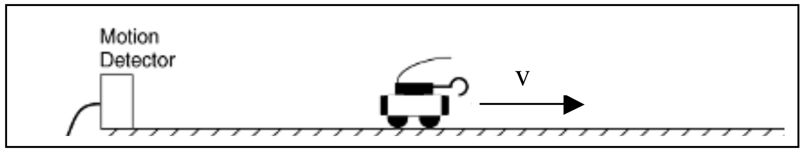
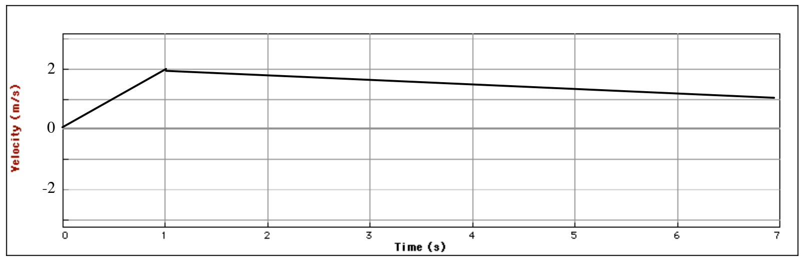
Name:___________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 300g cart initially at rest is given a push to the right along a level track beginning at t = 0s. The motion sensor plots the velocity as a function of time as shown in the graph at the right. Answer the following questions. Be sure to explain your answers for full credit.





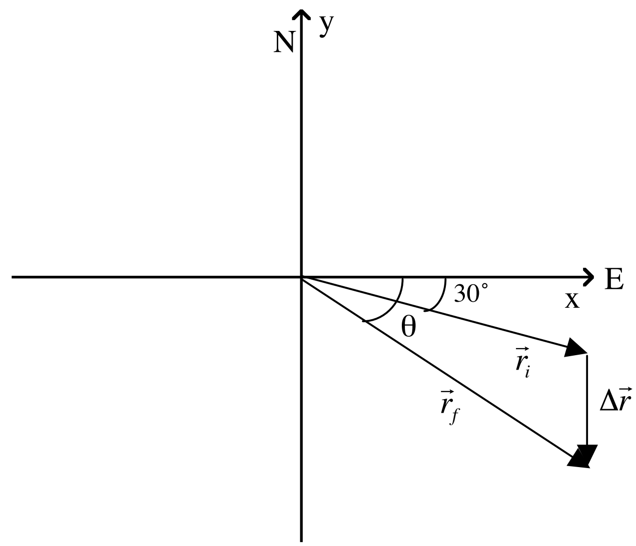
(a)How much time elapses while the cart being pushed? How do you know?
According to the graph, the object is speeding up for about $\boxed{1.0s}$, so this is the duration of the push.

(b)How far does the cart travel while being pushed? How do you know?
Using the definition of velocity, $v \equiv \frac{dx}{dt} \Rightarrow \Delta x = \int v dt$, the distance traveled must be the area under the velocity vs. time curve. $\Delta x \approx \frac{1}{2}(2.0)(1.0) \Rightarrow \boxed{\Delta x \approx 1.0m}$.

(c)What is the acceleration of the cart travel while being pushed? How do you know?
Using the definition of acceleration, $a \equiv \frac{dv}{dt}$, the acceleration must be the slope of the velocity vs. time curve. $a \approx \frac{2.0}{1.0} \Rightarrow \boxed{a \approx 2.0 m/s^2}$.

(d)Describe the motion of the cart travel after the push is complete.
The velocity slows gently as the cart continues to move away from the sensor. The slight deceleration is probably due to friction.

2. A cruise ship is 200km away from Houston at 30.0° south of east. The ship travels 100km due south. (a)Show the initial position, final position, and displacement at the right. (b)Find the final position (magnitude and direction) of the ship.



The head-to-tail rule suggests,
$\vec{r}_f = \vec{r}_i + \Delta\vec{r}$.
Breaking the vectors into components,
$r_{ix} = r_i \cos 30.0° = 200 \cos 30.0° = 173km$,
$r_{iy} = -r_i \sin 30.0° = -200 \sin 30.0° = -100km$,
$\Delta r_x = 0km$, and $\Delta r_y = -100km$.
Adding up the components,
$r_{fx} = r_{ix} + \Delta r_x = 173 + 0 = 173km$ and
$r_{fy} = r_{iy} + \Delta r_y = -100 + (-100) = -200km$
The magnitude can be found from the Pythagorean Theorem,
$r_f = \sqrt{r_{fx}^2 + r_{fy}^2} = \sqrt{173^2 + (-200)^2} \Rightarrow \boxed{r_f = 265km}$.
Using the definition of the tangent, $\theta = \arctan\left(\frac{-r_{fy}}{r_{fx}}\right) = \arctan\left(\frac{200}{173}\right) \Rightarrow \boxed{\theta = 49.1°}$.

3. Three rocks are each thrown from the edge of a 20.0m high cliff. They all end up at the base of the cliff. Rock A is thrown straight downward with an initial speed of 10.0m/s. Rock B is dropped from rest. Rock C this thrown upward with an initial speed of 10.0m/s. Assume that the coordinates use up as positive. Considering only the time from when they are released until just before they strike the ground, rank these rocks from largest to smallest according to:

(a)the distance they travel.
The total distance traveled is longer for the rock that travels upward initially, $\boxed{C>A=C}$.

(b)their displacement.
Displacement is just the final position minus the initial position, so it is the same for all, $\boxed{A=B=C}$.

(c)their initial speed.
Since speed doesn't include direction, so A and C are equal, $\boxed{A=C>B}$.

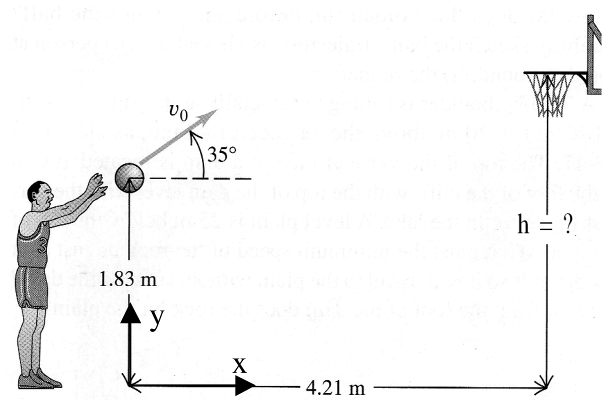
(d)their initial velocity.
Velocity includes a sign for direction so $\boxed{C>B>A}$.

(e)their final speed.
A and C end up with the same final speed at the bottom, while B will be moving more slowly $\boxed{A=C>B}$.

(f)rank these rocks from smallest to largest according to their final velocity.
They are all moving downward, so they all have the same sign, but A & C are more negative $\boxed{B>A=C}$.

(g)rank these rocks from smallest to largest according to their acceleration.
According to the Rule of Falling Bodies, they all have the same acceleration $\boxed{A=B=C}$.

4. A free throw is made by shooting the ball at 8.65m/s at 35.0° above horizontal from 1.83m above the ground. The basket is 4.21m away. Find (a)the time the ball is in the air and (b)the height of the basket.



$x_o = 0$	$y_o = 1.83m$
$x = 4.21m$	$y = ?$
$v_{ox} = 8.65\cos 35^\circ = 7.09m/s$	$v_{oy} = 8.65\sin 35^\circ = 4.96m/s$
$v_x = 7.09m/s$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	$t = ?$

Using the kinematic equation without v_x in the x-direction,

$x = x_o + v_{ox}t + \frac{1}{2}a_x t^2 \Rightarrow x = v_{ox}t \Rightarrow t = \dfrac{x}{v_{ox}}$ (You can put in numbers here if you like $t = \dfrac{4.21}{7.09} = 0.594s$).

Using the kinematic equation without v_x in the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow y = y_o + v_{oy}\left(\frac{x}{v_{ox}}\right) + \tfrac{1}{2}a_y\left(\frac{x}{v_{ox}}\right)^2.$$

Plugging in the numbers,

$$y = 1.83 + 4.96\left(\frac{4.21}{7.09}\right) - \tfrac{1}{2}(9.80)\left(\frac{4.21}{7.09}\right)^2 \Rightarrow \boxed{y = 3.05m}.$$

5. Earth has a radius of 6380km and spins around once every 24.0h day. (a)Find the acceleration (in m/s^2) of a person standing at the equator. (b)Find the radius of a planet that has the same spin rate as Earth, but the acceleration of a person on the equator is $9.80m/s^2$.

(a)The acceleration is centripetal, $a_c = \dfrac{v^2}{r}$.

The speed is the distance around divided by the period, $v = \dfrac{2\pi r}{T} \Rightarrow a_c = \dfrac{4\pi^2 r}{T^2}$.

Plugging in the numbers, $a_c = \dfrac{4\pi^2(6380x10^3 m)}{(24h\cdot\frac{3600s}{h})^2} \Rightarrow \boxed{a_c = 0.0337m/s^2}$.

(b)Again, the acceleration is centripetal, $a_c = \dfrac{v^2}{r}$,

and the speed is the distance around divided by the period, $v = \dfrac{2\pi r}{T} \Rightarrow a_c = \dfrac{4\pi^2 r}{T^2}$.

Solving for r, $r = \dfrac{a_c T^2}{4\pi^2} = \dfrac{(9.80)(24\cdot 3600)^2}{4\pi^2} \Rightarrow \boxed{r = 1.85x10^9 m}$.