Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

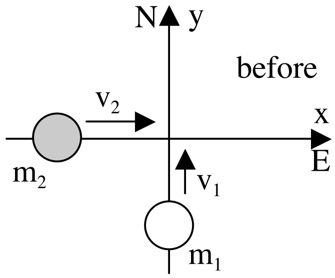
1. I saw the sketch below in the newspaper last summer. The author explained the benefits of using cable barriers instead of concrete on highways in simple terms. Now, you need to explain the same thing using the principles of physics.

The physics principle involved is the Impulse-Momentum Theorem.

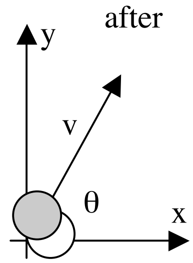
$$\Delta p = \int F dt$$

The change in momentum of a car colliding with the barrier is the same whether the barrier is made of concrete or cables because whatever the original momentum, it will presumably be reduced to zero. The difference between the barriers is on the right side of the equation. If the barrier is made of concrete, the time to bring the car to rest is very short compared to the time for a cable barrier. Therefore, the force exerted by the concrete barrier must be much larger than the force exerted by the cables. If the force on the car is smaller, damage to the vehicle and its passengers will be reduced.

2. A 75.0kg football player takes the ball and runs northward up the field at 5.00m/s. An 88.0kg tackler heading eastward across the field at 3.50m/s grabs the ball carrier. Find their combined speed and direction just as they collide.



$$p_{ox} = m_2 v_2$$
$$p_{oy} = m_1 v_1$$



$$p_x = (m_1 + m_2) v \cos\theta$$
$$p_y = (m_1 + m_2) v \sin\theta$$

Choosing the coordinates shown and finding the momentum along each direction separately allows the application of the Law of Conservation of Linear Momentum.

$$m_2 v_2 = (m_1 + m_2) v \cos\theta$$
$$m_1 v_1 = (m_1 + m_2) v \sin\theta$$

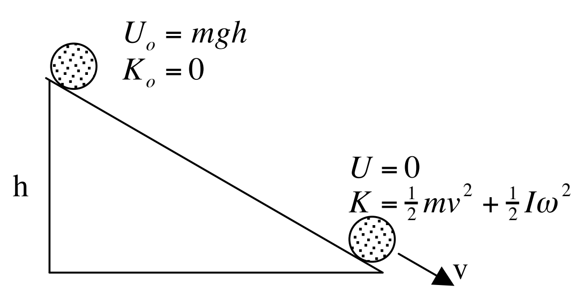
Solving the x-equation for v, $v = \dfrac{m_2 v_2}{(m_1 + m_2)\cos\theta}$, and substituting into the y-equation gives,

$$m_1 v_1 = (m_1 + m_2)\left[\frac{m_2 v_2}{(m_1 + m_2)\cos\theta}\right]\sin\theta = m_2 v_2 \frac{\sin\theta}{\cos\theta} = m_2 v_2 \tan\theta \Rightarrow \tan\theta = \frac{m_1 v_1}{m_2 v_2} \Rightarrow \theta = \arctan\left(\frac{m_1 v_1}{m_2 v_2}\right)$$

Plugging in the numbers, $\theta = \arctan\left(\dfrac{m_1 v_1}{m_2 v_2}\right) = \arctan\left[\dfrac{(75.0)(5.00)}{(88.0)(3.50)}\right] \Rightarrow \boxed{\theta = 50.6^\circ}$.

Using the equation for v, $v = \dfrac{m_2 v_2}{(m_1 + m_2)\cos\theta} = \dfrac{(88.0)(3.50)}{(75.0 + 88.0)\cos 50.6^\circ} \Rightarrow \boxed{v = 2.98 m/s}$.

3. A physics student is given a sphere and asked to determine whether it is hollow or solid. She makes a ramp 20.0cm high and lets the sphere roll from rest down the incline where she measures the final speed to be 1.53m/s. Find the type of sphere she was given.



Use the Law of Conservation of Energy to find the rotational inertia of the sphere,

$$\Delta U + \Delta K = 0 \Rightarrow (0 - mgh) + (\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2) = 0$$

Since the ball rolls without slipping, $\omega = \dfrac{v}{r}$.

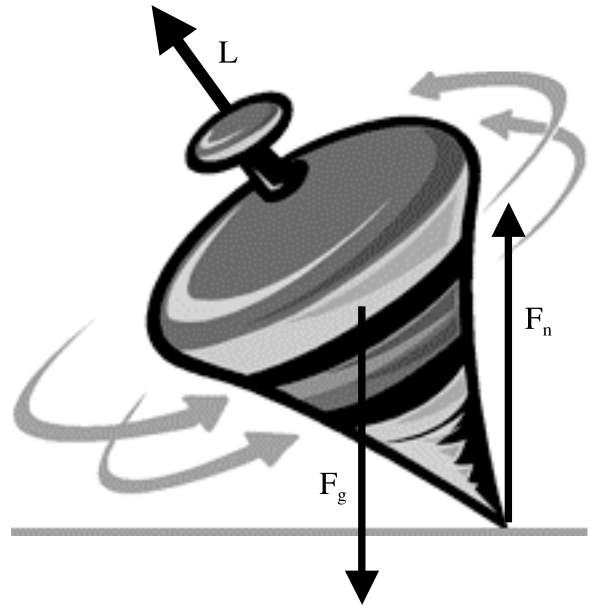
$$-mgh + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\frac{v^2}{r^2} = 0$$

Solving for the rotational inertia,

$$I = \left(\frac{2gh}{v^2} - 1\right)mr^2 = \left[\frac{2(9.80)(0.200)}{(1.53)^2} - 1\right]mr^2 \Rightarrow \boxed{I = (0.675)mr^2 \approx \tfrac{2}{3}mr^2}.$$

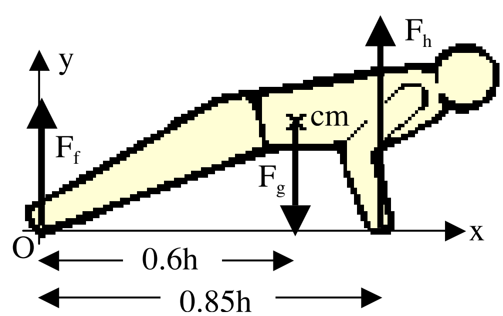
The rotational inertia of a solid sphere is, $I = \frac{2}{5}mr^2$ while for a hollow sphere it is, $I = \frac{2}{3}mr^2$. So this is most likely a $\boxed{\text{hollow sphere}}$.

4. The spinning top shown at the right is on a tabletop. There are two major forces acting on the top. (a)Name these forces. (b)Draw these forces where they act and clearly label them. (c)Describe the torque exerted by each force about the pivot point where the top touches the table. (d)Draw and clearly label the angular momentum vector for the top. (e)Explain why the top will precess and describe the direction of the precession.



(a)The two forces are gravity (weight) and the normal force.
(b)See drawing.
(c)The normal force exerts no torque because it acts at the pivot point. $\boxed{\tau_n = 0}$.
The torque exerted by gravity is $\boxed{\text{out of the page}}$ by the right hand rule.
(d)See drawing.
(e)Newton's Second Law for Rotation states that torques cause changes in angular momentum and that the change in angular momentum is in the direction of the torque. Since the torque is out of the page, the angular momentum vector will move toward me out of the page. This motion is called, "precession."

5. A 50.0kg athlete about to do a push-up lies horizontally with only her hands and toes touching the ground. Her center of mass is 60% of the way from her toes to her head and her hands are 85% of the way. Find (a)the force that ground exerts on her hands and (b)the force that her hands must exert on the ground.



(a)The forces that act on the athlete are shown at the right. Applying Newton's Second Law for the y-direction and for torques about the origin,

$$\Sigma F_y = ma_y \Rightarrow F_f + F_h - F_g = m(0) \Rightarrow F_f + F_h - mg = 0 \Rightarrow F_f = mg - F_h$$

$$\Sigma \tau_o = I\alpha \Rightarrow (0.85)F_h - (0.60)F_g = 0 \Rightarrow (0.85)F_h = (0.60)F_g \Rightarrow F_h = \frac{(0.60)F_g}{0.85} = \frac{(0.60)mg}{0.85}$$

Plugging the numbers into the torque equation,

$$F_h = \frac{(0.60)(50.0)(9.80)}{0.85} \Rightarrow \boxed{F_h = 346N}.$$

(b)By Newton's Third Law, the force that the ground exerts on her hands is equal (and opposite) to the force that her hands exert on the ground.