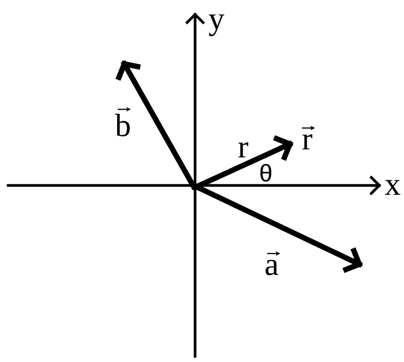
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The vector $\vec{a} = 4.00\hat{i} - 3.00\hat{j}$ and the vector $\vec{b} = -2.00\hat{i} + 4.00\hat{j}$. (a)Sketch the vectors $\vec{a}$ and $\vec{b}$. (b)Find the x and y components of $\vec{r} = \vec{a} + \vec{b}$. (c)Sketch $\vec{r}$. (d)Find the magnitude and direction of $\vec{r}$.



(a)see sketch

(b) $\vec{r} = \vec{a} + \vec{b} = (4\hat{i} - 3\hat{j}) + (-2\hat{i} + 4\hat{j}) = (4-2)\hat{i} + (-3+4)\hat{j}$

$$\boxed{\vec{r} = 2\hat{i} + \hat{j}}.$$

(c)see sketch

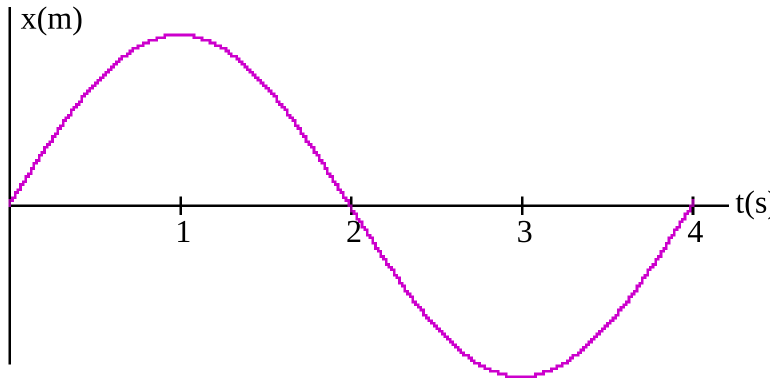
(d)Using the Pythagorean Theorem,

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{2^2 + 1^2} \qquad \boxed{r = 2.24}.$$

and the definition of tangent

$$\tan\theta \equiv \frac{r_y}{r_x} \qquad \theta = \arctan\frac{r_y}{r_x} = \arctan\frac{1}{2} \qquad \boxed{\theta = 26.6^\circ}.$$

2. Using the graph of position versus time for an object shown at the right, estimate the time or times when the object



(a)is to the right of the origin,

x>0 from t = 0s to t = 2s.

(b)is to the left of the origin,

x<0 from t = 2s to t = 4s.

(c)is at the origin, x = 0 when t = 0s, 2s, and 4s.

(d)has a positive velocity, The velocity is the slope. v>0 from t = 0s to t =1s and t = 3s to t = 4s.

(e)has a negative velocity, v<0 from t = 1s to t =3s.

(f)has a velocity of zero, v=0 at t = 1s and t =3s.

(g)has a positive acceleration, The acceleration is the rate at which the slope is changing.

The slope is increasing (a>0) from t = 2s to t = 4s.

(h)has a negative acceleration, The slope is decreasing (a<0) from t = 0s to t = 2s.

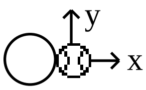
(i)has a acceleration of zero. The slope must not be changing (a = 0) in between at t = 2s.

The acceleration is also probably zero at t = 0s and 4s as well.

(j)Find the total displacement of the object.

The object winds up where it started so the total displacement must be zero.

3. A baseball is thrown at 40.0m/s. Mark McGwire hits it back in the opposite direction at a speed of 65.0m/s. The ball is in contact with the bat for 1.20ms. Find (a)the acceleration of the ball assuming it is constant and (b)the distance the ball travels while it is in contact with the bat.



$x_o = 0$
$x = ?$
$v_o = -40.0\text{m/s}$
$v = +65.0\text{m/s}$
$a = ?$
$t = 1.20\text{ms}$

(a)Use the kinematic equation without the final position,

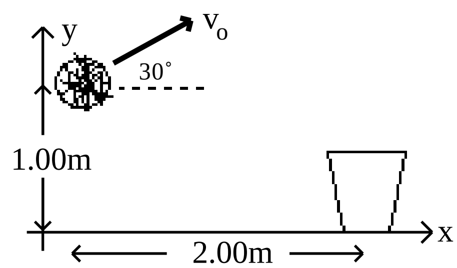
$$v = v_o + at \qquad a = \frac{v - v_o}{t} = \frac{65 - (-40)}{1.2\text{x}10^{-3}} \qquad \boxed{a = 8.75\text{x}10^{4}\,\text{m/s}^2}.$$

(b)Use the kinematic equation without the acceleration,

$$\frac{x - x_o}{t} = \frac{1}{2}(v_o + v) \qquad x = \left(\frac{v_o + v}{2}\right)t$$

$$x = \left(\frac{-40 + 65}{2}\right)(1.2\text{m}) \qquad \boxed{x = 15.0\text{mm}}.$$

4. A wad of paper is tossed into a wastebasket 2.00m away. It is released from a height of 1.00m at an angle of 30.0˚. Find the initial speed required for it to land in the center of the bottom assuming that air resistance is negligible.



$x_o = 0$	$y_o = 1.00\text{m}$
$x = 2.00\text{m}$	$y = 0$
$v_{ox} = v_o \cos 30^\circ$	$v_{oy} = v_o \sin 30^\circ$
$v_x = v_{ox}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80\text{m/s}^2$
$t = ?$	

Using the kinematic equation for the final x-position,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad x = v_{ox}t = v_o t\cos 30^\circ \qquad t = \frac{x}{v_o \cos 30^\circ}$$

and the kinematic equation for the final y-position,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \qquad 0 = y_o + v_o t\sin 30^\circ + \tfrac{1}{2}a_y t^2 .$$

Substituting for t from above,

$$0 = y_o + v_o \frac{x}{v_o \cos 30^\circ}\sin 30^\circ + \tfrac{1}{2}a_y\left(\frac{x}{v_o \cos 30^\circ}\right)^2 = y_o + x\frac{\sin 30^\circ}{\cos 30^\circ} + \tfrac{1}{2}a_y\left(\frac{x}{v_o \cos 30^\circ}\right)^2 .$$

Solving for the initial speed,

$$y_o + x\frac{\sin 30^\circ}{\cos 30^\circ} = -\tfrac{1}{2}a_y\left(\frac{x}{v_o \cos 30^\circ}\right)^2 \qquad v_o = \sqrt{\frac{-a_y x^2}{2y_o \cos^2 30^\circ + 2x\sin 30^\circ \cos 30^\circ}}$$

Putting in the numbers,

$$v_o = \sqrt{\frac{-(-9.8)(2)^2}{2(1)\cos^2 30^\circ + 2(2)\sin 30^\circ \cos 30^\circ}} \qquad \boxed{v_o = 3.48\text{m/s}}.$$

5. A television satellite must appear stationary in the sky so that the satellite dish doesn't have to move as it orbits. This means that the satellite completes precisely one orbit each day. The radius of the satellites orbit is $4.22\text{x}10^4$km. Find the acceleration due to gravity felt by this satellite. Is your answer consistent with the Rule of Falling Bodies? Explain.

The period of the orbit is, $T = (1\text{day})\left(\frac{24\text{h}}{\text{day}}\right)\left(\frac{3600\text{s}}{\text{h}}\right) = 8.64\text{x}10^4\text{s}$.

The speed of orbit can be found from the definition of speed,

$$v \equiv \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T} = \frac{2\pi(4.22\text{x}10^7)}{8.64\text{x}10^4} = 3070\text{m/s}.$$

The centripetal acceleration is,

$$a_c = \frac{v^2}{r} = \frac{(3070)^2}{4.22\text{x}10^7} \qquad \boxed{a_c = 0.223\text{m/s}^2}.$$

The Rule of Falling Bodies only applies to objects near the surface of Earth. This object is far away compared to the radius of Earth ($6.37\text{x}10^6$m).