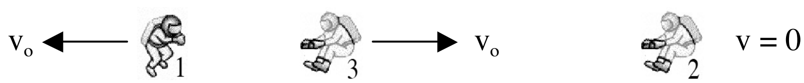
Name:_______________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You <u>must</u> show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. Three equally massive and equally strong astronauts are outside their ship in outer space. Two of them get the bright idea to play a game of catch by throwing the third one back and forth. Suppose the game begins with the first astronaut throwing the third astronaut toward the second astronaut at a speed v_o. Describe the rest of the game. This means that you must find the velocity of each astronaut after each catch and after each throw. Sketches of the game at each stage might be the best way to explain your answer. Be sure to state the principle or principles you use.

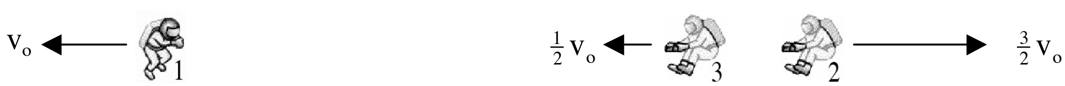
The main principle is the Law of Conservation of Linear Momentum.

The game begins with astronaut 1 pushing astronaut 3 to the right at v_o. Due to the Law of Conservation of Momentum, astronaut 1 recoils with a speed v_o to the left. Astronaut 2 is at rest.



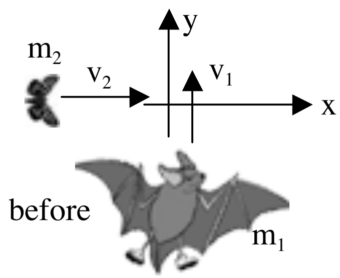
When astronaut 3 is caught by astronaut 2, the Law of Conservation of Momentum requires that they head off to the right at one-half of v_o.

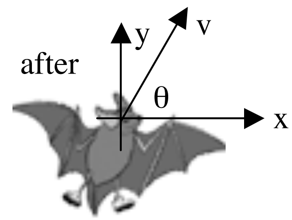
Since the astronauts are equally strong, when astronaut 2 pushes astronaut 3 back towards astronaut 1, he can only change astronaut 3's velocity by v_o. As a result, astronaut 3 heads back to the left at one-half v_o. According to the Law of Conservation of Momentum, astronaut 2 will move to the right at three-halves v_o.



At this point the game ends because astronaut3 can never catch up to astronaut 1.

2. A 100g bat (the animal) flying northward at 0.800m/s gulps down a 20.0g moth heading eastward at 3.50m/s. Find the speed and direction of the bat and its full belly just after his meal.





$p_{ox} = m_2 v_2$ $\quad$ $p_x = (m_1 + m_2)v\cos\theta$

$p_{oy} = m_1 v_1$ $\quad$ $p_y = (m_1 + m_2)v\sin\theta$

Applying the Law of Conservation of Linear Momentum along each axis separately,

$$m_2 v_2 = (m_1 + m_2)v\cos\theta$$
$$m_1 v_1 = (m_1 + m_2)v\sin\theta$$

Dividing one equation by the other,

$$\frac{\sin\theta}{\cos\theta} = \frac{m_1 v_1}{m_2 v_2} \Rightarrow \theta = \tan^{-1}\left(\frac{m_1 v_1}{m_2 v_2}\right)$$

Plugging in the numbers,

$$\theta = \tan^{-1}\left(\frac{(100)(0.800)}{(20.0)(3.50)}\right) \Rightarrow \boxed{\theta = 48.8^\circ}.$$

Using this angle and the equation for the x-direction then solving for the speed,

$$v = \frac{m_2 v_2}{(m_1 + m_2)\cos\theta} = \frac{(20.0)(3.50)}{(100 + 20.0)\cos 48.8^\circ} \Rightarrow \boxed{v = 0.886 m/s}.$$

3. A basketball has a mass of 0.450kg, a radius of 12.0cm, and is hollow. The basketball player shown at the right wants to get the basketball spinning by exerting an average torque of 0.0300N·m for 3.00s. Find (a)the rotational inertia of the ball and (b)the rate at which it will be spinning.

(a)The rotational inertia of a hollow sphere is, $I = \frac{2}{3}mr^2$.

Plugging in the numbers, $I = \frac{2}{3}(0.450)(0.120)^2 \Rightarrow \boxed{I = 4.32x10^{-3}\, kg \cdot m^2}$.

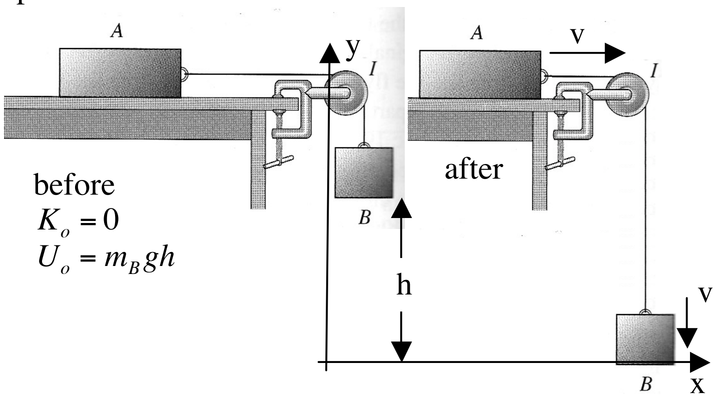
(b)Using the 2nd Law for Rotation, $\Sigma\tau = I\alpha \Rightarrow \tau = I\alpha$.

Using the definition of angular acceleration, $\alpha \equiv \frac{d\omega}{dt} \approx \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_o}{t} = \frac{\omega}{t}$.

Substituting into the 2nd Law, $\tau = I\frac{\omega}{t}$.

Solving for the rotation rate, $\omega = \frac{\tau \cdot t}{I} = \frac{(0.0300)(3.00)}{4.32x10^{-3}} \Rightarrow \boxed{\omega = 20.8 rad/s = 3.32 rev/s = 199 rpm}$.

4. The system shown below starts from rest. Block A slides across a frictionless table and has a mass of 2.00kg. The pulley has a mass of 1.00kg and a radius of 5.00cm. Block B has a mass of 5.00kg. Find the speed of block B after it has fallen 1.00m.



The final potential energy is zero, but the kinetic energy includes the motion of the masses and the pulley,

$U = 0$ and $K = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}I\omega^2$.

Since the tangential velocity of the pulley is the same as the velocity of the masses,

$$v = r\omega \Rightarrow K = \tfrac{1}{2}(m_A + m_B)v^2 + \tfrac{1}{2}I\frac{v^2}{r^2}.$$

The rotational inertia of a disk is,

$I = \frac{1}{2}mr^2 \Rightarrow K = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{4}mv^2$.

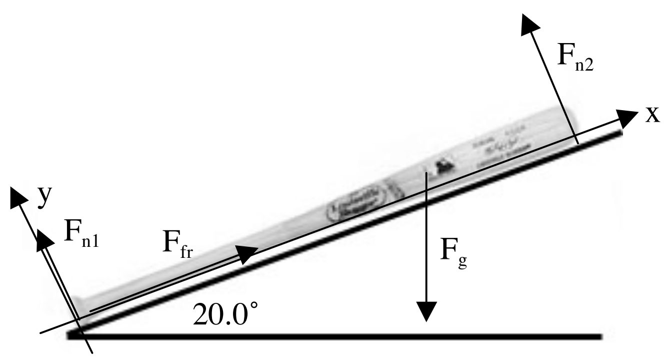
Applying the Law of Conservation of Energy,

$\Delta K + \Delta U = 0 \Rightarrow (K - K_o) + (U - U_o) = 0 \Rightarrow \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{4}mv^2 - m_B gh = 0$.

Solving for the speed,

$$\tfrac{1}{2}(m_A + m_B + \tfrac{1}{2}m)v^2 = m_B gh \Rightarrow v = \sqrt{\frac{2m_B gh}{m_A + m_B + \frac{1}{2}m}} = \sqrt{\frac{2(5.00)(9.80)(1.00)}{2.00 + 5.00 + \frac{1}{2}(1.00)}} \Rightarrow \boxed{v = 3.61 m/s}.$$

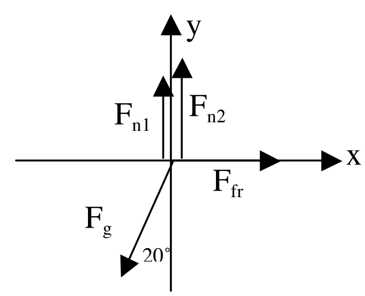
5. The slope of a pitcher's mound makes a 20.0° angle with the horizontal. A 15.0N – 90.0cm baseball bat rests on the mound in such a way that only the ends are actually in contact with the mound. The center of mass of the bat is 60.0cm from the skinny end. Find the magnitudes of each of the normal forces and the total frictional force that the ground exerts on the bat.



Using the free body diagram to apply the 2nd Law along the axes,

$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_g \sin 20° = 0$ and

$\Sigma F_y = ma_y \Rightarrow F_{n1} + F_{n2} - F_g \cos 20° = 0$.



Finding the torques about the origin and applying the 2nd Law for Rotation,

$\Sigma\tau_o = I\alpha \Rightarrow \ell F_{n2} - \frac{2}{3}\ell F_g \cos 20° = 0$.

Solving the x-equation for the frictional force,

$F_{fr} = F_g \sin 20° = (15.0)\sin 20° \Rightarrow \boxed{F_{fr} = 5.13N}$.

Solving the torque equation for the second normal force,

$F_{n2} = \frac{2}{3}F_g \cos 20° = \frac{2}{3}(15.0)\cos 20° \Rightarrow \boxed{F_{n2} = 9.40N}$.

Solving the y-equation for the first normal force,

$F_{n1} = F_g \cos 20° - F_{n2} = (15.0)\cos 20° - 9.40 \Rightarrow \boxed{F_{n1} = 4.70N}$.