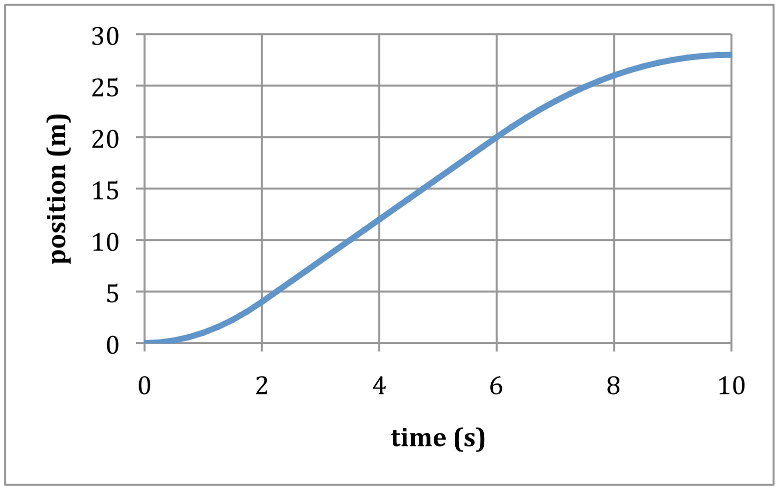
Name:____Answer Key________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

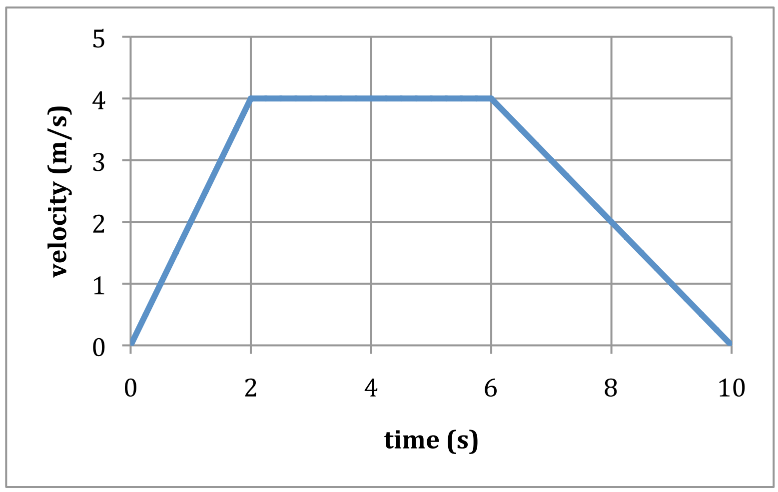
1. At the right is a graph of the velocity of a toy car as a function of time. Draw the graphs of position versus time and acceleration versus time. For full credit you need to explain your reasoning.



The definition of acceleration,

$$a \equiv \frac{dv}{dt},$$

means that the slope of the velocity time graph is the acceleration. So, the slope has three different constant values and is graphed at the right.



The definition of velocity,

$$v \equiv \frac{dx}{dt},$$

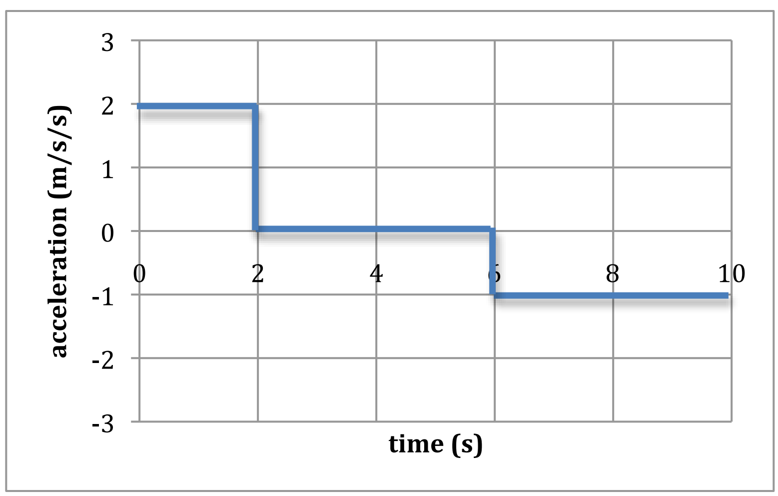
can be rewritten as,

$$dx = vdt .$$

Integrating,

$$\int dx = \int v\,dt \Rightarrow \Delta x = \int v\,dt ,$$

which means that the area under the velocity time graph is equal to the displacement. Note that the area grows as t^2 from t = 0 to t=2s and from t=6 to t=10s. The area grows linearly from t=2 to t=6s.



2. Six balls with different masses are thrown straight upward with different speeds. The masses and speeds are indicated for each ball. They are all released from the same height and you can assume the air resistance is negligible. Rank them from greatest to least based upon. (a)the maximum height they reach, (b)their speed when they reach maximum height, (c)their acceleration on the way up, (d)their acceleration on the way down, and (e)their acceleration when they are at maximum height. Explain your reasoning for full credit.

A	B	C	D	E	F
0.046kg	0.13kg	7.2kg	0.40kg	0.25kg	0.60kg
5m/s	8m/s	10m/s	8m/s	5m/s	12m/s

By the Rule of Falling Bodies, the mass of the balls don't make any difference since they all will have the same acceleration.

(a)The maximum height will only depend upon their initial speed. So,
F > C > B = D > A = E.

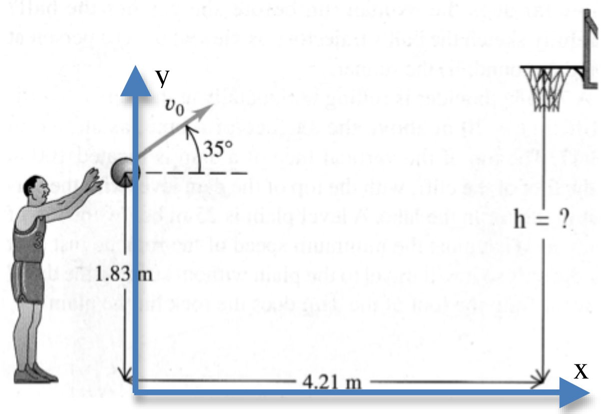
(b)The speed at the maximum height must be zero for all the balls or they would still be moving upward.
A = B = C = D = E = F.

(c)According to the Rule of Falling Bodies, they will all have the same acceleration,
A = B = C = D = E = F.

(d)According to the Rule of Falling Bodies, they will all have the same acceleration,
A = B = C = D = E = F.

(e)At the top of their motion, the velocity is zero for an instant, but they are changing speed, so they are still accelerating. According to the Rule of Falling Bodies, they will all have the same acceleration,
A = B = C = D = E = F.

3. A free throw is made by shooting the ball at 8.65m/s at 35.0˚ above horizontal from 1.83m above the ground. The basket is 4.21m away. Find (a)the time the ball is in the air and (b)the height of the basket.



Given:

$x_0 = 0$	$y_0 = 1.83m$
$x = 4.21m$	$y = ?$
$v_{ox} = (8.65)\cos 35^\circ$ $= 7.09m/s$	$v_{oy} = (8.65)\sin 35^\circ$ $= 4.96m/s$
$v_x = 7.09m/s$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	

Find: t = ? and y = ?

(a)Use the kinematic equation without the final speed along the x-direction,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 = 0 + v_{ox}t + 0 = v_{ox}t\ .$$

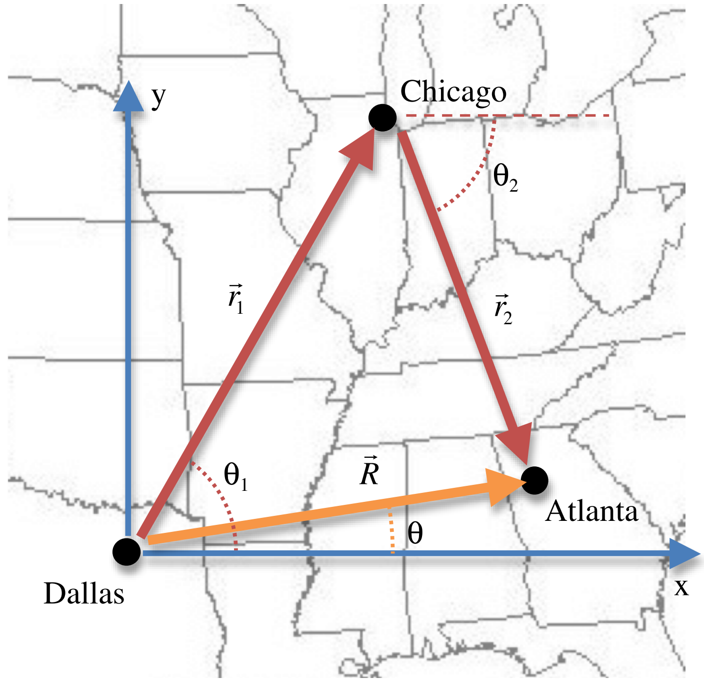
Solve for the time,

$$x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{4.21}{7.09} \Rightarrow \boxed{t = 0.594s}.$$

(b)Use the kinematic equation without the final speed along the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 = 1.83 + (4.96)(0.594) - \tfrac{1}{2}(9.8)(0.594)^2 \Rightarrow \boxed{y = 3.05m}.$$

4. A plane flies 788miles at 48.0° north of east to go from Dallas to Chicago. The plane then travels 560miles at 69.0° south of east to get to Atlanta. (a)Draw the two displacement vectors on the map at the right. (b)Find the distance and direction that a plane would have to travel to go directly from Dallas to Atlanta.



Given:
$r_1 = 788mi$, $r_2 = 560mi$, $\theta_1 = 48.0°$,
and $\theta_2 = 69.0°$
Find: R = ? and θ = ?

(a)see sketch

(b)From the sketch we can see that $\vec{R} = \vec{r}_1 + \vec{r}_2$.
Finding the components of each vector,

$$r_{1x} = r_1 \cos\theta_1 = 788\cos 48° = 527mi \qquad r_{1y} = r_1 \sin\theta_1 = 788\sin 48° = 586mi$$

$$r_{2x} = r_2 \cos\theta_2 = 560\cos 69° = 201mi \qquad r_{2y} = r_2 \sin\theta_2 = -560\sin 69° = -523mi$$ (minus because it is along –y)

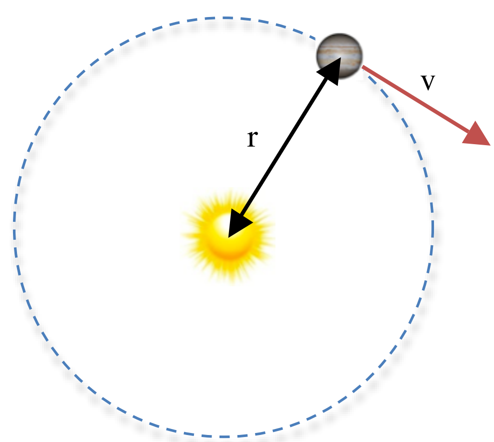
Adding the components,

$R_x = r_{1x} + r_{2x} = 527 + 201 = 728mi$ and $R_y = r_{1y} + r_{2y} = 586 + (-523) = 63mi$.

Using the Pythagorean Theorem and the Definition of Tangent,

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{728^2 + 63^2} \Rightarrow \boxed{R = 731mi} \quad \text{and} \quad \theta = \arctan\left(\frac{R_y}{R_x}\right) = \arctan\left(\frac{63}{728}\right) \Rightarrow \boxed{\theta = 4.9°}.$$

5. We are now beginning to find many planets that orbit other stars. One such planet is found to have an orbital period of $3.00x10^7$s and an orbital speed of $2.40x10^4$m/s. Find (a)the radius of the planet's orbit and (a)the acceleration of the planet in its orbit.



Given: $T = 3.00x10^7$s and $v = 2.40x10^4$m/s
Find: r = ? and a = ?

(a)Using the definition of speed,

$$v \equiv \frac{dx}{dt} = \frac{2\pi r}{T} \Rightarrow r = \frac{vT}{2\pi} = \frac{(2.4x10^4)(3x10^7)}{2\pi} \Rightarrow \boxed{r = 1.15x10^{11}m}.$$

(b)Using the centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{(2.4x10^4)^2}{7.2x10^{11}} \Rightarrow \boxed{a_c = 5.02x10^{-3}\,\tfrac{m}{s^2}}.$$