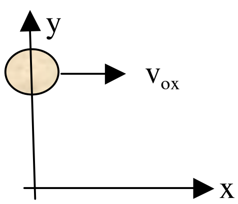
Name:___________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You <u>must</u> show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. Pumpkins dropped from the edge of Butte Hall fall 62.0m to the ground below. The crowd is 10.0m back from the edge of the building. Find the maximum horizontal velocity the pumpkins can have when released so that they can't reach the crowd.



$x_o = 0$	$y_o = 62.0m$
$x = 10.0m$	$y = 0$
$v_{ox} = ?$	$v_{oy} = 0$
$v_x = v_{ox}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	

Using the kinematic equation without the final speed along y,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2$$

Putting in the zero values and solving for t,

$$0 = y_o + \tfrac{1}{2}a_y t^2 \Rightarrow t = \sqrt{\frac{-2y_o}{a_y}}$$

Using the kinematic equation without the final speed along x,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2$$

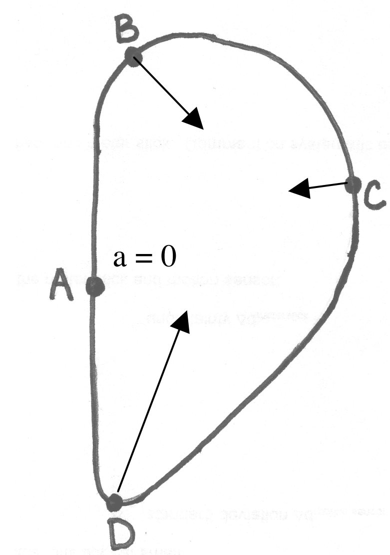
Substituting and solving for the initial speed,

$$x = v_{ox}\sqrt{\frac{-2y_o}{a_y}} \Rightarrow v_{ox} = x\sqrt{\frac{a_y}{-2y_o}}$$

Putting in the numbers,

$$v_{ox} = (10.0)\sqrt{\frac{-9.80}{-2(62.0)}} \Rightarrow \boxed{v_{ox} = 2.81m/s}.$$

2. A racecar goes completely around the racetrack shown at the right moving at a constant speed of 200km/h the entire way. Indicate in the diagram the direction of the acceleration at each of the labeled points and rank the accelerations from largest to smallest. Explain your thinking.



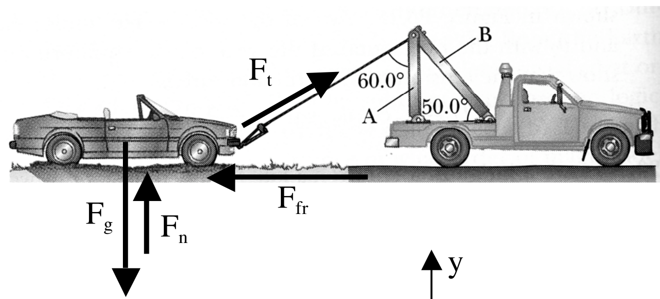
Since the speed of the car is constant there will be no tangential acceleration. The centripetal acceleration is given by,

$$a_c = \frac{v^2}{r}.$$

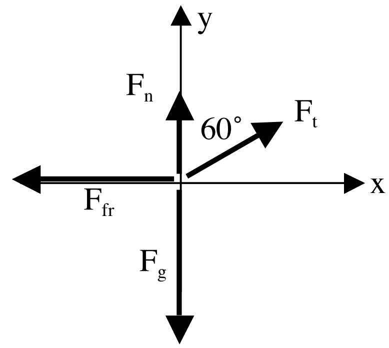
Since v is constant, the acceleration will be the largest when the radius of the turn is the smallest. Where the track is straight, the radius is infinite and the centripetal acceleration is zero. So the ranking will be,

$$\boxed{D>B>C>A=0}.$$

3. The car at the right has a mass of 750kg. The wheels of the car are broken and cannot roll. They will only slide. The coefficient of static friction between the tires and the road is 0.800. Find the tension in the cable needed to move the car.



Note that the normal force in the sketch is the sum of all the normal forces on all the tires. The same is true for the friction.



Applying the Second Law to each direction separately,

$\Sigma F_x = ma_x \Rightarrow F_t \sin 60° - F_{fr} = 0 \Rightarrow F_{fr} = F_t \sin 60°$

$\Sigma F_y = ma_y \Rightarrow F_t \cos 60° + F_n - F_g = 0 \Rightarrow F_n = F_g - F_t \cos 60°$

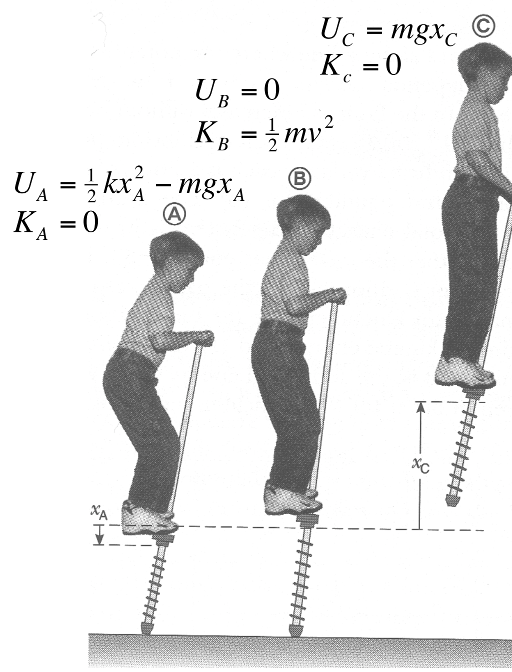
Using the definition of the COSF and substituting from above,

$$\mu \equiv \frac{F_{fr}}{F_n} = \frac{F_t \sin 60°}{F_g - F_t \cos 60°} = \frac{F_t \sin 60°}{mg - F_t \cos 60°}$$

Doing the mountain of algebra to solve for the tension,

$$F_t = \frac{\mu mg}{\sin 60° + \mu \cos 60°} = \frac{(0.800)(750)(9.80)}{\sin 60° + (0.800)\cos 60°} \Rightarrow \boxed{F_t = 4640N}.$$

4. The pogo stick at the right uses a spring (k = 25.0kN/m). At position A the child is at rest while the spring is compressed 10.0cm. At position B, the spring is relaxed and the child is moving upward. At position C the child is at the top of the jump. Find (a)the speed at position B and (b)the height if the jump. The mass of the child and pogo stick is 50.0kg.



(a)Applying the Law of Conservation of Energy between positions A and B,

$\Delta U + \Delta K = 0 \Rightarrow (U_B - U_A) + (K_B - K_A) = 0 \Rightarrow K_B = U_A$

Putting in the energies and solving,

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_A^2 - mgx_A \Rightarrow v = \sqrt{\tfrac{k}{m}x_A^2 - 2gx_A}$$

$$v = \sqrt{\tfrac{25000}{50.0}(0.100)^2 - 2(9.80)(0.100)} \Rightarrow \boxed{v = 1.74m/s}.$$

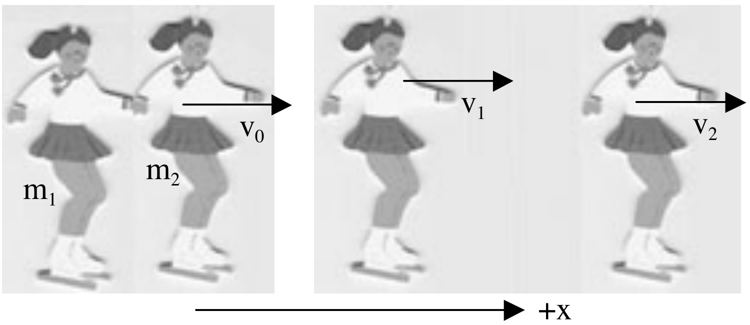
(b)Applying the Law of Conservation of Energy between positions A and C,

$\Delta U + \Delta K = 0 \Rightarrow (U_C - U_A) + (K_C - K_A) = 0 \Rightarrow U_C = U_A$

Putting in the energies and solving,

$$mgx_C = \tfrac{1}{2}kx_A^2 - mgx_A \Rightarrow x_C = \frac{kx_A^2}{2mg} - x_A = \frac{(25000)(0.100)^2}{2(50.0)(9.80)} - 0.100 \Rightarrow \boxed{x_C = 0.155m}.$$

5. Two ice skaters are moving along together at 3.00m/s. The first skater has a weight of 700N. The first skater pushes the second skater forward so she speeds up to 4.00m/s. As a result, the first skater slows to 2.25m/s. Find the weight of the second skater.



The linear momentum before the push is,

$$p_o = (m_1 + m_2)v_o.$$

The momentum after the collision is,

$$p = m_1 v_1 + m_2 v_2.$$

Using the Law of Conservation of Linear Momentum,

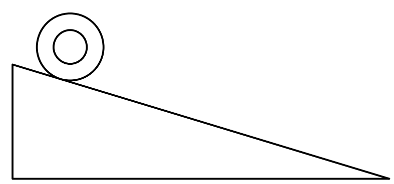
$$(m_1 + m_2)v_o = m_1 v_1 + m_2 v_2$$

Solving for the mass of the second skater then multiplying both sides by the acceleration due to gravity,

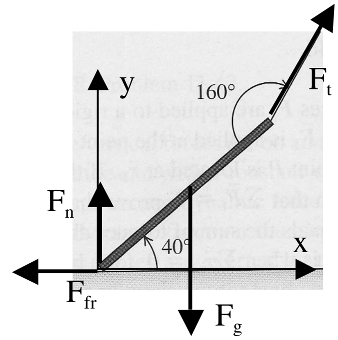
$$m_2 = m_1 \frac{v_o - v_1}{v_2 - v_o} \Rightarrow m_2 g = m_1 g \frac{v_o - v_1}{v_2 - v_o} \Rightarrow m_2 g = (700N)\frac{3.00 - 2.25}{4.00 - 3.00} \Rightarrow \boxed{m_2 g = 525N}.$$

6. A frictionless 500g air puck (object A) slides down an incline. In addition, the four objects described in the table below roll down the same incline. Rank from shortest to longest the time for the object to go from the top to the bottom. Explain your reasoning.

Object	*Mass*	*Inner radius*	*Outer radius*	*Length*
B. Solid Rod	450g	None	3.00cm	12.0cm
C. Hoop	600g	3.00cm	5.00cm	6.00cm
D. Disk	200g	None	5.00cm	10.0cm
E. Ring	300g	10.0cm	10.0cm	3.00cm



Using the Law of Conservation of Energy, the change in potential energy must equal the change in kinetic energy for each object. Since the change in potential energy is the same for each object, the change in kinetic energy will also be the same. The kinetic energy is the sum of the translational kinetic energy and the rotational kinetic energy. Since the rotational kinetic energy depends upon the rotational inertia, the object with the smallest rotational inertia will have the smallest rotational KE and the largest translational KE. It is important to note that the radius and mass cancel in the problem and make no difference. The winner of the race is the air puck since it spends no KE on rotation and the loser is the ring which spends the largest fraction on rotational KE. The ranking is $\boxed{A<B=D<C<E}$.



7. A 9.00m - 1500kg tree is cut down by a logging company. A machine uses a short cable to lift it by one end until it is in the position shown at the right. Find the magnitude and show the direction of each force that acts on the log.

Applying the Second Law,

$\Sigma F_x = ma_x \Rightarrow F_t \cos(40^\circ + 20^\circ) - F_{fr} = 0 \Rightarrow F_{fr} = F_t \cos 60^\circ$

$\Sigma F_y = ma_y \Rightarrow F_t \sin(40^\circ + 20^\circ) + F_n - F_g = 0 \Rightarrow F_n = F_g - F_t \sin 60^\circ$

$\Sigma \tau_o = I\alpha \Rightarrow \ell F_t \sin 20^\circ - \frac{\ell}{2} F_g \cos 40^\circ = 0$

Using the mass weight rule, $F_g = mg = (1500)(9.8) \Rightarrow \boxed{F_g = 14.7kN}$.

Solving the torque equation for the tension, $F_t = \dfrac{F_g \cos 40^\circ}{2\sin 20^\circ} = \dfrac{(14.7)\cos 40^\circ}{2\sin 20^\circ} \Rightarrow \boxed{F_t = 16.5kN}$.

Substitute into the y equation and solve for the normal force,
$F_n = 14.7 - 16.5\sin 60^\circ \Rightarrow \boxed{F_n = 0.443kN = 443N}$.

Finally, solve the y equation for the friction, $F_{fr} = F_t \cos 60^\circ = 16.5\cos 60^\circ \Rightarrow \boxed{F_{fr} = 8.25kN}$.

8. The machine in the previous problem continues to lift the log until it is off the ground. It gently swings back and forth pivoted at the end of the cable. Find the period of oscillation.

The log acts like a physical pendulum. The angular frequency should be given by, $\omega = \sqrt{\frac{mgr}{I}}$.

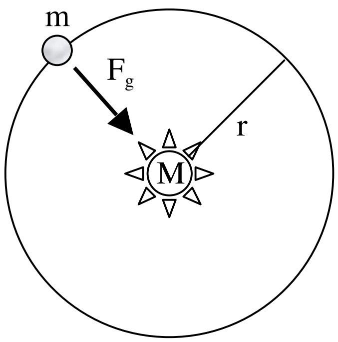
The rotational inertia of a stick about one end is, $I = \frac{1}{3}m\ell^2 \Rightarrow \omega = \sqrt{\frac{3mgr}{m\ell^2}} = \sqrt{\frac{3gr}{\ell^2}}$.

The r is from the pivot point to the cm so it is half the length, $r = \frac{\ell}{2} \Rightarrow \omega = \sqrt{\frac{3g\ell}{\ell^2 2}} = \sqrt{\frac{3g}{2\ell}}$

The period is related to the angular frequency by, $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\ell}{3g}}$.

Plugging in the numbers, $T = 2\pi\sqrt{\frac{2(9.00)}{3(9.80)}} \Rightarrow \boxed{T = 4.92s}$.

9. The following is a quote from a periodical I read over the summer, “It is a bit bigger than Pluto, an astounding 14.5 billion kilometers from the sun, and the most distant object ever seen in the solar system. Last week's discovery of a "10th planet"….” Find the time for this “new planet” to orbit the sun.



Applying Newton’s Second Law to the planet, $\Sigma F = ma \Rightarrow F_g = ma$.
Using the Universal Law of Gravitation, $F_g = G\frac{mM}{r^2} \Rightarrow G\frac{mM}{r^2} = ma \Rightarrow G\frac{M}{r^2} = a$.
Since the motion is circular use the centripetal acceleration,

$$a = \frac{v^2}{r} \Rightarrow G\frac{M}{r^2} = \frac{v^2}{r} \Rightarrow G\frac{M}{r} = v^2.$$

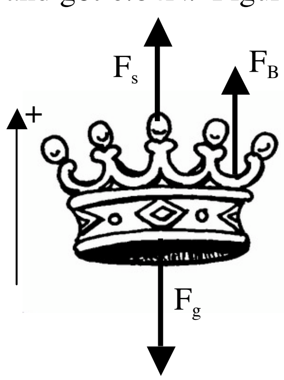
The speed will just be the circumference of the orbit divided by the period,

$$v = \frac{2\pi r}{T} \Rightarrow G\frac{M}{r} = \left(\frac{2\pi r}{T}\right)^2.$$

Solving for the period and plugging in the numbers,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2(14.5x10^{12})^3}{(6.67x10^{-11})(1.99x10^{30})}} \Rightarrow \boxed{T = 3.01x10^{10}s = 955\,years}.$$

10. Legend has it that Archimedes was asked by the king to determine whether his crown was made of solid gold without damaging the crown in any way. Archimedes knew that the density of gold was $19.3x10^3$ kg/m^3. So he weighted the crown in air and got 7.84N. He then weighted the crown in water and got 6.84N. Figure out what Archimedes told the king.



When the crown is in the water the three forces on it are the scale pulling upward, gravity pulling downward, and the upward buoyant force. Applying the Second Law,

$$\Sigma F = ma \Rightarrow F_s + F_B - F_g = 0 \Rightarrow F_B = F_g - F_s.$$

The buoyant force is given by Archimedes’ Principle, $F_B = \rho_w V g$.
The volume of the crown that displaces the water can be written in terms of the density and mass of the crown,

$$\rho \equiv \frac{m}{V} \Rightarrow V = \frac{m}{\rho}.$$

Substituting into the equation for the buoyant force, $F_B = \rho_w \frac{m}{\rho} g = \frac{\rho_w}{\rho} F_g$.

Substituting into the equation from the Second Law, $\frac{\rho_w}{\rho} F_g = F_g - F_s$.

Solving for the density of the crown,

$$\frac{\rho_w}{\rho} = \frac{F_g - F_s}{F_g} \Rightarrow \rho = \rho_w \frac{F_g}{F_g - F_s} = (1000)\frac{7.84}{7.84 - 6.84} \Rightarrow \boxed{\rho = 7.84x10^3 kg/m^3}.$$

Who wants to tell the king that he has been ripped off?