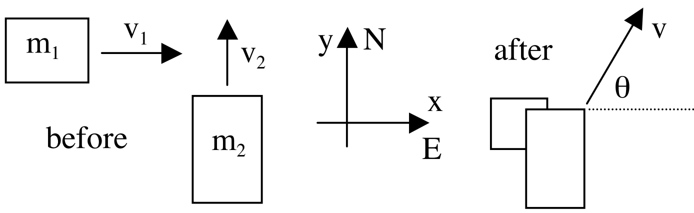
Name:_____________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles shown on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 1200kg car heading east on an icy road at 35.0km/h collides with a 2200kg pick-up truck going north at 50.0km/h. Assuming the vehicles stick together after the collision find their combined velocity.



The initial momentum components are: $p_{ox} = m_1 v_1$ and $p_{oy} = m_2 v_2$.
The final momentum components are: $p_{ox} = (m_1 + m_2) v \cos\theta$ and $p_{ox} = (m_1 + m_2) v \sin\theta$.

According to the Law of Conservation of Linear Momentum the components must match,

$$m_1 v_1 = (m_1 + m_2) v \cos\theta \Rightarrow v\cos\theta = \frac{m_1 v_1}{m_1 + m_2} \text{ and } m_2 v_2 = (m_1 + m_2) v \sin\theta \Rightarrow v \sin\theta = \frac{m_2 v_2}{m_1 + m_2}.$$

Combining these equations, $\frac{v\sin\theta}{v\cos\theta} = \frac{m_2 v_2}{m_1 + m_2}\frac{m_1 + m_2}{m_1 v_1} \Rightarrow \tan\theta = \frac{m_2 v_2}{m_1 v_1} \Rightarrow \theta = \arctan\frac{m_2 v_2}{m_1 v_1}$.

Plugging in the numbers, $\theta = \arctan\frac{m_2 v_2}{m_1 v_1} = \arctan\frac{(2200)(50.0)}{(1200)(35.0)} \Rightarrow \boxed{\theta = 69.1^\circ}$.

Substituting back in,

$$v\cos\theta = \frac{m_1 v_1}{m_1 + m_2} \Rightarrow v = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta} = \frac{(1200)(35.0)}{(1200 + 2200)\cos 69.1^\circ} \Rightarrow \boxed{v = 34.6 km/h}.$$

2. The basketball player shown at the right wants to get the basketball spinning at 200rpm from rest in 3.00s. The ball has a mass of 0.450kg, a radius of 12.0cm and is hollow. Find the average torque that she must exert.

Using the 2nd Law for Rotation, $\Sigma\tau = I\alpha \Rightarrow \tau = I\alpha$.
The rotational inertia of a hollow sphere is, $I = \frac{2}{3} m r^2$.

Using the definition of angular acceleration, $\alpha \equiv \frac{d\omega}{dt} \approx \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_o}{t} = \frac{\omega}{t}$.

Substituting into the 2nd Law, $\tau = \frac{2 m r^2 \omega}{3t}$.

Plugging in the numbers, $\tau = \frac{2(0.450)(0.120)^2 (200\,\frac{2\pi}{60})}{3(3.00)} \Rightarrow \boxed{\tau = 0.0302 N\cdot m}$.

3. A hollow ball and a solid ball of equal mass and radius each roll without slipping up an incline with the same initial speed. Decide which ball will go the furthest up the incline and explain your answer. Be sure your explanation includes the names of important physical principles.

Applying the *Law of Conservation of Energy*, the ball with the most kinetic energy at the bottom will have the most potential energy at the top and therefore go furthest up the ramp.

Since both balls have the same translational kinetic energy, the one with the most rotational kinetic energy will be the winner.

The *rotational kinetic energy depends upon the rotational inertia and the angular speed.* Since they are rolling without slipping they both have the same angular speed, so the winner is the one with the highest rotational inertia.

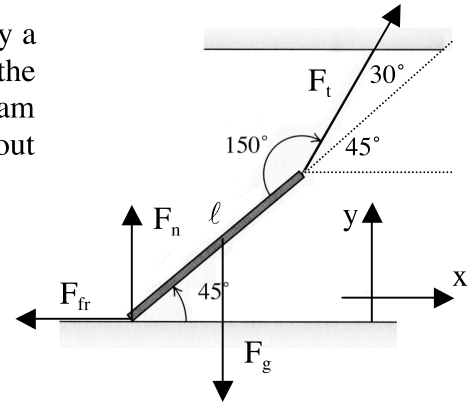
The $\boxed{\text{hollow ball}}$ will go the highest.

4. (a)Find the angular momentum of the basketball in problem 2. (b)Divide the answer to this problem by your answer to problem 2 and explain the result.

(a)Using the definition of angular momentum, $L \equiv I\omega$.
The rotational inertia is on the equation sheet, $I = \frac{2}{3}mr^2$.
Putting it all together, $L = \frac{2}{3}mr^2\omega$.
Putting in the values, $L = \frac{2}{3}(0.450)(0.120)^2(200\frac{2\pi}{60}) \Rightarrow \boxed{L = 0.0905\frac{kg\cdot m^2}{s}}$.
(b)OK…here goes, $\frac{L}{\tau} = \frac{0.0905\frac{kg\cdot m^2}{s}}{0.0302N\cdot m} \Rightarrow \boxed{\frac{L}{\tau} = 3.00s}$.
This is because the Second Law for Rotation can also be written in terms of angular momentum instead of angular acceleration,

$$\Sigma\tau = \frac{\Delta L}{\Delta t} \Rightarrow \Delta t = \frac{\Delta L}{\Sigma\tau}$$

5. The uniform 200kg beam show at the right is supported by a cable connected to the ceiling while the lower end rests on the floor. In the drawing, sketch each force that acts on the beam and find the size of each. Be sure you are very clear about labeling angles.



Using the mass weight rule,
$F_g = mg = (200)(9.80) \Rightarrow \boxed{F_g = 1960N}$.

Applying the Second Laws using the contact point with the ground for the pivot point,

$$\Sigma F_x = ma_x \Rightarrow F_t \cos 75^\circ - F_{fr} = 0$$
$$\Sigma F_y = ma_y \Rightarrow F_t \sin 75^\circ + F_n - F_g = 0$$
$$\Sigma \tau = I\alpha \Rightarrow F_t \ell \sin 30^\circ - F_g \frac{\ell}{2} \cos 45^\circ = 0$$

Notice how careful you need to be about the angles….

Solving the torque equation for the tension,
$$F_t = \frac{F_g \cos 45^\circ}{2\sin 30^\circ} = \frac{1960 \cos 45^\circ}{2 \sin 30^\circ} \Rightarrow \boxed{F_t = 1390N}.$$

Solving the x-equation for the friction,
$$F_{fr} = F_t \cos 75^\circ = 1390 \cos 75^\circ \Rightarrow \boxed{F_{fr} = 359N}.$$

Solving the y-equation for the normal force,

$$F_n = F_g - F_t \sin 75^\circ = 1960 - 1390 \sin 75^\circ \Rightarrow \boxed{F_n = 617N}.$$