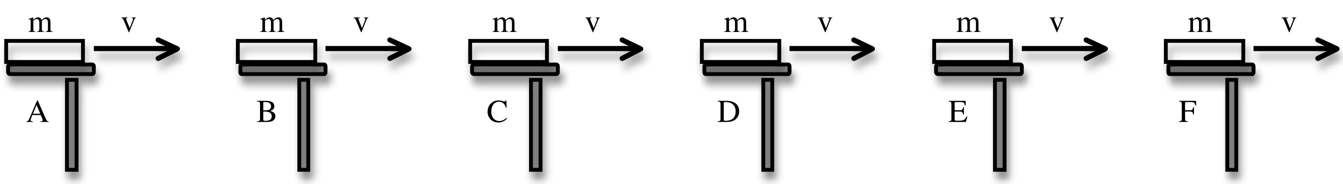
Name:_________________________________ PC _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied and clearly stated physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. Six different books slide off the edge of a table one at a time. The mass and speed along the table are given. Rank the books from greatest to least based upon (a)the time to hit the ground and (b)the horizontal distance from the edge of the table to the point where the book lands. You must explain your reasoning for full credit.




| A | B | C | D | E | F |
|---|---|---|---|---|---|
| m = 1kg | m = 2kg | m = 1kg | m = 2kg | m = 1kg | m = 2kg |
| v = 1m/s | v = 1m/s | v = 2m/s | v = 2m/s | v = 3m/s | v = 3m/s |

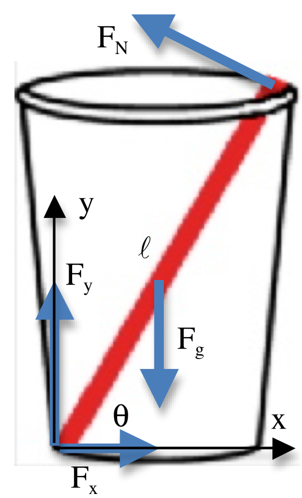
(a)Since the horizontal motion is independent of the vertical motion and the Rule of Falling Bodies tells us they all have the same downward acceleration, they all take the same amount of time to fall.

**A = B = C = D = E = F**

(b)Since they all take the same time to fall, the ones moving the fastest travel the most horizontally.

**E = F > C = D > A = B**

2. A 5.00g straw is just long enough to rest across a drinking glass. It is 12.0cm long and makes a 60° angle with the base of the cup. Assume the top of the glass is smooth and rounded. Find the horizontal and vertical components of the force that the bottom of the cup exerts on the straw.




*Given*: m = 0.00500kg, $\ell$ = 0.120m, and θ = 60°
*Find*: $F_x$ = ? and $F_y$ = ?

Applying the 2nd Laws using the origin as the pivot point,

$$\Sigma F_x = ma_x \Rightarrow F_x - F_N \sin\theta = 0$$

$$\Sigma F_y = ma_y \Rightarrow F_y + F_N \cos\theta - mg = 0$$

$$\Sigma\tau_o = I\alpha \Rightarrow \ell F_N - \tfrac{\ell}{2} F_g \cos\theta = 0$$

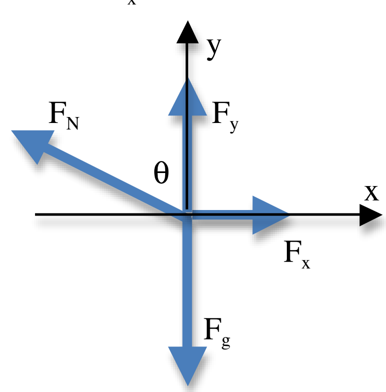
Using the torque equation, $F_N = \frac{1}{2}mg\cos\theta$.

Using the x equation, $F_x = F_N \sin\theta = \frac{1}{2}mg\cos\theta\sin\theta$.

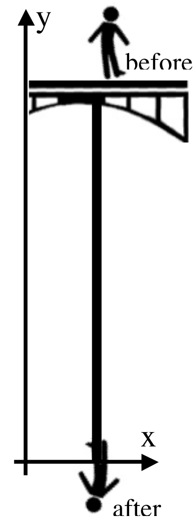
Using the y equation, $F_y = mg - F_N\cos\theta = mg - \frac{1}{2}mg\cos^2\theta$.

Plugging in, $F_x = \frac{1}{2}(0.005)(9.8)\cos 60°\sin 60° \Rightarrow \boxed{F_x = 0.0106N}$

and $F_y = (0.005)(9.8) - \frac{1}{2}(0.005)(9.8)\cos^2 60° \Rightarrow \boxed{F_y = 0.0429N}$.



3. A 60.0kg bungee jumper steps off a 55.0m high bridge. The un-stretched length of the cord is 30.0m and it stretches an additional 20.0m when the jumper is at the lowest point. Find the spring constant of the cord assuming both air resistance and the mass of the cord can be neglected.




*Given*: m = 60.0kg, h = 55.0m, $y_o$ = 50.0m, $x_o$ = 30.0m, and x = 20.0m.
*Find*: k = ?

At the top there is no kinetic energy but there is gravitational potential energy.

$$K_o = 0 \text{ and } U_o = mgy_o$$

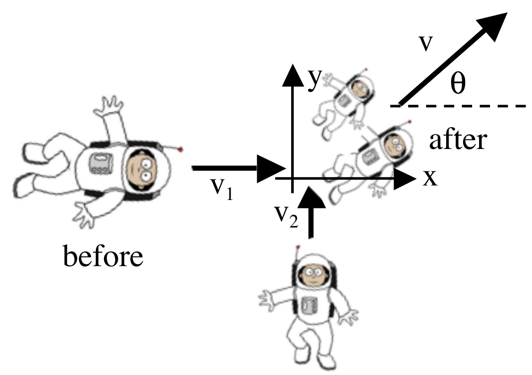
At the bottom there is also no kinetic energy, the gravitational potential energy is zero, and there is now spring potential energy.

$$K = 0 \text{ and } U = \tfrac{1}{2}kx^2$$

Applying the Law of Conservation of Energy,

$$K_o + U_o = K + U \Rightarrow 0 + mgy_o = 0 + \tfrac{1}{2}kx^2 \Rightarrow k = \frac{2mgy_o}{x^2} = \frac{2(60)(9.8)(50)}{(20)^2} \Rightarrow \boxed{k = 147\,\tfrac{N}{m}}.$$

4. A 110kg astronaut heading to the right collides with and holds on to an 80.0kg astronaut moving upward. After the collision, they move off together at 2.20m/s at an angle of 49° as shown. Find the speed of each astronaut before the collision.




*Given:* M = 110kg, m = 80.0kg, v = 2.20m/s, and θ = 49°.
*Find:* $v_1$ = ? and $v_2$ = ?

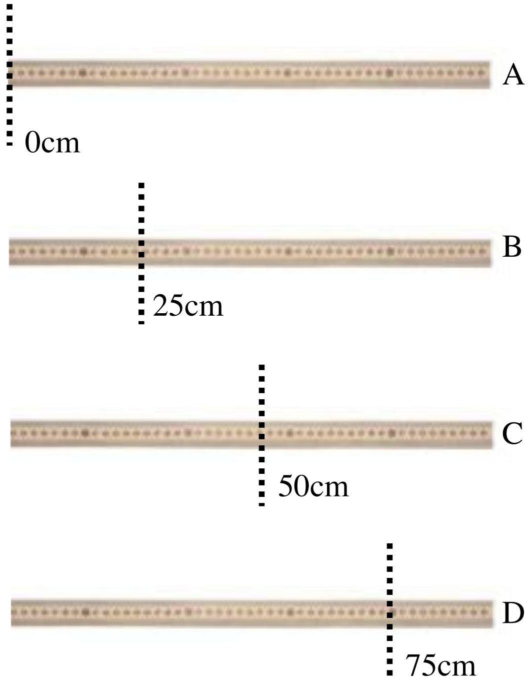
The initial momentum along each axis is
$p_x = Mv_1$ and $p_y = mv_2$.

The final momentum along each axis is $p_x = (M+m)v\cos\theta$ and $p_y = (M+m)v\sin\theta$.

By the Law of Conservation of Momentum, the initial momentum along each axis is equal to the final momentum along each axis:
$Mv_1 = (M+m)v\cos\theta$ and $mv_2 = (M+m)v\sin\theta$.

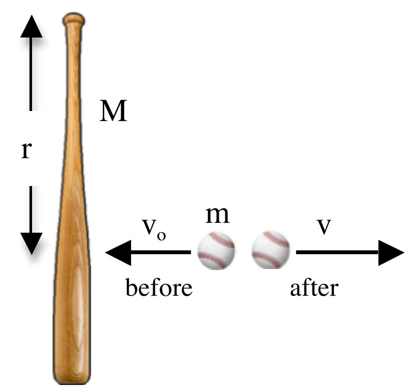
Solving for the speeds and plugging in the numbers,
$v_1 = (\frac{M+m}{M})v\cos\theta = (\frac{110+80}{110})(2.2)\cos 49° \Rightarrow \boxed{v_1 = 2.49\,\tfrac{m}{s}}$ and
$v_2 = (\frac{M+m}{m})v\sin\theta = (\frac{110+80}{80})(2.2)\sin 49° \Rightarrow \boxed{v_2 = 3.94\,\tfrac{m}{s}}$..

5. A meterstick is rotated about an axis perpendicular to the stick. In each sketch, the axis is located at a different point along the stick as shown. The location is given. Rank these situations from greatest to least based upon the rotational inertia of the meterstick.




Rotational inertia depends upon how far the mass is located from the axis of rotation. In A, the mass is as far away as possible while in C it is as close as possible. B and D have the same rotational inertia because they are mirror images of each other. So the ranking is $\boxed{A > B = D > C}$.

6. A baseball has a mass of 150g while a bat has a mass of 1.0kg and a rotational inertia of 0.45kg·m$^2$. A fastball heads toward the batter at 92mph (41m/s) and leaves the bat at 110mph (49m/s) in the opposite direction. The ball struck the bat at 75cm from the knob end when the bat is perpendicular to the velocity of the ball. Find (a)the initial angular momentum of the ball about the knob, (b)the final angular momentum of the ball about the knob, (c)the change in angular momentum of the ball, and (d)the change in angular momentum of the bat.




*Given*: m = 0.15kg, M = 1.0kg, I = 0.45kg·m$^2$, $v_o$ = 41m/s, v = 49m/s, and r = 0.75m.
*Find*: $L_o$ = ?, L = ?, $\Delta L_{ball}$ = ?, and $\Delta L_{bat}$ = ?

(a)The ball is a point particle so using the definition of angular momentum, $\vec{L} = \vec{r} \times \vec{p}$. The linear momentum is perpendicular to the distance r and the cross product is into the page or negative. So,

$L_o = -rmv_o = -(0.75)(0.15)(41) \Rightarrow \boxed{L_o = -4.61 \frac{kg \cdot m^2}{s}}$.

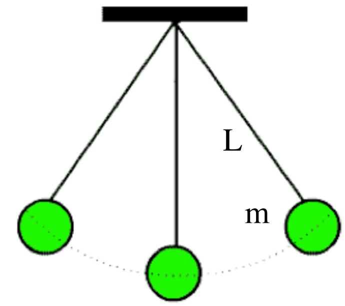
(b)The final angular momentum is out of the page or positive,

$L = rmv = -(0.75)(0.15)(49) \Rightarrow \boxed{L = 5.51 \frac{kg \cdot m^2}{s}}$.

(c)The change in the ball's angular momentum is, $\Delta L_{ball} = L - L_o = 5.51 - (-4.61) \Rightarrow \boxed{\Delta L_{ball} = 10.1 \frac{kg \cdot m^2}{s}}$.

(d)The Law of Conservation of Angular Momentum requires the change on angular momentum of the ball-bat system to be zero. So, if the angular momentum of the ball increase, the angular momentum of the bat must decrease. So, $\boxed{\Delta L_{bat} = -10.1 \frac{kg \cdot m^2}{s}}$.

7. A 1.00m long string has a small 250g mass at the end. The mass is allowed to swing back and forth. Find the period.



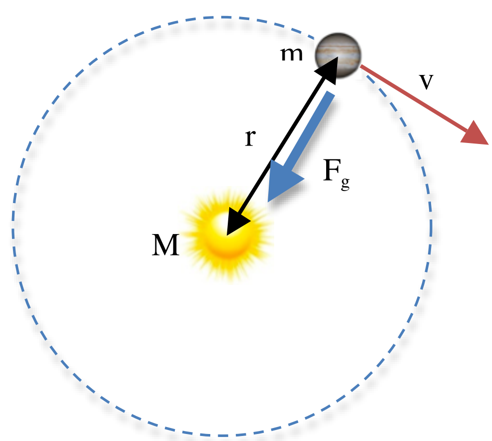

*Given*: L = 1.00m and m = 0.250kg
*Find*: T = ?

The angular frequency for a simple pendulum is given by, $\omega = \sqrt{\frac{g}{L}}$.

The angular frequency is related to the period by, $\omega = 2\pi f = \frac{2\pi}{T}$.

So, $\frac{2\pi}{T} = \sqrt{\frac{g}{L}} \Rightarrow T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1}{9.8}} \Rightarrow \boxed{T = 2.01s}$.

8. We are now beginning to find many planets that orbit other stars. One such planet is found to have an orbital period of $3.00x10^7$s and an orbital speed of $2.40x10^4$m/s. Find (a)the radius of the planet's orbit, (b)the acceleration of the planet in its orbit, and(c)the mass of the star it orbits.




*Given*: T = $3.00x10^7$s and v = $2.40x10^4$m/s
*Find*: r = ?, a = ?, and M = ?

(a)Using the definition of speed,

$$v \equiv \frac{dx}{dt} = \frac{2\pi r}{T} \Rightarrow r = \frac{vT}{2\pi} = \frac{(2.4x10^4)(3x10^7)}{2\pi} \Rightarrow \boxed{r = 1.15x10^{11}m}.$$

(b)Using the centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{(2.4x10^4)^2}{1.15x10^{11}} \Rightarrow \boxed{a_c = 5.02x10^{-3}\,\tfrac{m}{s^2}}.$$

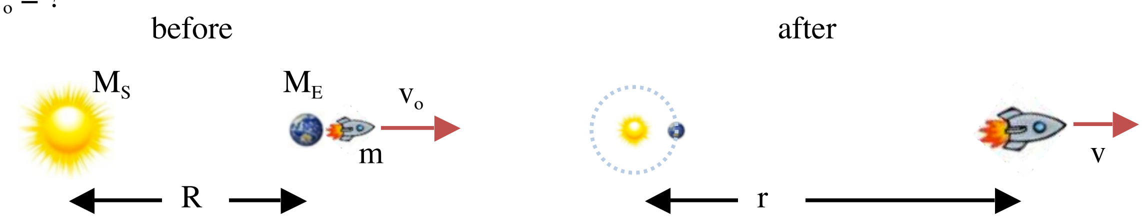
(c)Applying the Second Law and the Law of Universal Gravitation, $\Sigma F = ma \Rightarrow G\frac{mM}{r^2} = ma$.

The mass of the planet cancels. Solving for the mass of the star,

$$M = \frac{ar^2}{G} = \frac{(5.02x10^{-3})(1.15x10^{11})^2}{6.67x10^{-11}} \Rightarrow \boxed{M = 9.95x10^{29}kg}.$$

9. We have sent a space craft out of the solar system. Find the speed that such a craft would need starting at Earth so that it can escape the sun's gravitational field.

*Given*: $M_S = 1.99x10^{30}$kg, $M_E = 5.98x10^{24}$kg, and $R = 1.50x10^{11}$m
*Find*: $v_o$ = ?




The rocket near Earth has $K_o = \frac{1}{2}mv_o^2$ and $U_o = -G\frac{M_s m}{R}$ where we have neglected the small contribution due to the potential energy near Earth.

When the rocket is far away $K = \frac{1}{2}mv^2$ and $U = -G\frac{M_S m}{r}$. Since the rocket is very far away $r \to \infty$ so the potential energy is zero. Since we want the minimum launch speed, the final speed should be zero. So, $K \approx 0$ and $U \approx 0$. Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow \left[0 - \tfrac{1}{2}mv_o^2\right] + \left[0 - \left(-G\frac{M_S m}{R}\right)\right] = 0 \Rightarrow \tfrac{1}{2}mv_o^2 = G\frac{M_S m}{R}.$$

The mass of the rocket cancels. So,

$$v_o = \sqrt{G\frac{2M_S}{R}} = \sqrt{(6.67x10^{-11})\frac{2(1.99x10^{30})}{1.50x10^{11}}} \Rightarrow \boxed{v_o = 42.1\tfrac{km}{s}}.$$

10. Explain how we know Dark Matter exists.

The stars near the edge of our galaxy (or any other for that matter) rotate around the center of the galaxy at a rate that is much higher than that predicted using the Law of Gravitation and the known mass of the visible matter in the galaxy.

Also, we have seen the bending of light round galazies that is far more than the ordinary matter would create.