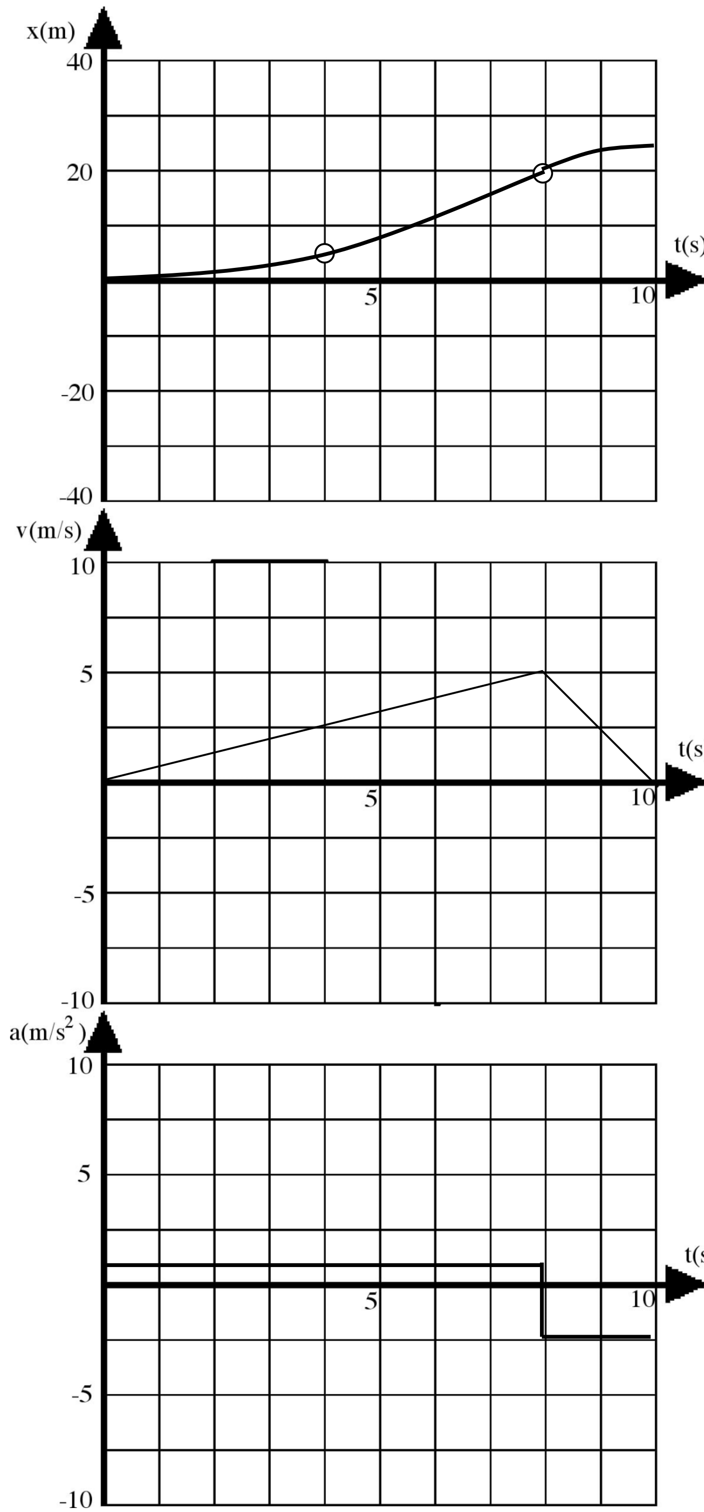
Name:___________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The velocity of a car as a function of time is shown in the middle graph at the right. (a)Explain how you would go about sketching the position versus time graph (top) and draw the curve. (b)Explain how you would go about sketching the acceleration versus time graph (bottom) and draw the curve.



(a)The definition of velocity is $\vec{v} \equiv \frac{d\vec{r}}{dt}$.

For one-dimensional motion, $v = \frac{dx}{dt} \Rightarrow dx = vdt$.

Therefore, the change in position is the area under the velocity versus time graph.

Each box of the velocity graph represents, $(2.5m/s)(1s) = 2.5m$

After 4s the area is 2 boxes = 5.00m
After 8s the area is 8 boxes = 20.0m
Between 8s and 10s 2 more boxes are added totaling 10 boxes = 25.0m

(b)The definition of acceleration is $\vec{a} \equiv \frac{d\vec{v}}{dt}$.

Therefore, the acceleration is the slope of the velocity versus time graph.

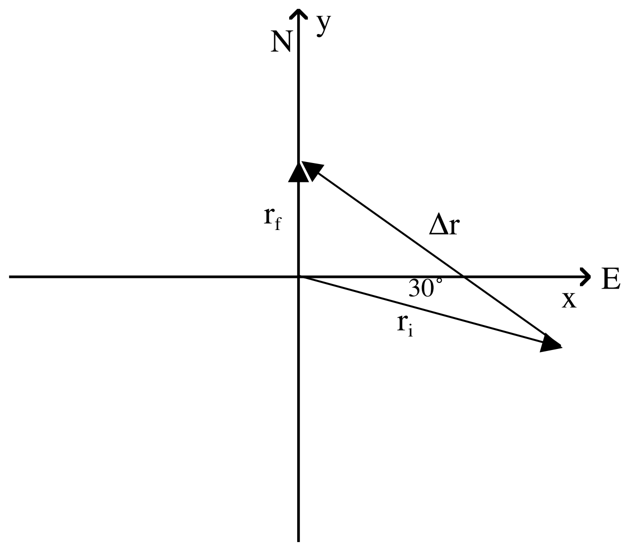
From t=0s to t=8s the slope is roughly constant,

$$a = \frac{\Delta v}{\Delta t} = \frac{5.0-0}{8-0} = 0.63m/s^2.$$

From t=8s to t=10s the slope is also roughly constant but negative,

$$a = \frac{\Delta v}{\Delta t} = \frac{0-5.0}{10-8} = -2.5m/s^2.$$

2. A tropical storm was centered 400km away from Honolulu at 30.0° south of east. Six hours later the storm is centered 200km due north. (a)Show the initial position, final position, and displacement at the right. (b)Find the displacement (magnitude and direction) of the storm during this time and (c)find the average velocity of the storm.



(b)Breaking the position vectors into components,
$\vec{r}_i = r_i \cos 30° \hat{i} - r_i \sin 30° \hat{j}$
$\vec{r}_i = 400 \cos 30° \hat{i} - 400 \sin 30° \hat{j} \Rightarrow \vec{r}_i = 346\hat{i} - 200\hat{j}$
and $\vec{r}_f = 200\hat{j}$.
The definition of displacement is,
$\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i \Rightarrow \Delta\vec{r} = (200\hat{j}) - (346\hat{i} - 200\hat{j})$
Finally, $\Delta\vec{r} = -346\hat{i} + 400\hat{j}$

Using the Pythagorean Theorem and the definition of tangent,
$\Delta r = \sqrt{346^2 + 400^2} = 529$ and $\tan\theta \equiv \frac{400}{346} \Rightarrow \theta = \arctan\frac{400}{346} = 49.1°$.
The answer is **529km at 49.1° north of west**.

(c)The definition of velocity is, $\vec{v} \equiv \frac{d\vec{r}}{dt} \Rightarrow v = \frac{529km}{6h} = 88.2km/h$.
So the answer is **88.2km/h at 49.1° north of west**.

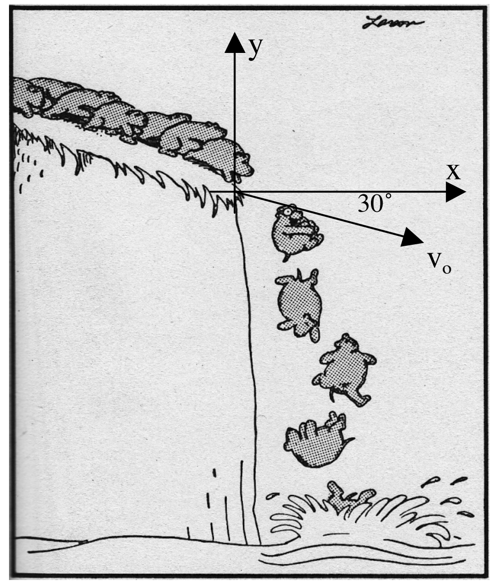
3. A sprinter starting from rest reaches their top speed of 7.00m/s in a distance of 25.0m. Find (a)their acceleration and (b)the time to reach this top speed.

$x_o = 0$
$x = 25.0m$
$v_o = 0$
$v = 7.00m/s$
$a = ?$
$t = ?$

(a)Using the kinematic equation without the time,
$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow v^2 = 2ax$.
Solving for the acceleration,
$$a = \frac{v^2}{2x} = \frac{(7.00)^2}{2(25.0)} \Rightarrow \boxed{a = 0.980 m/s^2}.$$

(b)Using the kinematic equation with the acceleration,
$$\frac{x - x_o}{t} = \frac{1}{2}(v + v_o) \Rightarrow \frac{x}{t} = \frac{1}{2}v.$$
Solving for the time,
$$t = \frac{2x}{v} = \frac{2(25.0)}{7} \Rightarrow \boxed{t = 7.14s}.$$

4. In the cartoon at the right, the next lemming to leave the cliff has an initial velocity of 2.00m/s at 30.0° below the horizontal. He will land 1.00m out from the base of the cliff. Find (a)the time he is in the air and (b)the height of the cliff.



$x_o = 0$	$y_o = 0$
$x = 1.00m$	$y = ?$
$v_{ox} = 2.00\cos 30° = 1.73 m/s$	$v_{oy} = -2.00\sin 30° = -1.00 m/s$
$v_x = v_{ox}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80 m/s^2$
$t = ?$	$t = ?$

(a)Using the kinematic equation without the final speed in the x-direction,
$x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$,
solve for the time, $x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{1.00}{1.73} \Rightarrow \boxed{t = 0.578s}$.

(b) Using the kinematic equation without the final speed in the y-direction, $y = y_o + v_{oy}t + \frac{1}{2}a_y t^2$,
solve for the height, $y = y_o + v_{oy}t + \frac{1}{2}a_y t^2 = 0 + (-1.00)(0.578) - \frac{1}{2}(9.80)(0.578)^2 \Rightarrow \boxed{y = -2.22m}$.
The minus sign is due to the choice of coordinates.

5. The six figures below depict six different satellites in circular orbit around Earth. The satellites differ in their orbital radius, speed, and mass. The specific orbital radius, speed, and mass for each satellite is given in each figure. Rank these satellites, from greatest to least, on the basis of their acceleration. You must carefully explain your reasoning for full credit.






A	B	C	D	E	F
R = 12800km	R = 12800km	R = 12800km	R = 19200km	R = 19200km	R = 16000km
V = 5.7km/s	V = 5.7km/s	V = 5.7km/s	V = 4.6km/s	V = 4.6km/s	V = 5.1km/s
M = 20kg	M = 50kg	M = 100kg	M = 20kg	M = 50kg	M = 100kg

The equation for the centripetal acceleration is $a_c = \frac{v^2}{r}$. Note it doesn't depend upon mass.

A: $a_c = \frac{(5700)^2}{12.8x10^6} = 2.54 m/s^2$, B: same as A, C: also same as A,
D: $a_c = \frac{(4600)^2}{19.2x10^6} = 1.10 m/s^2$, E: same as D, F: $a_c = \frac{(5100)^2}{16.0x10^6} = 1.63 m/s^2$.
The answer is, $\boxed{A=B=C>F>D=E}$.