Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles shown on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A bug flying northward at 8.00m/s collides with the windshield of a car traveling southward at 20.0m/s. Answer the following questions. For full credit, you must explain your thinking. Be sure to cite any relevant principles of physics. Which object, the bug or the car:

(a)feels the greater force during the collision? BOTH THE SAME

Newton's Third Law states that if one object exerts a force on a second object the second object always exerts an equal force back on the first object.

(b)has the greater acceleration during the collision? BUG

Newton's Second Law states that the acceleration of an object is proportional to the net force acting on it and inversely proportional to its mass. Since both objects feel the same size force, the one with the smaller mass will have the larger acceleration.

(c)has the greater impulse on it during the collision? BOTH THE SAME

The definition of impulse is the product of force and time. Since the bug and the car both feel the same force over the same time, they both feel the same impulse.

(d)has the greater change in momentum during the collision? BOTH THE SAME

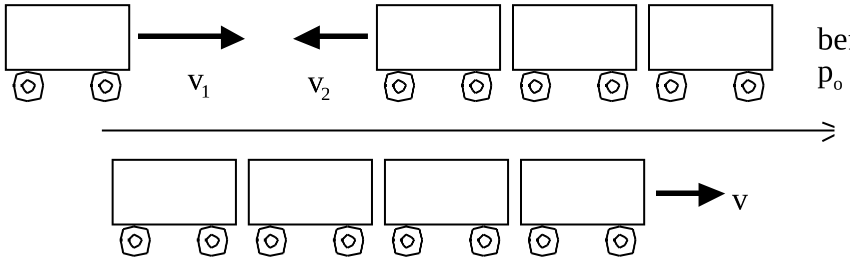
The impulse-momentum theorem states that the impulse equals the change in momentum. Since they both feel the same impulse, they both have the same change in momentum.

(e)has the greater momentum after the collision? CAR

The definition of momentum is the product of mass and velocity. After the bug is splattered on the windshield, it has the same speed as the car. Since the car has a much greater mass and is going at the same speed the bug, it must have the higher momentum.

2. A railroad car traveling east at a speed of 4.00m/s collides and couples with three identical cars traveling in the opposite direction at 2.00m/s. Find the velocity of the fours coupled cars just after the collision. Is the collision elastic? Explain you answer quantitatively.

Given:
v_1 = 4.00 m/s
v_2 = 2.00 m/s
Find:
v = ?



before
$p_o = mv_1 - 3mv_2$

after
p = 4mv

(a)Using the Law of Conservation of Momentum,

$$mv_1 - 3mv_2 = 4mv \qquad v = \tfrac{1}{4}(v_1 - 3v_2) = \tfrac{1}{4}(4 - 3\ 2) \qquad \boxed{v = -0.500\,m/s}$$. This is toward the west.

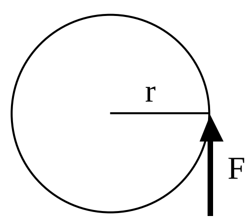
(b)Using the definition of kinetic energy,

$$K_o = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}3mv_2^2 = \tfrac{1}{2}m(4)^2 + \tfrac{1}{2}3m(2)^2 = 14m \text{ and } K = \tfrac{1}{2}4mv^2 = \tfrac{1}{2}4m(\tfrac{1}{2})^2 = \tfrac{1}{2}m.$$

Since the kinetic energy isn't conserved, the collision is $\boxed{\text{inelastic}}$.

3. A 4.00m diameter merry-go-round is sped up from rest to 10.0rpm in 8.00s by a force of 120N exerted tangentially on its edge. Assuming that there is no friction, find the rotational inertia of the merry-go-round.

Given:
r = 2.00m
$\omega_o = 0$
ω = 10.0rpm = 1.05rad/s
Δt = 8.00s
F = 120N
Find:
I = ?



Using the definition of angular acceleration,

$$\alpha \equiv \frac{d\omega}{dt} = \frac{\omega - \omega_o}{\Delta t} = \frac{\omega}{\Delta t}.$$

Using the Second Law for Rotation,

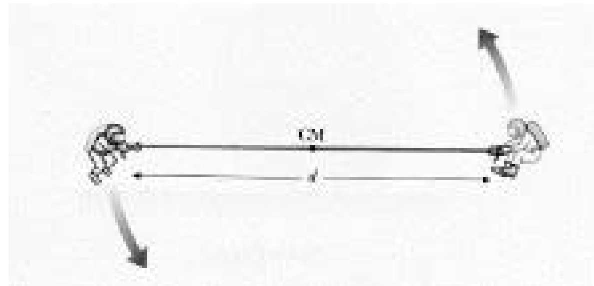
$$\Sigma\tau = I\alpha \qquad \tau = I\frac{\omega}{\Delta t},$$

and the definition of torque,

$$\tau = rF \qquad rF = I\frac{\omega}{\Delta t} \qquad I = \frac{rF\Delta t}{\omega}.$$

Plugging in the numbers, $I = \dfrac{(2.00)(120)(8.00)}{1.05}$ $\boxed{I = 1830\,kg\ m^2}$.

4. Two astronauts each have a mass of 75.0kg are initially connected by a 10.0m long rope of negligible mass. They are isolated in space and orbit their center of mass with a speed of 5.00m/s. They then begin to pull in on the rope until they are only 5.00m apart. Find (a)their initial kinetic energy (b)their final speed, (c)their final kinetic energy and (d)the work they have done.



(a)Using the definition of kinetic energy,

$K \equiv \frac{1}{2}mv^2 \qquad K_o = \frac{1}{2}mv_o^2 + \frac{1}{2}mv_o^2 = mv_o^2 = (75.0)(5.00)^2 \qquad \boxed{K_o = 1880J}$.

(b)Using the definition of angular momentum,

$\vec{L} \equiv \vec{r} \times \vec{p} \qquad L_o = r_o m v_o + r_o m v_o = 2r_o m v_o$ and $L = rmv + rmv = 2rmv$.

Applying the Law of Conservation of Angular Momentum,

$L = L_o \qquad 2rmv = 2r_o m v_o \qquad v = v_o \frac{r_o}{r} = (5.00)\frac{5.00}{2.50} \qquad \boxed{v = 10.0m/s}$.

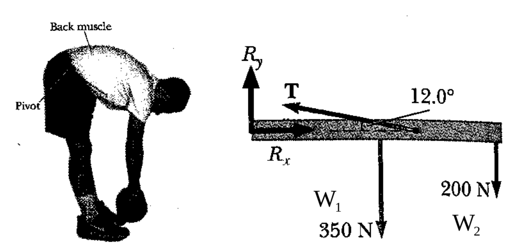
(c)Again, using the definition of kinetic energy,

$K \equiv \frac{1}{2}mv^2 \qquad K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 = (75.0)(10.0)^2 \qquad \boxed{K = 7500J}$.

(d)Using the work-energy theorem,

$W = \Delta K \qquad W = K - K_o = 7500 - 1880 \qquad \boxed{W = 5620J}$.

5. A person bends over and lifts a 200N weight as shown with his back in a horizontal position. The back muscle is attached two-thirds of the way up the spine and makes a 12° angle with the spine. Assuming the weight of the upper part of the body is 350N and acts at the center of the spine, find the tension in the back muscle and the horizontal and vertical components of the force exerted by the base of the spine.



Using the Second Law and taking the torques about the base of the spine,

$\Sigma F_x = ma_x \qquad R_x - T\cos 12° = 0 \qquad R_x = T\cos 12°$

$\Sigma F_y = ma_y \qquad R_y + T\sin 12° - W_1 - W_2 = 0 \qquad R_y = W_1 + W_2 - T\sin 12°$

$\Sigma\tau_o = I\alpha \qquad \frac{2}{3}\ell T\sin 12° - \frac{1}{2}\ell W_1 - \ell W_2 = 0 \qquad T = \frac{\frac{1}{2}W_1 + W_2}{\frac{2}{3}\sin 12°} = \frac{\frac{1}{2}(350) + 200}{\frac{2}{3}\sin 12°} \qquad \boxed{T = 2710N}$.

Substituting back into the force equations,

$R_x = T\cos 12° = (2710)\cos 12° \qquad \boxed{R_x = 2650N}$.

$R_y = W_1 + W_2 - T\sin 12° = 350 + 200 - 2710\sin 12° \qquad \boxed{R_y = -13.4N}$.

Note that this is opposite to the direction shown in the sketch.