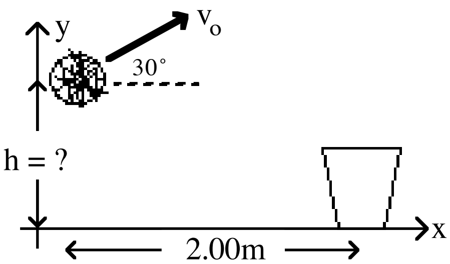
Name:___________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A wad of paper is tossed into a wastebasket 2.00m away. It is released with an initial speed of 4.00m/s at an angle of 30.0°. Assume that it lands in the center of the bottom and that air resistance is negligible. Find (a)the time the wad is in the air and (b)the height from which it was released.



$x_o = 0$
$x = 2.00m$
$v_{ox} = 4.00 \cos 30° = 3.46m/s$
$v_x = 3.46m/s$
$a_x = 0$
$t = ?$

$y_o = h$
$y = 0$
$v_{oy} = v_o \sin 30° = 2.00m/s$
$v_y = ?$
$a_y = -9.80m/s^2$

(a)Using the kinematic equation for the final x-position,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \Rightarrow x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{2.00}{3.46} \Rightarrow \boxed{t = 0.577s}.$$

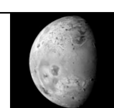
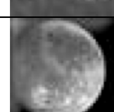
(b)Using the kinematic equation for the final y-position,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow 0 = h + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow h = -(v_{oy}t + \tfrac{1}{2}a_y t^2).$$

Putting in the numbers,

$$h = -[(2.00)(0.577) + \tfrac{1}{2}(-9.80)(0.577)^2] \Rightarrow \boxed{h = 0.479m}.$$

2. Below is a table of some data about four moons of Jupiter discovered by Galileo. Rank these moons in order of the magnitude of their acceleration. That is, rank first the one with the highest acceleration and rank last the one with the smallest acceleration. Explain your reasoning.

Moon	Orbital Radius ($x10^6$m)	Orbital Speed ($x10^3$m/s)	
Io	422	17.3	
Callisto	1883	8.20	
Ganymede	1070	10.9	
Europa	671	13.7	

These moon are in circular orbit around Jupiter so the experience centripetal acceleration which is given by,

$$a_c = \frac{v^2}{r}.$$

Io: $a_c = \frac{v^2}{r} = \frac{(17.3)^2}{422} = 0.709$

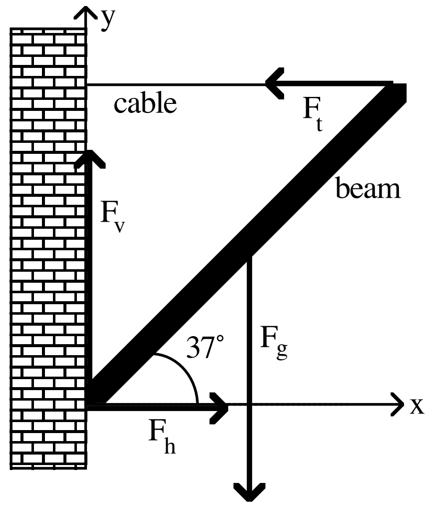
Callisto: $a_c = \frac{(8.20)^2}{1883} = 0.036$

Ganymede: $a_c = \frac{v^2}{r} = \frac{(10.9)^2}{1070} = 0.111$

Europa: $a_c = \frac{v^2}{r} = \frac{(13.7)^2}{671} = 0.280$

The ranking is: Io >Europa> Ganymede> Callisto

3. A 5.00kg beam 2.00m long is hinged at one end and held at a 37° angle above the horizontal by a horizontal cable. Find the tension in the cable and the horizontal and vertical components of the force that the hinge exerts on the beam.



Applying the Second Law along each direction,

$$\Sigma F_x = ma_x \Rightarrow F_h - F_t = 0 \Rightarrow F_h = F_t$$
$$\Sigma F_y = ma_y \Rightarrow F_v - F_g = 0 \Rightarrow F_v = F_g = mg.$$

Plugging in the numbers for the vertical component,

$$F_v = mg = (5.00)(9.80) \Rightarrow \boxed{F_v = 49.0\text{N}}.$$

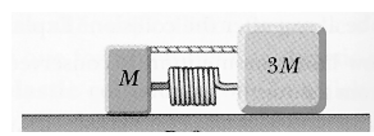
Applying the Second Law for Rotation about the origin,

$$\Sigma\tau_o = I\alpha \Rightarrow F_t \ell \sin 37° - F_g \frac{\ell}{2}\cos 37° = 0 \Rightarrow F_t \ell \sin 37° = F_g \frac{\ell}{2}\cos 37° \Rightarrow F_t = \frac{F_g \cos 37°}{2\sin 37°}.$$

Substituting into the equation for the x-direction and putting in the numbers,

$$F_h = F_t = \frac{F_g \cos 37°}{2\sin 37°} = \frac{mg}{2\tan 37°} = \frac{(5.00)(9.80)}{2\tan 37°} \Rightarrow \boxed{F_h = 32.5\text{N}}.$$

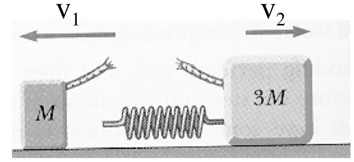
4. Two blocks of masses 100g and 300g are placed on a horizontal, frictionless surface. A light spring of spring constant 8.00N/m is attached to one of them and the blocks as pushed together compressing the spring 5.00cm. A cord initially holding the blocks together is burned. Find the speed of the blocks assuming the energy stored in the spring is completely transferred to the blocks motion.



before

$p_o = 0 \quad U_o = \frac{1}{2}kx^2 \quad K_o = 0$

v_1 v_2



after

$p = Mv_2 - mv_1 \quad U = 0$

$K = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$

Applying the Law of Conservation of Momentum,

$$0 = Mv_2 - mv_1 \Rightarrow v_2 = \tfrac{m}{M}v_1.$$

Applying the Law of Conservation of Energy,

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}Mv_2^2 \Rightarrow kx^2 = mv_1^2 + Mv_2^2.$$

Substituting from the momentum equation into the energy equation,

$$kx^2 = mv_1^2 + M\left(\tfrac{m}{M}v_1\right)^2 \Rightarrow kx^2 = \tfrac{m}{M}\left(m + M\right)v_1^2 \Rightarrow v_1 = \sqrt{\tfrac{M}{m}\cdot\tfrac{k}{m+M}}\,x$$

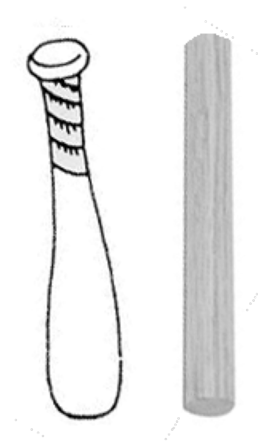
Plugging in the numbers,

$$v_1 = \sqrt{\tfrac{300}{100}\cdot\tfrac{8.00}{0.100+0.300}}(0.0500) \Rightarrow \boxed{v_1 = 0.387 m/s}.$$

Using the momentum equation again,

$$v_2 = \tfrac{m}{M}v_1 = \tfrac{100}{300}(0.387) \Rightarrow \boxed{v_2 = 0.129 m/s}.$$

5. A baseball bat and a uniform stick of wood both have the same mass and length. Explain which one has the higher rotational inertia about the top end shown in the sketch at the right.

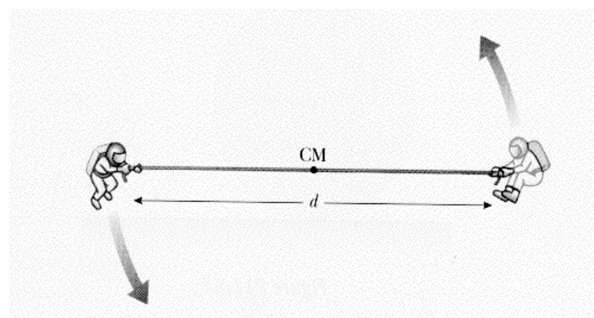


The rotational inertia of an object is defined to be,

$$I \equiv \int r^2 dm.$$

This means that objects that have a greater fraction of their mass further away from the axis of rotation have a larger rotational inertia. Since a baseball bat is designed to get wider at the "business" end. It has more mass further away from the axis of rotation than a uniform stick. Therefore, the bat has a larger rotational inertia than the stick.

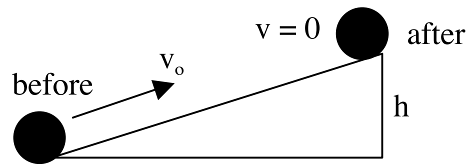
6. Two astronauts each have a mass of 75.0kg are connected by a 10.0m long rope of negligible mass. They are isolated in space and orbit their center of mass with a speed of 5.00m/s. They then begin to work their way toward each other along the rope until they are only 5.00m apart. Find (a)their combined angular momentum about their CM initially and (b)their combined kinetic energy initially. (c)Explain which should stay constant as they decrease their separation and why. Find (d)their combined angular momentum about their CM afterward and (e)their combined kinetic energy afterward.



(a)The angular momentum of "point particles" is, $\vec{L} = \vec{r} \times \vec{p} \Rightarrow L = mv_o r_o$.
Since both masses are the same distance from the center,
$L_o = 2mv_o r_o = 2(75.0)(5.00)(5.00) \Rightarrow \boxed{L_o = 3750 kg \cdot m^2/s}$.
(b)Using the definition of kinetic energy, $K_o = 2(\frac{1}{2}mv_o^2) = (75.0)(5.00)^2 \Rightarrow \boxed{K_o = 1880J}$.
(c)The kinetic energy will not stay constant because the astronauts are supplying energy to pull themselves inward. The Law of Conservation of Angular Momentum applies because the forces they exert are toward the center and create no torque on the system.
(d)By the Law of Conservation of Angular Momentum, $\boxed{L = 3750 kg \cdot m^2/s}$.
(e)We can find their new speed from $L_o = L \Rightarrow 2mv_o r_o = 2mvr \Rightarrow v = \frac{r_o}{r} v_o$.

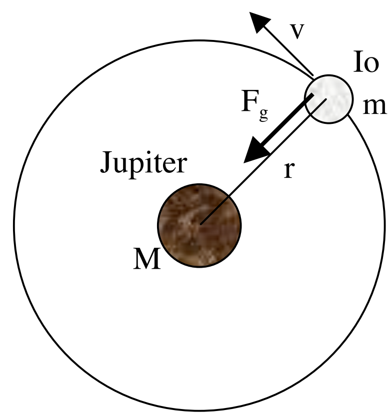
So the new kinetic energy is, $K = 2(\frac{1}{2}mv^2) = mv_o\left(\frac{r_o}{r}\right)^2 = K_o\left(\frac{r_o}{r}\right)^2 = (1880)(2)^2 \Rightarrow \boxed{K = 7500J}$.

7. A bowling ball rolls along a flat surface at 5.00m/s when it comes to a ramp. Find the minimum height of the ramp needed to stop the bowling ball.



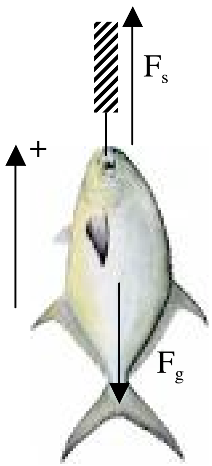
The energy situation initially is,
$U_o = 0$ and $K_o = \frac{1}{2}mv_o^2 + \frac{1}{2}I\omega^2$.
At the top of the ramp,
$U = mgh$ and $K = 0$.
Applying the Law of Conservation of Energy,
$\Delta U + \Delta K = 0 \Rightarrow (mgh - 0) + (0 - \frac{1}{2}mv_o^2 - \frac{1}{2}I\omega^2) \Rightarrow mgh = \frac{1}{2}mv_o^2 + \frac{1}{2}I\omega^2$
The rotational inertia of a bowling ball is a solid sphere, $I = \frac{2}{5}mr^2$.
If the ball rolls without slipping, $v_{cm} = r\omega \Rightarrow \omega = \dfrac{v_o}{r}$.
The energy equation becomes, $mgh = \frac{1}{2}mv_o^2 + \frac{1}{2}\frac{2}{5}mr^2\left(\frac{v_o}{r}\right)^2 \Rightarrow mgh = \frac{7}{10}mv_o^2$
Solving for the height,
$$h = \frac{7v_o^2}{10g} = \frac{7(5.00)^2}{10(9.80)} \Rightarrow \boxed{h = 1.79m}.$$

8. Use the data from problem 2 to find the mass of Jupiter.



Applying the Second Law to Io,
$$\Sigma F = ma \Rightarrow F_g = ma.$$
The force of gravity is given by the Law of Gravitation,
$$G\frac{Mm}{r^2} = ma \Rightarrow G\frac{M}{r^2} = a.$$
Note that the mass of the moon cancels out.
The acceleration is centripetal so,
$$G\frac{M}{r^2} = \frac{v^2}{r} \Rightarrow M = \frac{rv^2}{G}.$$
Plugging in the numbers from problem 2,
$$M = \frac{(422x10^6)(17.3x10^3)^2}{6.67x10^{-11}} \Rightarrow \boxed{M = 1.89x10^{27}kg}.$$

9. A fisherman hangs his trophy catch of mass 65.0kg from a spring that is stretched 12.0cm when it comes to rest. Find (a)the spring constant of the spring and (b)the frequency of the oscillations in the spring before it came to rest.



(a)Applying the Second Law to the fish,

$$\Sigma F = ma \Rightarrow F_s - F_g = ma.$$

The fish is at rest. Using the mass/weight rule and Hooke's Rule,

$$F_s - F_g = 0 \Rightarrow F_s = F_g \Rightarrow kx = mg \Rightarrow k = \frac{mg}{x}$$

Putting in the numbers,

$$k = \frac{(65.0)(9.80)}{0.120} \Rightarrow \boxed{k = 5310 N/m}.$$

(b)For an oscillating spring,

$$\omega = \sqrt{\tfrac{k}{m}}.$$

Converting from angular frequency,

$$2\pi f = \sqrt{\tfrac{k}{m}} \Rightarrow f = \tfrac{1}{2\pi}\sqrt{\tfrac{k}{m}} = \tfrac{1}{2\pi}\sqrt{\tfrac{5310}{65}} \Rightarrow \boxed{f = 1.44 Hz}.$$

10. At the right is a picture of a levee before a flood. During flood conditions the water rises. Levees tend to give out at the bottom instead of the top. Explain this in terms of the principles of physics.

The pressure in a fluid depends on the depth. The pressure on the bottom of the levee will be higher than on the top. That is why they are thicker on the bottom than the top. None the less, they greater pressure at the bottom often causes a failure near the bottom.

If you want to put numbers to it for say a 10m high levee, the pressure in the fluid is,

$$P = \rho g h = (1000)(10)(10) = 100{,}000 Pa \approx 1 atm.$$

So a 30ft levee feels about 15psi on the bottom due to the water.