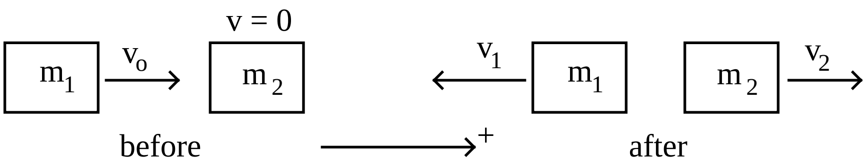
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 0.350kg block slides along a frictionless surface at 2.00m/s when it makes a head-on elastic collision with a 1.15kg block. Find the velocity of each block after the collision. *Note: The algebra is very messy so you may want to just set this problem up to get most of the credit and complete the algebra later if you have time.*



$p_o = m_1 v_o$ $\qquad$ $p = m_2 v_2 - m_1 v_1$

$K_o = \frac{1}{2} m_1 v_o^2$ $\qquad$ $K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

The Law of Conservation of Momentum requires, $m_1 v_o = m_2 v_2 - m_1 v_1$.

The fact that the collision is elastic means, $\frac{1}{2} m_1 v_o^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$.

Solving the Conservation of Momentum equation for v_1 gives, $v_1 = \frac{m_2}{m_1} v_2 - v_o$.

Substituting into the elasticity equation,

$$\frac{1}{2} m_1 v_o^2 = \frac{1}{2} m_1 \left[\frac{m_2}{m_1} v_2 - v_o\right]^2 + \frac{1}{2} m_2 v_2^2 \qquad v_o^2 = \left[\frac{m_2}{m_1} v_2 - v_o\right]^2 + \frac{m_2}{m_1} v_2^2 .$$

Completing the algebra,

$$v_o^2 = \left(\frac{m_2}{m_1}\right)^2 v_2^2 - 2\left(\frac{m_2}{m_1}\right) v_o v_2 + v_o^2 + \left(\frac{m_2}{m_1}\right) v_2^2 \qquad 0 = \left(\frac{m_2}{m_1}\right)^2 v_2^2 - 2\left(\frac{m_2}{m_1}\right) v_o v_2 + \left(\frac{m_2}{m_1}\right) v_2^2 ,$$

$$0 = \left(\frac{m_2}{m_1}\right) v_2 - 2v_o + v_2 \qquad \left(\frac{m_2}{m_1} + 1\right) v_2 = 2v_o \qquad v_2 = \frac{2v_o}{\frac{m_2}{m_1} + 1} .$$

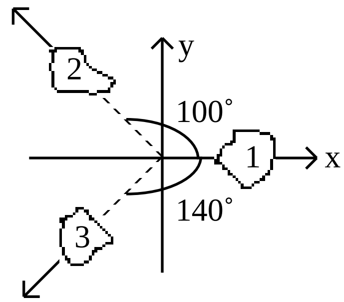
Putting in the numbers,

$$v_2 = \frac{2(2.00)}{\frac{1.15}{0.350} + 1} \qquad \boxed{v_2 = 0.933\text{m/s}}.$$

The solutions for v_1 can be found by substitution back into

$$v_1 = \frac{m_2}{m_1} v_2 - v_o = \frac{1.15}{0.350}(0.933) - 2.00 \qquad \boxed{v_1 = 1.07\text{m/s}}.$$

2. A pumpkin collides with the ground and breaks into three pieces of equal mass. Considering only the horizontal motion, the first piece heads off with a speed of 20.0m/s. The second moves away at a 100˚ angle with respect to the first and the third heads off at 140˚ as shown in the sketch. Find the speed of the second and third pieces, respectively.



The initial momentum in the x and y directions are both zero. The final momentum can be found by adding the components from the three pieces,

$$p_{ox} = 0 \qquad p_x = m_1v_1 - m_2v_2\cos 80^\circ - m_3v_3\cos 40^\circ$$
$$p_{oy} = 0 \qquad p_y = 0 + m_2v_2\sin 80^\circ - m_3v_3\sin 40^\circ.$$

Using the Law of Conservation of Momentum,

$$0 = m_1v_1 - m_2v_2\cos 80^\circ - m_3v_3\cos 40^\circ \qquad v_1 = v_2\cos 80^\circ + v_3\cos 40^\circ \qquad (1)$$
$$0 = m_2v_2\sin 80^\circ - m_3v_3\sin 40^\circ \qquad v_2\sin 80^\circ = v_3\sin 40^\circ \qquad (2)$$

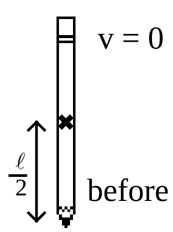
Solving eq. 2 for v_3, $v_3 = v_2\frac{\sin 80^\circ}{\sin 40^\circ}$ (3).
Substituting into eq. 1 and solving for v_2,

$$v_1 = v_2\cos 80^\circ + v_2\frac{\sin 80^\circ}{\sin 40^\circ}\cos 40^\circ \qquad v_2 = \frac{v_1}{\cos 80^\circ + \frac{\sin 80^\circ}{\sin 40^\circ}\cos 40^\circ}.$$

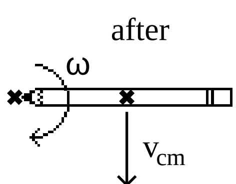
Plugging in the numbers, $v_2 = \dfrac{20.0}{\cos 80^\circ + \frac{\sin 80^\circ}{\sin 40^\circ}\cos 40^\circ}$ $\boxed{v_2 = 14.8\text{m/s}}$.

Substituting back into eq. 3, $v_3 = (14.8)\frac{\sin 80^\circ}{\sin 40^\circ}$ $\boxed{v_3 = 22.7\text{m/s}}$.

3. A 12.0cm long pencil is balanced temporarily on its point. The mass of the pencil is 20.0g. The pencil tips and the point of contact stays fixed as it fall. Find the speed of the center of mass just as it lands on the horizontal tabletop.



$K_o = 0$
$U_o = mg\frac{\ell}{2}$



$K = \frac{1}{2}I\omega^2$
$U = 0$

Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \qquad \left(\tfrac{1}{2}I\omega^2 - 0\right) + \left(0 - mg\tfrac{\ell}{2}\right) = 0 \qquad \tfrac{1}{2}I\omega^2 = mg\tfrac{\ell}{2}$$

The rotational inertia about the point of contact is,

$$I = \tfrac{1}{3}m\ell^2.$$

Since the pencil rotates without slipping,

$$v_{cm} = \tfrac{\ell}{2}\omega.$$

Substituting and solving for the speed of the center of mass,

$$\tfrac{1}{2}\left(\tfrac{1}{3}m\ell^2\right)\left(\frac{v_{cm}}{\frac{\ell}{2}}\right)^2 = mg\tfrac{\ell}{2} \qquad \tfrac{2}{3}v_{cm}^2 = g\tfrac{\ell}{2} \qquad v_{cm} = \sqrt{\tfrac{3}{4}g\ell}.$$

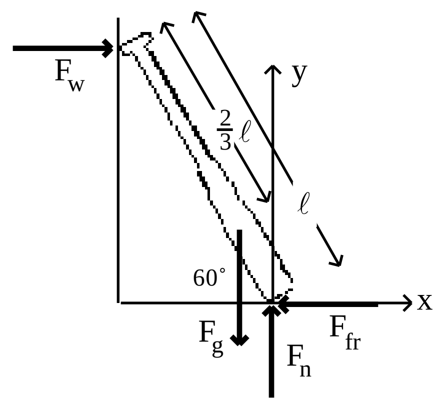
Plugging in the numbers,

$$v_{cm} = \sqrt{\tfrac{3}{4}(9.80)(0.120)} \qquad \boxed{v_{cm} = 0.939\text{m/s}}.$$

4. Between ice ages the polar ice caps melt substantially and the resulting liquid water is distributed throughout the world's oceans. Since the ice caps are near the poles and the oceans are uniformly distributed over the entire globe (more-or-less) the length of the day changes. Explain why and state whether the days get longer or shorter.

The water moves from near the axis of rotation at the poles to further away from the axis. This means that some of the mass of Earth is now further away from the axis. Therefor, Earth's rotational inertia becomes larger. Applying the Law of Conservation of Angular Momentum, the total angular momentum of Earth must remain fixed. Since angular momentum is the product of rotational inertia and angular speed and the rotational inertia has increased, the angular velocity of Earth must decrease. Since Earth is now spinning slower, the day must be longer.

5. A baseball bat leans against a smooth wall making a 60° angle with the ground. The center of mass is two-thirds of the way down the bat. Find the minimum coefficient of static friction needed to keep the bat in place.



Applying the Second Law,

$$\Sigma F_x = ma_x \qquad F_w - F_{fr} = 0 \qquad F_w = F_{fr} \qquad (1)$$
$$\Sigma F_y = ma_y \qquad F_n - F_g = 0 \qquad F_n = F_g = mg \qquad (2)$$
$$\Sigma \tau_o = I\alpha \qquad F_g(\tfrac{1}{3}\ell)\cos 60^\circ - F_w \ell \sin 60^\circ = 0 \qquad F_w = \frac{mg}{3\tan 60^\circ} \quad (3)$$

Using the definition of the coefficient of friction, eq. 1 and eq. 2,

$$\mu \equiv \frac{F_{fr}}{F_n} = \frac{F_w}{mg}.$$

Substituting eq. 3 and the numbers,

$$\mu = \frac{\left(\frac{mg}{3\tan 60^\circ}\right)}{mg} = \frac{1}{3\tan 60^\circ} \qquad \boxed{\mu = 0.192}.$$