the best value, we have the mean,

$$\bar{T} = \frac{\sum_{i=1}^{10} T_i}{10} = 1.435 \text{ s.}$$

To estimate the uncertainty, we calculate s.d. from Eq. (4):

$$\text{s.d.} = \sqrt{\frac{\sum_{i=1}^{10} (T_i - \bar{T})^2}{10}} = 0.02 \text{ s.}$$

But since we made 10 measurements, we can report the uncertainty as

$$\Delta T \approx \text{S.D.} \approx \frac{\text{s.d.}}{\sqrt{10}} \approx 0.006 \text{ s.}$$

Our result for T thus appears as

T = 1.435 s ± 0.006 s (with absolute uncertainty)
or
T = 1.435 s ± 0.5 % (with relative uncertainty)

Significant Figures. A decision must always be made, consciously or otherwise, as to how many significant figures should be used to report a quantity x and its uncertainty Δx. Since uncertainties are merely estimates, *it is seldom necessary to report Δx to more than 1 significant figure!* The number of significant figures with which you report x is often taken to imply its uncertainty, therefore this number *must be consistent with the explicitly stated uncertainty Δx*.

Rule of thumb for significant figures: *The measurement and the uncertainty should have their last digits in the same location relative to the decimal point*. Example: $(1.06 \pm 0.01) \times 10^3$ m.

Thus, a length stated as 1.0 m ± 0.002 m would present a problem because the stated uncertainty would imply that the observer is withholding information by not reporting more decimal places in the length. A result stated as 1.06 m ± 0.2 m reports a length with too much accuracy, given the uncertainty. If x and Δx imply different uncertainties, one of them should be adjusted so that they are both consistent with the largest of the two uncertainties.

Guesstimating Uncertainties. The uncertainty to assign a given single measurement of x depends on the technique employed in the measurement, the quality of the equipment used, and the care with which the measurement is made. The number you come up with as Δx for a particular measurement is often quite subjective, because only **you** know what happened during **your** measurement. There are a couple of options for estimating Δx:

1. Take several measurements and calculate s.d., as in the example above for T;

2. Estimate Δx from the nature of the measurement. For example, when reading a scale, the uncertainty is **sometimes**, but **not always**, half of the smallest scale division, but you will encounter situations when the scale divisions have nothing to do with Δx. In other words, you will always have to use your judgment.

You will find yourself using option #2 quite often; it's a real timesaver.

5. PROPAGATION OF UNCERTAINTIES

Quite often the quantity of interest is measured indirectly, i.e., derived from one or more quantities which are each measured directly and reported with uncertainties, as discussed above. In these cases, the estimate of the uncertainty in the indirectly measured quantity must reflect the uncertainties associated with all of the relevant directly measured ones.

For example, suppose we wish to infer the acceleration of gravity (g) from direct measurement of both T_f (the time taken for an object to fall) and of L (the height from which it is dropped). The equations of kinematics tell us that g can be calculated from T_f and L:

$$g = \frac{2L}{T_f^2}.$$

How, then, do the uncertainties in L and T_f contribute to Δg, the uncertainty in g?

You will need to know the answer to this question, and to others just like it. We first give the general expression for propagating the uncertainties in directly measured quantities to find the uncertainty in the derived result. We then treat two special cases which occur quite often.

General Case: Let z be a quantity to be derived from the direct measurement of two independent quantities x and y by using the relationship z = f(x,y). Suppose we were to make small definite (known) errors Δx and Δy in the measurements of x and y respectively. These would show up as an error Δz in z that would be given by

$$\Delta z \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y\,.$$

But because Δx and Δy are actually **random** uncertainties, a more realistic estimate of the uncertainty Δz is

$$\Delta z \approx \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2}\,, \qquad \textbf{(8)}$$

as can be demonstrated from considerations of probability theory. Given uncertainties in the two quantities Δx and Δy, Eq. (8) can be used to estimate the propagated uncertainty Δz. The generalization to more than two directly measured quantities is straightforward.

Origin of Eq. (8) (Optional): Consider two independent random variables P and Q, each with an average of zero and with s.d.'s of ΔP and ΔQ, respectively. The s.d. of the sum P + Q (denoted Δ(P+Q)) is, according to the definition of Eq. (4),

$$\Delta(P+Q) = \sqrt{\overline{(P+Q)^2}}$$

If we compute the square of this quantity in terms of ΔP and ΔQ, we get

$$(\Delta(P+Q))^2 = \overline{(P+Q)^2} = \overline{P^2 + 2PQ + P^2}$$
$$= \overline{P^2} + \overline{Q^2} = (\Delta P)^2 + (\Delta Q)^2,$$

from which the s.d. for P + Q is

$$\Delta(P+Q) = \sqrt{(\Delta P)^2 + (\Delta Q)^2}.$$

Now considering the function z = f(x,y): the uncertainty in z (Δz), will be due to the combined effects of the uncertainty in x (Δx) and the uncertainty in y (Δy). If P and Q represent the effect on z of changes in x and y, respectively, then ΔP and ΔQ, the s.d.'s for P and Q, will be given by

$$\Delta P = \frac{\partial f}{\partial x}\Delta x; \quad \Delta Q = \frac{\partial f}{\partial y}\Delta y,$$

where Δx and Δy are the s.d.'s, or uncertainties, in x and y. The spread, or uncertainty Δz in z will then be $\Delta(P+Q)$, given above. Substitution for ΔP and ΔQ results directly in Eq. (8) for Δz. It should be noted that this same argument can also be applied to derive Eq. (6) above for the uncertainty (S.D.) in the average.

Many times, it is not necessary to use Eq. (8) directly because the form of the expression for z will fit one of two **special cases**.

Special Case I: The quantity of interest (z) is given in terms of two directly measured quantities x and y, each with their respective uncertainties Δx and Δy, as $\mathbf{z = Ax + By}$. In this case, the absolute uncertainty in z is given by

$$\Delta z = \sqrt{(A\,\Delta x)^2 + (B\,\Delta y)^2 + \cdots} \qquad \textbf{(8a)}$$

where the "$+\cdots$" indicates that this expression is easily generalized to the case where z is given as the sum of three or more quantities.

Example: We wish to determine the mass of water, m_w, contained in a beaker from independent measurements of the mass of the beaker alone, m_b, and the combined mass of the beaker + water, m_t. Then $m_w = m_t - m_b$ and the uncertainty in the derived quantity m_w is

$$\Delta m_w = \sqrt{(\Delta m_t)^2 + (\Delta m_b)^2}\ ,$$

where Δm_t and Δm_b must either be estimated or determined from repeated measurements of these quantities.

Special Case II: The derived quantity (z) is the product of powers of measured quantities x and y: $\mathbf{z = K\, x^a\, y^b}$, where K is a constant. In this case we can give the fractional (relative) uncertainty in z as

$$\frac{\Delta z}{z} = \sqrt{\left[a\frac{\Delta x}{x}\right]^2 + \left[b\frac{\Delta y}{y}\right]^2} \qquad \textbf{(8b)}$$

where we can again generalize to any number of quantities, as indicated for Eq. (8a).

Example: If the acceleration of gravity g is given, as discussed above, in terms of L and T_f as $g = 2L/T_f^2$, what is Δg in terms of the uncertainties ΔL and ΔT_f? If g is rewritten as $g = 2(L^1)(T_f^{-2})$, then we have something that fits the form of special case II with x = L, a = 1 and y = T_f, b = -2. So for the relative uncertainty $\Delta g/g$ we get

$$\frac{\Delta g}{g} = \sqrt{\left[\frac{\Delta L}{L}\right]^2 + 4\left[\frac{\Delta T_f}{T}\right]^2}\ .$$

Note that because g depends on the second power of T_f but on only the first power of L, the uncertainty in g is much more "sensitive" to the uncertainty in T_f than to the uncertainty in L.

If neither of the special cases applies to the quantity being investigated, it is sometimes possible to derive the uncertainty by applying Eq. (8a) and (8b) in combination; otherwise, use the general expression of Eq. (8) directly.

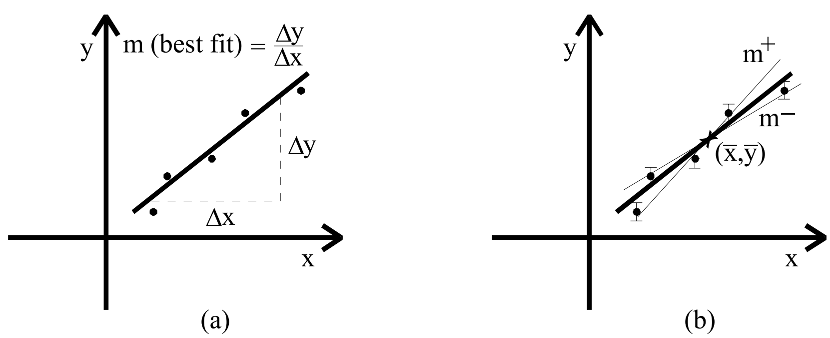
Graphs and Uncertainties. Physical laws often predict that two quantities (x and y) that can be measured (directly or indirectly) are proportional to each other. For example, Newton's second law in one dimension (F = Ma) predicts that the acceleration (a) of an object is proportional to the force F (the mass M being the constant of proportionality); Ohm's law (V = IR) predicts, for certain materials, that the voltage V and current I are proportional. It is often the constant of proportionality between these measured quantities (M or R in the above examples) that is to be determined by the experiment, rather than just individual pairs of values for x and y. The linear relationship between x and y can be expressed as

slope

$$y = mx + b$$

y-intercept

If y is plotted vs. x, as in graph (a) below, then the result **should** be a line with a slope m, calculated as $\Delta y/\Delta x$. (Note that here Δy and Δx refer to the rise and run, respectively, **not** to the uncertainties in y and x!).



Of course, experiments being what they are, the data points (as indicated in graph (a)) will not all be on a single line; it is up to you to draw the "best" line that "fits" the data so that the slope m (or y-intercept b) may be determined. There is a well-known method for determining the best line for a given set of points (x,y) -- the least squares fit (see below). For the purposes of this lab, however, it will often be sufficient to use a straight-edge and your eyes to draw the line which represents the best compromise, i.e., "comes as close to as many points as possible."

NOTE: If the slope or the intercept is to be measured directly from the graph, it is most often advantageous to use *as much of the page as possible to draw the graph*!

If the slope m is the quantity to be reported as the result of the experiment, then the uncertainty Δm must be reported along with it: we need $m \pm \Delta m$. The uncertainty Δm can be thought of as arising from the uncertainties in the position of each plotted point on the graph, since each y value may be characterized by an uncertainty Δy. One way of representing these uncertainties is by means of the "error bars" drawn for each point on graph (b) above. As with the slope m, there is a well known (and somewhat complex) expression that can be used to determine Δm, but for our purposes it is sufficient to "eye ball" the range of slopes. As indicated in graph (b), this can be done by using the error bars to estimate m^+, the highest possible slope for a line that passes through or near most of the error bars, and m^-, the lowest slope. The uncertainty Δm is then **very roughly** given by

$$\Delta m \approx \frac{m^+ - m^-}{2}.$$

To estimate the slope (m) and its uncertainty (Δm) by eye:

1. Plot the data points (x,y) on graph paper. Use as much of the page as possible.

2. Place error bars through each point to indicate roughly the extent to which the location of that point on the graph is uncertain. The size of these bars can be estimated from the discussion under the "uncertainty" section in each experiment. An alternative to bars is to enlarge your dot to indicate its "fuzziness." If the uncertainty is too small to show up on your graph, state this in the report.

3. Plot the point $(\bar{x}, \bar{y})$ on your graph (where $\bar{x}$ and $\bar{y}$ are the means of the x and y values, respectively). It so happens that this point always lies on the "best line" defined below by the least squares fit procedure.

4. Using a straight edge, draw the line passing through $(\bar{x}, \bar{y})$ that "comes closest" to the data points. Measure the slope m as $\frac{\text{rise}}{\text{run}}$, using as much of the line as possible and *paying attention to the units on the axis scales*.

5. Estimate the uncertainty Δm by drawing 2 dotted lines through $(\bar{x}, \bar{y})$ and passing within most of the error bars--one with the minimum slope, m^-, and one with the maximum slope, m^+. Estimate Δm from the expression in the paragraph above.

<u>Least Squares Fit</u> (Linear Regression): Though somewhat more involved and less intuitive than the graphical method described above for finding the slope and intercept of a best fit line, the analytic method of linear regression finds the best fit by minimizing the sum of squares of deviations of y values from the fitted straight line. For this reason, it is called the *method of least squares*. Given N data points x_1, y_1; x_2, y_2; ...; x_N, y_N, the slope m and y-intercept b of the best fit line (given by y = mx + b) are calculated as:

$$m = \frac{\sum_{i=1}^{N}(x-\bar{x})(y-\bar{y})}{N\sigma_x{}^2}; \quad b = \bar{y} - m\bar{x}, \qquad \textbf{(9)}$$

where $\bar{x}$ and $\bar{y}$ are the means of the x and y values respectively, given by Eq. (3), and σ_x is the s.d. of the x values as given by Eq. (4).

As an alternative to the graphical method described above for estimating the uncertainty Δm for the slope, you may use the least squares methodology to estimate this quantity along with the uncertainty in the intercept, Δb. The expressions for these are

$$\Delta m = \frac{1}{\sigma_x}\sqrt{\frac{\sum_{i=1}^{N} d^2}{N(N-2)}}; \qquad \Delta b = \Delta m \sqrt{{\sigma_x}^2 + \overline{x}^2}\,, \qquad \textbf{(10)}$$

in which $d_i \equiv y_i - (mx_i + b)$ is the y deviation of the point (x_i,y_i) from the best fit straight line $y = mx + b$. If your calculator does linear regression calculations which result in the least squares fit slope m and correlation coefficient r, then it may be simpler to calculate the uncertainty in the slope (Δm) with the equation*

$$\Delta m = |m|\frac{\tan(\arccos(r))}{\sqrt{N-2}} = \frac{|m|}{r}\sqrt{\frac{1-r^2}{N-2}}\,. \qquad \textbf{(11)}$$

References and further reading:

1. Lichten, William, Data and Error Analysis in the Introductory Physics Laboratory (Allyn and Bacon, Inc.)

* J. Higbie, *American Journal of Physics*, **59**(2) 1991

Appendix B: Meter and Oscilloscope Uncertainties

Fluke 179: The rightmost column of the table below indicates the uncertainties to be associated with the readings of the Fluke 179 multimeter. The term [Counts] in the expression for the "Accuracy" at the top refers to the value of the least significant figure displayed, which is also the entry in the "Resolution" column. Example: For a reading of V = 1.500 V (on the 6.000 V range), ΔV = (0.09% of 1.500 V) + 2(0.001 V) ≈ 0.003 V. So record V = 1.500 ± 0.003 V.

Function	Range [1]	Resolution	Accuracy ± ([% of Reading] + [Counts]) Model 175	Model 177	Model 179
AC Volts [2,3]	600.0 mV 6.000 V 60.00 V 600.0 V 1000 V	0.1 mV 0.001 V 0.01 V 0.1 V 1 V	1.0 % + 3 (45 Hz to 500 Hz) 2.0 % + 3 (500 Hz to 1 kHz)	1.0 % + 3 (45 Hz to 500 Hz) 2.0 % + 3 (500 Hz to 1 kHz)	1.0 % + 3 (45 Hz to 500 Hz) 2.0 % + 3 (500 Hz to 1 kHz)
DC mV	600.0 mV	0.1 mV	0.15 % + 2	0.09 % + 2	0.09 % + 2
DC Volts	6.000 V 60.00 V 600.0 V	0.001 V 0.01 V 0.1 V	0.15 % + 2	0.09 % + 2	0.09 % + 2
	1000 V	1 V	0.15 % + 2	0.15 % + 2	0.15 % + 2
Continuity	600 Ω	1 Ω	Meter beeps at < 25 Ω, beeper turns off at > 250 Ω; detects opens or shorts of 250 µs or longer.		
Ohms	600.0 Ω 6.000 kΩ 60.00 kΩ 600.0 kΩ 6.000 MΩ 50.00 MΩ	0.1 Ω 0.001 kΩ 0.01 kΩ 0.1 kΩ 0.001 MΩ 0.01 MΩ	0.9 % + 2 0.9 % + 1 0.9 % + 1 0.9 % + 1 0.9 % + 1 1.5 % + 3	0.9 % + 2 0.9 % + 1 0.9 % + 1 0.9 % + 1 0.9 % + 1 1.5 % + 3	0.9 % + 2 0.9 % + 1 0.9 % + 1 0.9 % + 1 0.9 % + 1 1.5 % + 3
Diode test	2.400 V	0.001 V	1 % + 2		
Capacitance	1000 nF 10.00 µF 100.0 µF 9999 µF [4]	1 nF 0.01 µF 0.1 µF 1 µF	1.2 % + 2 1.2 % + 2 1.2 % + 2 10 % typical	1.2 % + 2 1.2 % + 2 1.2 % + 2 10 % typical	1.2 % + 2 1.2 % + 2 1.2 % + 2 10 % typical
AC Amps [5] (True RMS) (45 Hz to 1 kHz)	60.00 mA 400.0 mA (600 mA for 18 hrs) 6.000 A 10.00 A (20 A for 30 s)	0.01 mA 0.1 mA 0.001 A 0.01 A	1.5 % + 3	1.5 % + 3	1.5 % + 3

1. All AC voltage and AC current ranges are specified from 5 % of range to 100 % of range.
2. Crest factor of ≤ 3 at full scale up to 500 V, decreasing linearly to crest factor ≤ 1.5 at 1000 V.
3. For non-sinusoidal waveforms, add -(2% reading + 2% full scale) typical, for crest factors up to 3.
4. In the 9999 µF range for measurements to 1000 µF, the measurement accuracy is 1.2 % + 2 for all models.
5. Amps input burden voltage (typical): 400 mA input 2 mV/mA, 10 A input 37 mV/A.

Function	Range [1]	Resolution	Accuracy ± ([% of Reading] + [Counts]) Model 175	Model 177	Model 179
DC Amps [5]	60.00 mA 400.0 mA (600 mA for 18 hrs) 6.000 A 10.00 A (20 A for 30 s)	0.01 mA 0.1 mA 0.001 A 0.01 A	1.0 % + 3	1.0 % + 3	1.0 % + 3
Hz (AC- or DC- coupled, V or A [2, 3, 4] input)	99.99 Hz 999.9 Hz 9.999 kHz 99.99 kHz	0.01 Hz 0.1 Hz 0.001 kHz 0.01 kHz	0.1 % + 1	0.1 % + 1	0.1 % + 1
Temperature	-40 °C to +400 °C -40 °F to +752 °F	0.1 °C 0.1 °F	NA	NA	1 % + 10 1 % + 18
MIN MAX AVG	For DC functions, accuracy is the specified accuracy of the measurement function ± 12 counts for changes longer than 275 ms in duration. For AC functions, accuracy is the specified accuracy of the measurement function ± 40 counts for changes longer than 1.2 s in duration.				

1. All AC voltage and AC current ranges are specified from 5 % of range to 100 % of range.
2. Frequency is specified from 2 Hz to 99.99 kHz in Volts and from 2 Hz to 30 kHz in Amps.
3. Frequencies < 10 kHz are not specified in 600 mV AC, 60 mA AC, and 6 A AC ranges.
4. Below 2 Hz, the display shows zero Hz.
5. Amps input burden voltage (typical): 400 mA input 2 mV/A, 10 A input 37 mV/A.

<u>SENCORE LC103</u>:
Uncertainties for capacitance measurements:
For $C \leq 1990\ \mu F$: $\pm(1\% + 1\ pF + 1$ digit[30]$)$
For $2000\ \mu F \leq C \leq 19.99\ F$: $\pm(5\% + 0.1\%$ of range full scale[31]$)$

Uncertainties for inductance measurements: $\pm(2\% + 1$ digit $+ 0.1\ \mu H)$

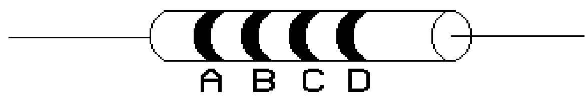
RESOLUTION AND RANGES: 1

0.1 pF	**1.0 pF to 199.9 pF**
1.0 pF	**200 pF to 1999 pF**
0.00001 uF	**0.002 uF to 0.01999uF**
0.0001 uF	**0.02 uF to 0.1999 uF**
0.001 uF	**0.2 uF to 1.999 uF**
0.01 uF	**2.0uF to 19.99 uF**
0.1 uF	**20.0 uF to 199.9 uF**
1.0 uF	**200 uF to 1,999 uF**
10 uF	**2,000 uF to 19,990 uF**
100 uF	**20,000 uF to 199,900 uF**
0.001 F	**0.2 F to 1.999 F**
0.01 F	**2.0 F to 19.99 F**

[30] The term "1 digit" refers to the change in the value when the least significant figure changes by 1. This is the same as the "resolution" shown above with the full scale readings.
[31] See above for the full scale values corresponding to your readings.

Appendix C: Resistor Color Code

Resistors come in many different sizes, shapes and compositions. Some are marked with four colored bands indicating their resistances in Ohms (Ω), as illustrated below:



The colored bands labeled A, B, C, and D each represent a digit, and the resistance is then given in terms of A, B and C as

$$R = AB \times 10^C \ \Omega$$

and the tolerance (uncertainty) ΔR/R is indicated by D. The table below gives the color code for the digits:

Black	0	Green	5
Brown	1	Blue	6
Red	2	Violet	7
Orange	3	Gray	8
Yellow	4	White	9

Gold	(C only)	-1
Silver	(C only)	-2

Tolerance (D) code:

Gold	5%
Silver	10%
No band	20%

Example:

A white	(= 9)	B brown	(= 1)
C red	(= 2)	D silver	(10%)

$$R = 91 \text{ x } 10^2; \quad \frac{\Delta R}{R} = 0.1$$

The maximum amount of power that a resistor can dissipate without overheating depends on the surface area of its body, as indicated in the table below for carbon-composition resistors:

LENGTH (")	DIAMETER (")	POWER RATING (W)
1/4	0.090	1/4
3/8	9/64	1/2
9/16	7/32	1
11/16	5/16	2