 Extreme value statistics and probabilities of flooding

Kathryn Roscoe
Deltares
Vulnerability of The Netherlands to flooding

- Below water-level:
  - Approximately 9 mln people and 70% of GDP
  - 60% of the land
Dutch reclamation and water engineering
1945 Germans blow up levee
1953 Flood

- Spring tide + large storm
- 600 square miles flooded
- 1,800 people died
- 30,000 animals drowned
- Damage = 450 million Euros
The Delta Works

- One of the Seven Wonders of the Modern World by the American Society of Civil Engineers
Dikes, dunes and barriers
Protection levels – exceedance probabilities

The Netherlands
Safety Standard per Dike-ring area

Legend
- 1 number of dike-ring area
- 1/10,000 per year
- 1/4,000 per year
- 1/2,000 per year
- 1/1,250 per year

High grounds (also outside The Netherlands)
Primary water defence outside The Netherlands

North Sea
Belgium
Germany

Deltarese
Mathematical description of failure

$Z = R - S$

- $Z \rightarrow$ limit state function
- $Z > 0 \rightarrow$ no failure
- $R \rightarrow$ resistance
- $Z < 0 \rightarrow$ failure
- $S \rightarrow$ hydraulic load
- $Z = 0 \rightarrow$ limit state
Limit state: water level = crest level

Failure: water level > crest level

Z = R - S = h_c - h

h_c: crest level
h: water level
From deterministic to probabilistic approach

Deterministic approach

\[ Z = R - S \rightarrow \text{Failure yes/no} \]

<table>
<thead>
<tr>
<th>Load</th>
<th>Strength</th>
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Probabilistic approach

\[ Z = R - S \rightarrow \text{Failure} = \text{probability}(Z < 0) \]

\[ P(\text{load}>\text{strength}) \]
Quantifying failure probabilities

Rock weighs 8 kg
Rope can carry 10 kg
Rope holds
2.2 Quantifying failure probabilities

Load (kg) vs. Strength (kg)

Z<0: failure
Z=0
Z>0: no failure

Z = Strength - Load
2.2 Quantifying failure probabilities

- Rock weighs 8 kg or 12 kg
- Rope can carry 6 kg or 10 kg
- Every combination is equally likely
2.2 Quantifying failure probabilities

Load (kg) vs. Strength (kg) diagram:

- Z<0
- Z>0: no failure
- P(Z<0) = 0.75
2.2 Quantifying failure probabilities

Rock weighs 8 kg

Rope load bearing capacity is normally distributed: \( N(10,2) \)
2.2 Quantifying failure probabilities

Monte Carlo simulation

Load (kg) vs. Strength (kg)

- Z<0: failure
- Z>0: no failure

Z=0

Load = 8 kg

Monte Carlo simulation
2.2 Quantifying failure probabilities

![Graph showing probability density, load, and strength with weight in kg on the x-axis.]

- Load
- Strength
- Weight (kg)
2.2 Quantifying failure probabilities

\[ P(Z < 0) \]

\[ Z = \text{Strength} - \text{Load} \]
2.2 Quantifying failure probabilities

Rock weight is normally distributed: N(8,2)

Rope load bearing capacity is normally distributed: N(10,2)
2.2 Quantifying failure probabilities

- Load (kg)
- Strength (kg)

Z<0: failure
Z>0: no failure
Z=0
2.2 Quantifying failure probabilities

![Graph showing probability density distribution for load and strength against weight (kg).]
2.2 Quantifying failure probabilities

\[ Z = \text{Strength} - \text{Load} \]

\[ \mu_Z = \mu_{\text{Strength}} - \mu_{\text{Load}} = 2 \]

\[ \sigma_Z^2 = \sigma_{\text{Strength}}^2 + \sigma_{\text{Load}}^2 = \frac{8}{\pi} \]
Loads

Probabilistic approach

Conditions measured at station locations

Hydrodynamic models translate ‘global’ conditions to local conditions

Loads at the defenses and their probabilities
Measurements and “global” statistics
Measurements
### Estimating probabilities from measurements

<table>
<thead>
<tr>
<th>Rank</th>
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<th>Water level</th>
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<tbody>
<tr>
<td>1</td>
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<td>3.36</td>
</tr>
<tr>
<td>2</td>
<td>1976</td>
<td>3.04</td>
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<td>3</td>
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![Water level diagram](image-url)
Estimating probabilities from measurements

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Frequency plot

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...  ...

- Water Level (m)
- Return period

Deltaris
Extreme value theory

Extreme values statistics

Let \( X_1, X_2, \ldots \) be iid random variables (e.g. water levels) with an unknown CDF \( F(x) = \Pr\{X_i \leq x\} \).

Define \( M_n = \max(X_1, \ldots, X_n) \) as the maximum observation in a sample of \( n \) water levels. From the iid assumption, the CDF of \( M_n \) is:

\[
\Pr\{M_n \leq x\} = \Pr\{X_1 \leq x, \ldots, X_n \leq x\} = \prod_{i=1}^{n} F(x) = F^n(x)
\]

\( \xrightarrow{\text{Compounded error}} \)

Generalized extreme value distribution

\[
H(x; \mu, \sigma, \xi) = \exp\left\{ -\left[1 + \xi \left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\xi}} \right\} \quad \xi \neq 0
\]

\[
H(x; \mu, \sigma, \xi) = \exp\left[ -\exp\left[ -\left(\frac{x-\mu}{\sigma}\right)\right] \right] \quad \xi = 0
\]

\( \xi = 0 \)  Gumbel – exponential decrease
\( \xi < 0 \)  Weibull – finite tail
\( \xi > 0 \)  Frechet – polynomial decrease
Translation to local conditions

Hydrodynamic models:

Coastal example

River example

Sobek, WAQUA
Failure: example of overflow
Example non-tidal river – 1 variable

Q Lobith

Deltalres
Example tidal river – 2 variables

Local water level influenced by river discharge and sea water level.
Example tidal river – 2 variables

Contour lines differ per location
Example tidal river – 2 variables

contour lines differ per location

Sea

River

Deltas
Example tidal river – 2 variables

use hydrodynamic model to derive the contour lines

Sea
River
Deltas
Example tidal river – exceedance probability

Example: flooding occurs if $h > 4.5 \text{ m+NAP}$
Example tidal river – exceedance probability

local water level [m+NAP]

failure domain

sea water level [m+NAP]

discharge [1000 m³/s]

Sea

River

Deltares
Analogy: game of two dice

loss (failure): opponent throws 10 or more

\[
\begin{array}{cccccc}
\text{y} & 6 & \bullet & \bullet & \bullet & \bullet & \bullet \\
5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
4 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
3 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
2 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\end{array}
\]
step 2: probability of combinations ≥ 10

P[\cdot] = 1/36

P[x + y ≥ 10] = 6\times P[\cdot] = 6/36 = 1/6
Flood risk complexer than dice

- continuous values instead of discrete values
- number of variables may be larger than 2
- correlation between variables

- no analytical solution available
- estimation techniques required
Computing real failure probabilities

failure domain

local water level [m+NAP]

sea water level [m+NAP]

discharge [1000 m³/s]

Sea

River

Zeltears
Safety Assessment based on various failure mechanisms

- Wave overtopping
- Instability by infiltration and erosion
- Piping
- Heave
- Instability of inner slope
- Instability of outer slope
- Micro-instability
- Instability of revetment
- Instability of toe by sliding and settlement
Effect of failure mechanism on the load

Overtopping discharge

Dune Erosion

offshore waves

sea water level
Derive statistics of variables such as wind, discharge, water levels, waves at stations.

Use of hydrodynamic models to translate conditions at discrete locations to conditions along the defenses.

Use of statistics at the measurement locations to estimate the probabilities of combinations of variables at the defenses.

Combine the loads with the defenses to determine loads that lead to a failure probability equal to the safety standard.
Thanks for your attention!

Questions?
Comments?