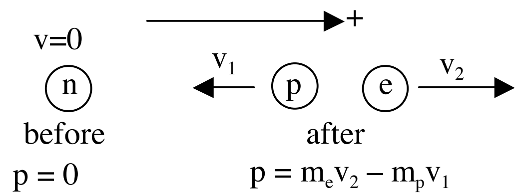
Name:_______________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles shown on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. It turns out that neutrons are not stable particles. A neutron will decay into a proton and an electron. The mass of a proton is 1833 times the mass of an electron. Suppose a neutron, initially at rest, breaks apart into an electron and a proton. The total kinetic energy of the electron and proton together is K. Find (a)the net momentum of the electron and the proton as a system and (b)the kinetic energy of the proton by itself as a fraction of the total kinetic energy.



(a)Since the neutron is initially at rest, the total momentum of the system is zero before the decay. By the Law of Conservation of Momentum, the momentum of the proton plus the electron will also be zero after the decay.

(b)Applying the Law of Conservation of Momentum,

$$0 = m_e v_2 - m_p v_1 \Rightarrow v_2 = \frac{m_p}{m_e} v_1$$

The kinetic energy is, $K = \frac{1}{2} m_e v_2^2 + \frac{1}{2} m_p v_1^2$.

Substituting, $K = \frac{1}{2} m_e \left(\frac{m_p}{m_e} v_1\right)^2 + \frac{1}{2} m_p v_1^2 = \frac{1}{2} \frac{m_p^2}{m_e} v_1^2 + \frac{1}{2} m_p v_1^2 = \frac{1}{2}\left(\frac{m_p}{m_e} + 1\right) m_p v_1^2 = \left(\frac{m_p}{m_e} + 1\right) K_p$

Solving for the kinetic energy of the proton, $K = \left(\frac{m_p}{m_e} + 1\right) K_p = (1833 + 1) K_p \Rightarrow \boxed{K_p = \frac{1}{1834} K}$.

2. Find the torque that a pitcher must exert on a 150g baseball with a radius of 3.50cm to get it to go from rest to a spin rate of 2400rpm in the 0.200s it takes to "snap" their wrist.

Using the definition of angular acceleration,

$$\alpha \equiv \frac{d\omega}{dt} \approx \frac{\omega - \omega_o}{\Delta t} = \frac{\omega}{\Delta t} = \frac{\left(2400 \frac{rev}{\min}\right)\left(\frac{\min}{60s}\right)\left(2\pi \frac{rad}{rev}\right)}{0.200s} \Rightarrow \alpha = 1260 \frac{rad}{s^2}.$$

Using the Second Law for Rotation,

$$\Sigma\tau = I\alpha \Rightarrow \tau = I\alpha.$$

The rotational inertia of a solid sphere is,

$$I = \frac{2}{5} mr^2.$$

Substituting,

$$\tau = \frac{2}{5} mr^2 \alpha = \frac{2}{5}(0.150)(0.0350)^2(1260) \Rightarrow \boxed{\tau = 0.0926 N \cdot m}.$$

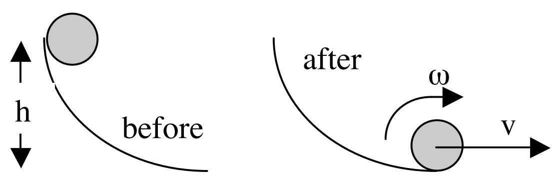
3. A basketball player cannot balance a basketball on her fingertip unless the ball is spinning. Explain this in terms of the relevant principles of physics.

When the ball is not spinning, it has no angular momentum. When is it spinning, it has an angular momentum vector pointing either upward or downward depending on the direction it spins. Second Law for Rotation states that the rate of change of angular momentum is equal to the torque.

Here, the torque is provided by gravity when the center of mass of the ball is not directly over the fingertip. When the ball is not spinning, this torque causes a change from zero, so the new angular momentum is about the fingertip causes the ball to rotate about a horizontal axis and fall off the finger.

When the ball starts with a very large angular momentum vector the torque causes a relatively small change in the direction of the angular momentum vector meaning that the torque must act for a much longer time to get the ball to rotate off the finger. This extra time gives the player enough time to correct the balance of the ball.

4. A 7.00g quarter (25¢ coin) has a radius of 1.20cm dropped into the slot of a vending machine rolls down a curved ramp 30.0cm high without slipping. At the bottom of the ramp find (a)the speed of its center of mass of the coin and (b)its angular speed about the center of mass.



$U_o = mgh$
$K_o = 0$

$U = 0$
$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Applying the law of Conservation of Energy,

$$\Delta U + \Delta K = 0 \Rightarrow (0 - mgh) + \left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 - 0\right) = 0$$

Doing some algebra,

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$

Using the rotational inertia of a disk, $I = \frac{1}{2}mr^2$

and the fact that the coin is rolling without slipping, $v = r\omega \Rightarrow \omega = \frac{v}{r}$,

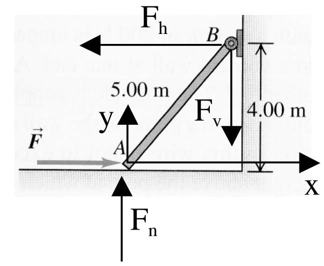
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{4}mv^2 = \tfrac{3}{4}mv^2 \Rightarrow$$

$$mgh = \tfrac{3}{4}mv^2 \Rightarrow v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.80)(0.300)}{3}} \Rightarrow \boxed{v = 1.98 m/s}.$$

(b)Rolling without slipping implies,

$$v = r\omega \Rightarrow \omega = \frac{v}{r} = \frac{1.98}{0.0120} \Rightarrow \boxed{\omega = 165 rad/s}.$$

5. The A end of bar AB rests on a frictionless horizontal surface while the B end is hinged. A horizontal 120N force is exerted on the A end. This force is so large that the weight of the bar can be ignored. Find (a)the horizontal component of the force exerted by the pivot and (b)the vertical component of the force exerted by the pivot.



Applying Newton's Second Law along the horizontal and vertical directions,

$$\Sigma F_x = ma_x \Rightarrow F - F_h = 0 \Rightarrow F = F_h \Rightarrow \boxed{F_h = 120N}.$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_v = 0 \Rightarrow F_n - F_v$$

Applying the Second Law for Rotation about the origin,

$$\Sigma \tau = I\alpha \Rightarrow 4F_h - 3F_v = 0 \Rightarrow 4F_h = 3F_v \Rightarrow F_v = \tfrac{4}{3}F_h = \tfrac{4}{3}(120N) \Rightarrow \boxed{F_v = 160N}.$$

Note that we have used the Pythagorean theorem to find the lever arm for the vertical force.