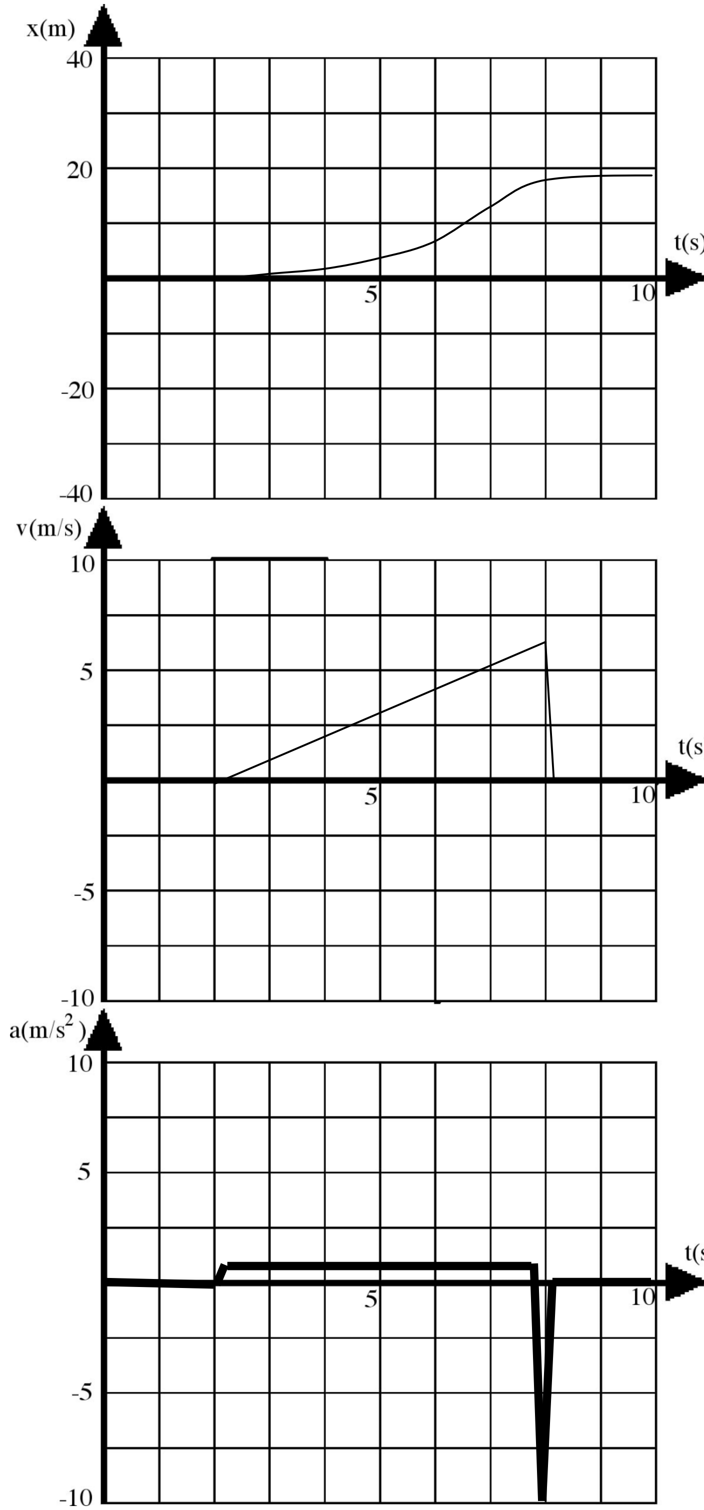
Name:_________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 300g cart initially at rest rolls down a track at a slight incline and strikes a wall at the end of the track. A motion sensor plots the acceleration as a function of time as shown in the graph at the bottom right.



(a)Estimate the value of the acceleration on the flat part of the curve between 2s and 8s.

Judging from the graph, it is about $1 m/s^2$.

(b)Explain the sudden negative spike in the acceleration graph.

This is the large deceleration caused by hitting the wall at the end of the track.

(c)Graph the velocity versus time curve. Explain your thinking for full credit.

Using the definition of acceleration and solving for the velocity,

$$a \equiv \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt \Rightarrow \Delta v = \int adt$$

So the change in velocity is equal to the area under the acceleration time curve. Since the acceleration is constant between 2 and 8s, the velocity is linear. The velocity presumably goes to zero when it hits the wall.

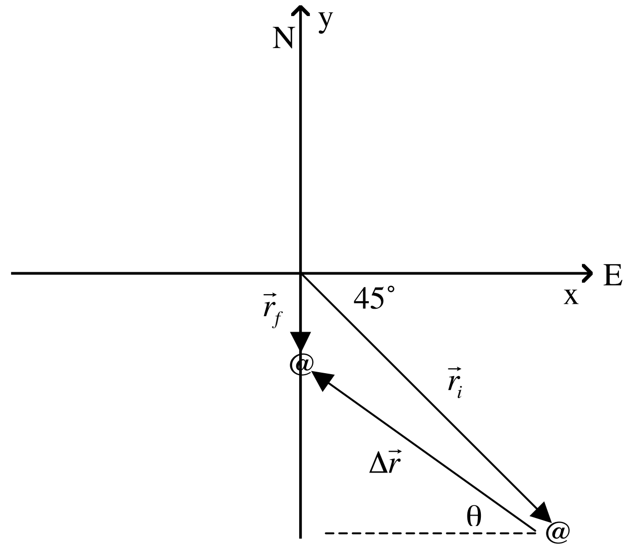
(d)Graph the position versus time curve. Explain your thinking for full credit.

Using the definition of velocity and solving for the displacement,

$$v \equiv \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt \Rightarrow \Delta x = \int vdt$$

So the change in position is equal to the area under the velocity time curve. Since the velocity is constant between 2 and 8s, the position can be found from the area of triangles. The result is a parabola.

2. A hurricane is 400km southeast of New Orleans. Six hours later it is 100km due south of New Orleans. (a)Show the initial and final positions of the hurricane at the right. Assume that New Orleans is at the origin. (b)Find the displacement of the hurricane during this time (magnitude and direction). (c)Find the average speed (not velocity) of the hurricane during this time.



(a)Breaking the vectors into components,

$$r_{ix} = r_i \cos 45^\circ = (400)\cos 45^\circ = 283km \qquad r_{fx} = 0$$

$$r_{iy} = -r_i \sin 45^\circ = (-400)\sin 45^\circ = -283km \quad r_{fy} = -100km$$

Using the definition of displacement,

$$\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i = \left(r_{fx}\hat{i} + r_{fy}\hat{j}\right) - \left(r_{ix}\hat{i} + r_{iy}\hat{j}\right)$$

$$\Delta\vec{r} = \left(r_{fx} - r_{ix}\right)\hat{i} + \left(r_{fy} - r_{iy}\right)\hat{j} \Rightarrow \begin{cases} \Delta r_x = r_{fx} - r_{ix} = 0 - 283 = -283km \\ \Delta r_y = r_{fy} - r_{iy} = -100 - (-283) = +183km \end{cases}$$

Using the Pythagorean Theorem and the definition of the tangent,

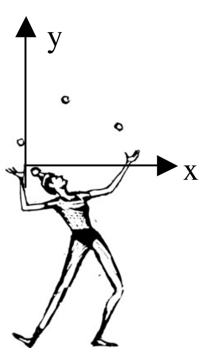
$$\Delta r = \sqrt{\Delta r_x^2 + \Delta r_y^2} = \sqrt{(-283)^2 + (+183)^2} \Rightarrow \boxed{\Delta r = 337km}.$$

$$\theta = \arctan\tfrac{\Delta r_y}{\Delta r_x} = \arctan\tfrac{183}{283} \Rightarrow \boxed{\theta = 32.9^\circ \text{ north of west}}.$$

(b)The speed is just the distance traveled per time,

$$v \equiv \tfrac{\Delta r}{\Delta t} = \tfrac{337}{6} \Rightarrow \boxed{v = 56.2km/h}.$$

3. A juggler needs 0.200s to catch and toss a given ball. If she is keeping three balls going, then two must be in the air while the third one is caught and tossed. Therefore, each ball must be in the air for at least 0.400s or, in other words, each ball must be tossed upward so it takes at least 0.200s to reach its maximum height. Find (a)the minimum initial vertical velocity for a ball and (b)the minimum height it must rise.



$y_o = 0$
$y = ?$
$v_o = ?$
$v = 0$
$a = -9.80 m/s^2$
$t = 0.200s$

(a)Using the kinematic equation without the final height,

$$v = v_o + at.$$

Setting the final velocity to zero and solving for the initial velocity,

$$0 = v_o + at \Rightarrow v_o = -at = -(-9.80)(0.200) \Rightarrow \boxed{v_o = 1.96m/s}.$$

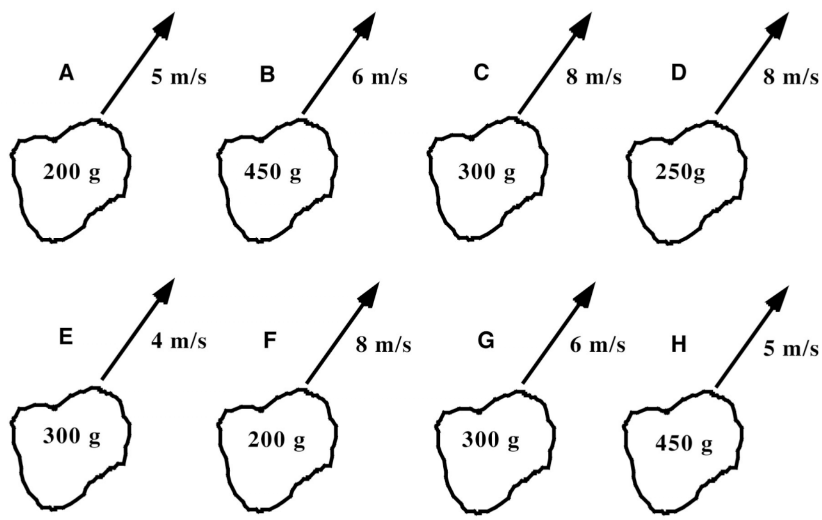
(b)Using the kinematic equation for the height,

$$y = y_o + v_o t + \tfrac{1}{2}at^2.$$

Plugging in the numbers,

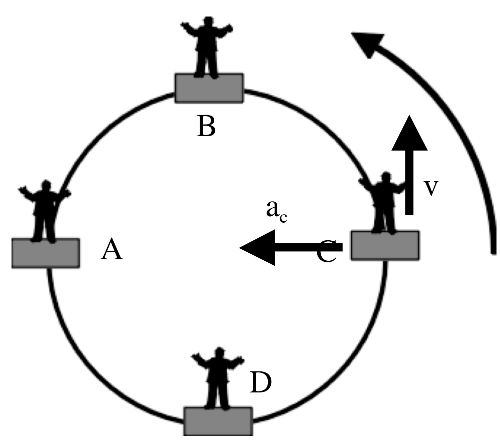
$$y = 0 + (1.96)(0.200) + \tfrac{1}{2}(-9.80)(0.200)^2 \Rightarrow \boxed{y = 0.196m}.$$

4. Shown below are eight rocks that have been thrown into the air. The rocks all have the same shape, but they have different masses. The rocks are all thrown at the same angle, but at different speeds. The masses of the rocks and their speeds, when released, are given in the figures. (We assume for this situation that the effect of air resistance can be ignored.) All start from the same height. Rank these rocks from greatest to least on the basis of the maximum heights the rocks reach. Be sure to explain your reasoning for full credit.



The Rule of Falling Bodies tells us that the mass doesn't matter. The principles of projectile motion indicate that only the vertical component of the velocity will affect the maximum height. Since the velocity vectors all point in the same direction, the vertical component will only depend on the magnitude of the vectors. So the one that goes the highest will be the one that is going fastest. Therefore, C=D=F>B=G>A=H>E.

5. A 12.0m diameter Ferris Wheel turns at a constant rate of 0.800 revolutions per minute. Find (a)the distance traveled by a passenger in one rotation, (b)the velocity (magnitude and direction) of the passenger when they are at position C half way up, and (c)the acceleration (magnitude and direction) at the same point. You may indicate directions in the drawing at the right.



(a)The circumference of a circle is,

$$c = 2\pi r = \pi d = (3.14)(12.0) \Rightarrow \boxed{c = 37.7m}.$$

(b)The time to go around once can be found from the frequency,

$$T \equiv \tfrac{1}{f} = \left(\tfrac{1m}{0.800rev}\right)\left(\tfrac{60s}{1m}\right) = 75.0s.$$

The definition of speed is distance traveled per time so, the speed is the circumference divided by the period.

$$v \equiv \tfrac{\Delta x}{\Delta t} = \tfrac{c}{T} = \tfrac{37.7}{75} \Rightarrow \boxed{v = 0.503m/s}.$$

(c)The acceleration is centripetal,

$$a_c = \tfrac{v^2}{r} = \tfrac{(0.503)^2}{6.00} \Rightarrow \boxed{a_c = 0.0421m/s^2}.$$