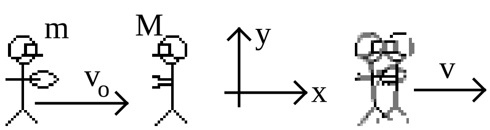
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 80.0kg running back traveling downfield at 10.0m/s slams into a stationary 95.0kg guard that grabs him to make a tackle. Assuming that the forces exerted by the ground on the athletes are small during their collision, find their combined speed just after the collision.



Before After

$p_o = mv_o$ $p = (m + M)v$

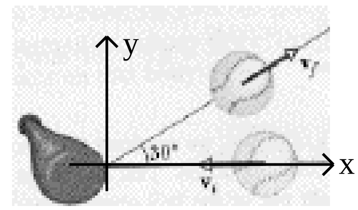
Assuming that there are no external forces during the collision, the Law of Conservation of Momentum applies,

$$mv_o = (m + M)v\,.$$

Solving for the final speed,

$$v = \frac{m}{m + M} v_o = \frac{80.0}{80.0 + 95.0}(10.0) \quad \boxed{v = 4.57 m/s}.$$

2. A pitcher throws a 150g ball so that it is traveling horizontally at 39.0m/s when it strikes the 1.20kg bat. The ball leaves the bat with a speed of 45.0m/s at an angle of 30.0° above the horizontal. Find the force that the bat exerts on the ball assuming that it is in contact with the ball for 1.20ms.



Using Newton's Second Law and the definition of momentum,

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F} = \frac{\Delta\vec{p}}{\Delta t} \qquad \vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{m}{\Delta t}\left(\vec{v}_f - \vec{v}_i\right).$$

Using the coordinates shown, the initial velocity is, $\vec{v}_i = -(39.0\text{m/s})\hat{i}$, and the final velocity is,

$$\vec{v}_f = (45.0\text{m/s})\cos 30°\,\hat{i} + (45.0\text{m/s})\sin 30°\,\hat{j} \qquad \vec{v}_f = (39.0\text{m/s})\hat{i} + (22.5\text{m/s})\hat{j}.$$

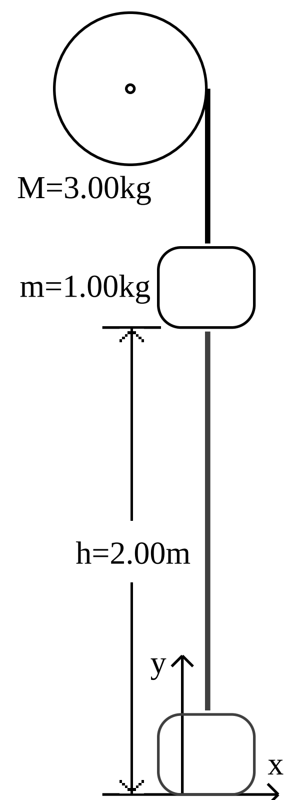
Substituting into the result from the Second Law,

$$\vec{F} = \frac{m}{\Delta t}\left(\vec{v}_f - \vec{v}_i\right) = \frac{0.150}{1.20\text{x}10^{-3}}\left((39.0\text{m/s})\hat{i} + (22.5\text{m/s})\hat{j} - \left[-(39.0\text{m/s})\hat{i}\right]\right).$$

Doing the math,

$$\vec{F} = 125\left[(78.0\text{m/s})\hat{i} + (22.5\text{m/s})\hat{j}\right] \qquad \boxed{\vec{F} = (9750\text{N})\hat{i} + (2810\text{N})\hat{j}}.$$

3. A 1.00kg mass hangs from a string that is wrapped around a 3.00kg cylinder with a 5.00cm radius. After the mass is released it falls as the string unwinds and the cylinder spins. After the mass has dropped 2.00m, find its velocity.



Initially all the energy is the potential energy of the hanging mass,

$$K_o = 0 \text{ and } U_o = mgh.$$

At the bottom, the potential energy has changed into kinetic energy of the cylinder and mass,

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \text{ and } U = 0.$$

Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \qquad (K - K_o) + (U - U_o) = 0$$

and substituting from above,

$$\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 - 0\right) + (0 - mgh) = 0 \qquad \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh.$$

The rotational inertia of the cylinder is, $I = \tfrac{1}{2}Mr^2$.

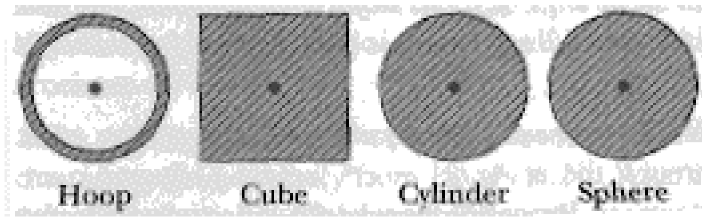
Since the tangential velocity of the cylinder must equal the velocity of the mass, $v = r\omega \qquad \omega = \dfrac{v}{r}$.

Substituting,

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}Mr^2\right)\left(\frac{v}{r}\right)^2 = mgh \qquad \tfrac{1}{2}mv^2 + \tfrac{1}{4}Mv^2 = mgh \qquad v^2 = \frac{gh}{\tfrac{1}{2}m + \tfrac{1}{4}M}$$

$$v = \left[\frac{gh}{\tfrac{1}{2}m + \tfrac{1}{4}M}\right]^{\frac{1}{2}} = \left[\frac{(9.80)(2.00)}{\tfrac{1}{2}(1.00) + \tfrac{1}{4}(3.00)}\right]^{\frac{1}{2}} \qquad \boxed{v = 3.96\text{m/s}}.$$

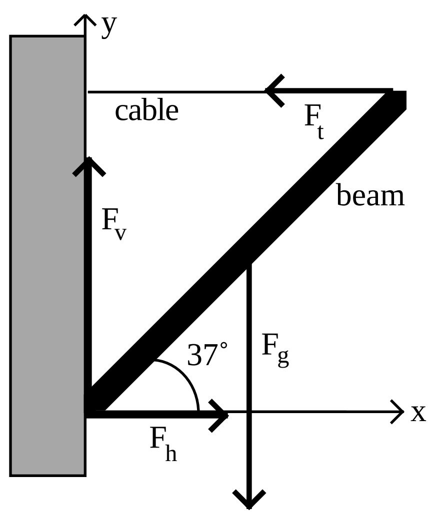
4. Four solid objects are shown below in cross section. They all have the same mass, equal heights, and equal widths (They will necessarily have different thickness). (a)Rank them from the highest rotational inertia to the lowest rotational inertia about the axis shown and (b)explain why you put the cube in the place that you chose. If your explanation only involves equations from the last page you will not get full credit.



(a)hoop, cube, cylinder, sphere

(b)The definition of rotational inertia is, $I \equiv \int r^2 dm$, which means that the rotational inertia is largest when the mass is farthest from the axis of rotation. In the case of the hoop, all the mass is far away from the axis so it must have the largest rotational inertia (mr^2). A sphere has all the mass is as close as it could be to the center so it must have the smallest rotational inertia ($\frac{2}{5}mr^2$). The cylinder must have a slightly higher rotational inertia than the sphere($\frac{1}{2}mr^2$) since it has proportionally more mass out at the radius. The cube must be a bit larger still because the corners are further away from the center than the radius of the cylinder.

5. A 5.00kg beam 2.00m long is hinged at one end and held at a 37˚ angle above the horizontal by a horizontal cable. Find the tension in the cable and the horizontal and vertical components of the force that the hinge exerts on the beam.



Applying the Second Law along the x-direction,

$$\Sigma F_x = ma_x \qquad F_h - F_t = 0 \qquad F_h = F_t \text{ and}$$

$$\Sigma F_y = ma_y \qquad F_v - F_g = 0 \qquad F_v = F_g = mg\,.$$

Plugging in the numbers,

$$F_v = mg = (5.00)(9.80) \qquad \boxed{F_v = 49.0\text{N}}.$$

Applying the Second Law for Rotation about the origin,

$$\Sigma\tau_o = I\alpha \qquad F_t \ell \sin 37^\circ - F_g \frac{\ell}{2}\cos 37^\circ = 0 \qquad F_t \ell \sin 37^\circ = F_g \frac{\ell}{2}\cos 37^\circ \qquad F_t = \frac{F_g \cos 37^\circ}{2\sin 37^\circ}\,.$$

Substituting from the equation for the x-direction and putting in the numbers,

$$F_h = F_t = \frac{F_g \cos 37^\circ}{2\sin 37^\circ} = \frac{mg}{2\tan 37^\circ} = \frac{(5.00)(9.80)}{2\tan 37^\circ} \qquad \boxed{F_h = 32.5\text{N}}.$$