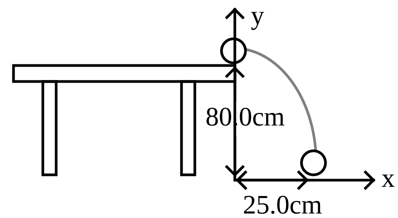
Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A marble rolls off the edge of an 80.0cm high table and strikes the ground 25.0cm horizontally from the edge of the table. Find (a)the time that the marble is in the air and (b)the speed of the marble as it left the table.



Use the kinematic equations separately for each direction:

$x_o = 0$	$y_o = 0.800\text{m}$
$x = 0.250\text{m}$	$y = 0$
$v_{ox} = ?$	$v_{oy} = 0$
$v_x = v_{ox}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80\text{m/s}^2$
$t = ?$	

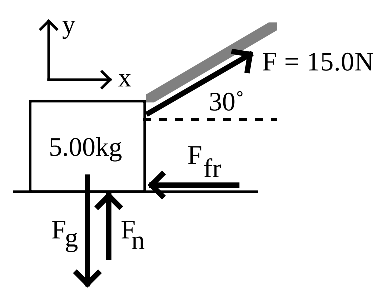
(a)Using the kinematic equation without the final speed for the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \qquad 0 = y_o + \tfrac{1}{2}a_y t^2 \qquad t = \sqrt{\frac{-2y_o}{a_y}} = \sqrt{\frac{-2(0.800)}{-9.80}} \qquad \boxed{t = 0.404\text{s}}.$$

(b)Using the kinematic equation without the final speed for the x-direction,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad x = v_{ox}t \qquad v_{ox} = \frac{x}{t} = \frac{0.250}{0.404} \qquad \boxed{v_{ox} = 0.619\text{m/s}}.$$

2. A 5.00kg block is pulled along a horizontal floor with an acceleration of 2.00m/s^2 by a cord that exerts a force of 15.0N at a 30.0˚ angle as shown. Find the coefficient of kinetic friction between the block and the table.



Applying the Second Law to each direction separately,

$$\Sigma F_x = ma_x \qquad F\cos30^\circ - F_{fr} = ma \qquad F_{fr} = F\cos30^\circ - ma\,,$$
$$\Sigma F_y = ma_y \qquad F\sin30^\circ + F_n - F_g = 0 \qquad F_n = F_g - F\sin30^\circ\,.$$

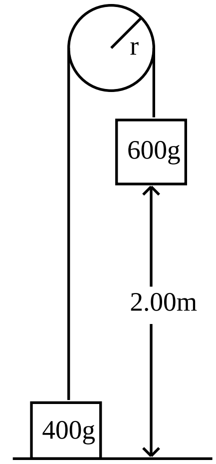
Substituting into the definition of coefficient of friction,

$$\mu \equiv \frac{F_{fr}}{F_n} = \frac{F\cos30^\circ - ma}{F_g - F\sin30^\circ} = \frac{F\cos30^\circ - ma}{mg - F\sin30^\circ}\,.$$

Plugging in the numbers,

$$\mu = \frac{(15.0)\cos30^\circ - (5.00)(2.00)}{(5.00)(9.80) - (15.0)\sin30^\circ} \qquad \boxed{\mu = 0.0721}.$$

3. An Atwood's Machine is made from a 600g mass, a 400g mass and a 200g pulley of radius 4.00cm. The 600g mass is released from rest and falls 2.00m. Find the angular speed of the pulley when the 600g mass reaches the ground.



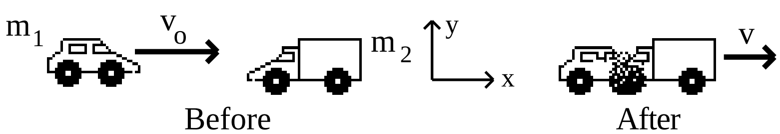
The initial energy of the system is,
$K_o = 0 \qquad U_o = m_1 gh$.
The final energy of the system is,
$K = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I\omega^2 \qquad U = m_2 gh$.

Using the Law of Conservation of Energy,
$\Delta K + \Delta U = W_{nc} \qquad (\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I\omega^2 - 0) + (m_2 gh - m_1 gh) = 0$.
Doing some algebra,
$\frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} I\omega^2 = (m_1 - m_2)gh$.

The linear speed of the masses is equal to the tangential speed of the pulley which is related to its angular speed,
$v = r\omega \qquad \frac{1}{2}(m_1 + m_2)(r\omega)^2 + \frac{1}{2} I\omega^2 = (m_1 - m_2)gh$.
Substituting the rotational inertia of the pulley which is a disk,
$I = \frac{1}{2} mr^2 \qquad \frac{1}{2}(m_1 + m_2)(r\omega)^2 + \frac{1}{2}\left(\frac{1}{2} mr^2\right)\omega^2 = (m_1 - m_2)gh$.
Solving for the angular speed,

$$\omega = \sqrt{\frac{4(m_1 - m_2)gh}{[2(m_1 + m_2) + m]r^2}} = \sqrt{\frac{4(0.600 - 0.400)(9.80)(2.00)}{[2(0.600 + 0.400) + 0.200](0.0400)^2}} \qquad \omega = 66.7\text{rad/s}$$

4. An 800kg car collides head-on with a stationary 1200kg truck. Experts estimate that the speed of the combined wreckage just after impact is 40±5km/h. Find (a)the experts estimate of the speed of the car before collision and (b)the experts estimate of the uncertainty in this speed.



(a)The momenta before and after the collision are $p_o = m_1 v_o$ and $p = (m_1 + m_2)v$.
Using the Law of Conservation of Linear Momentum,

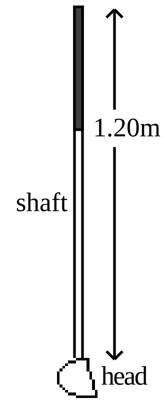
$$m_1 v_o = (m_1 + m_2)v \qquad v_o = \frac{m_1 + m_2}{m_1} v = \frac{800 + 1200}{800}(40) \qquad \boxed{v_o = 100\text{km/h}}.$$

(b)Using the multiplication rule for uncertainty,

$$\frac{\Delta v_o}{v_o} = \frac{\Delta v}{v} \qquad \Delta v_o = \frac{v_o}{v}\Delta v = \frac{100}{40}(5) \qquad \boxed{\Delta v_o = 12.5\text{km/h}}.$$

Using proper significant figures, $\boxed{v_o = 100 \pm 10\text{km/h}}$.

5. A golf club designer wants the rotational inertia about the handle end of the shaft to be $1.15\text{kg}\cdot\text{m}^2$. She assumes the shaft can be treated as a stick 1.20m long with a mass of 300g and the head can be treated as a point mass. Estimate the required mass of the head of the club.

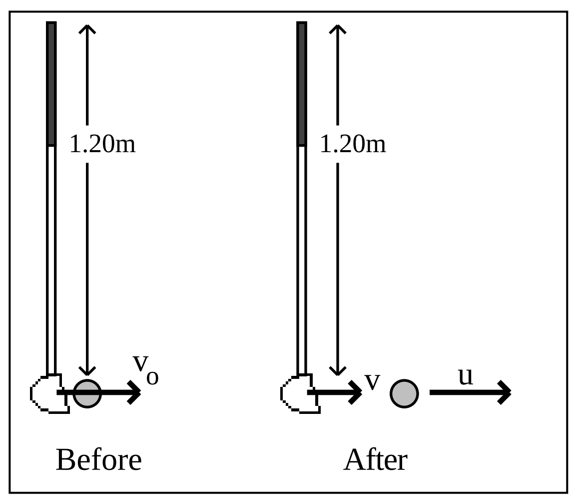


Rotational inertia adds so $I = I_{shaft} + I_{head}$.

The rotational inertia of the shaft is $I_{shaft} = \frac{1}{3} m\ell^2$.
The rotational inertia of the head is $I_{head} = Mr^2 = M\ell^2$.
Substituting,

$$I = \tfrac{1}{3} m\ell^2 + M\ell^2 \qquad M = \frac{I}{\ell^2} - \tfrac{1}{3} m = \frac{1.15}{(1.20)^2} - \tfrac{1}{3}(0.300) \qquad \boxed{M = 0.699\text{kg} = 699\text{g}}.$$

6. A 100g golf ball is struck by the club described in problem 5. The velocity of the club head is 40.0m/s just before the collision and 32.0m/s just after. Find (a)the angular momentum of the golf club about the end of the shaft just before the collision, (b)the angular momentum of the golf club about the end of the shaft just after the collision and (c)the velocity of the ball assuming that the torques exerted on the handle during the short time of the collision are small.



(a)The angular momentum of a rigid body is,

$$L_o = I\omega_o.$$

The angular speed of the club head is related to its linear speed,

$$v = r\omega \qquad L_o = I\frac{v_o}{\ell}.$$

Plugging in the numbers,

$$L_o = 1.15\frac{40.0}{1.20} \qquad \boxed{L_o = 38.3\text{kg}\tfrac{\text{m}^2}{\text{s}}}.$$

(b)Similarly,

$$L_a = I\frac{u}{\ell} = 1.15\frac{32.0}{1.20} \qquad \boxed{L_a = 30.7\text{kg}\tfrac{\text{m}^2}{\text{s}}}.$$

(c)The angular momentum of the golf ball after collision can be found from the definition of angular momentum,

$$\vec{L} \equiv \vec{r} \times \vec{p} \qquad L_b = mu\ell.$$

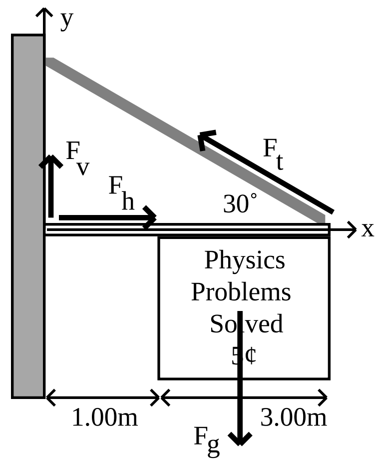
Using the Law of Conservation of Angular Momentum,

$$L_o = L_a + L_b \qquad L_o = L_a + mu\ell \qquad u = \frac{L_o - L_a}{m\ell}.$$

Plugging in the numbers,

$$u = \frac{38.3 - 30.7}{(0.100)(1.20)} \qquad \boxed{u = 63.3\text{m/s}}.$$

7. A 20.0kg sign 3.00m wide hangs from a horizontal rod that is supported by a cable that makes a 30.0˚ angle with the rod. Find (a)the tension in the cable and (b)the horizontal and vertical components of the force that the wall exerts on the rod. Ignore the mass of the rod.



Applying the Second Law,

$$\Sigma F_x = ma_x \qquad F_h - F_t\cos 30^\circ = 0 \qquad F_h = F_t\cos 30^\circ,$$

$$\Sigma F_y = ma_y \qquad F_t\sin 30^\circ + F_v - F_g = 0 \qquad F_v = F_g - F_t\sin 30^\circ,$$

$$\Sigma \tau_o = I\alpha \qquad 4F_t\sin 30^\circ - (2.5)F_g = 0 \qquad F_t = \frac{(2.5)F_g}{4\sin 30^\circ}.$$

(a)Solving the torque equation for the tension,

$$F_t = \frac{(2.5)F_g}{4\sin 30^\circ} = \frac{(2.5)mg}{4\sin 30^\circ} = \frac{(2.5)(20.0)(9.80)}{4\sin 30^\circ} \qquad \boxed{F_t = 245\text{N}}.$$

(b)Solving the equations from the force components,

$$F_h = F_t\cos 30^\circ = (245)\cos 30^\circ \qquad \boxed{F_h = 212\text{N}},$$

$$F_v = mg - F_t\sin 30^\circ = (20.0)(9.80) - (245)\sin 30^\circ \qquad \boxed{F_v = 73.5\text{N}}.$$

8. The International Space Station that is just starting to be built will orbit at an altitude of 350km. Find the period of orbit for the space station.

Using the Second Law and the Law of Universal Gravitation,

$$\Sigma F = ma \qquad G\frac{Mm}{r^2} = m\frac{v^2}{r} \qquad G\frac{M}{r} = v^2,$$

where the equation for centripetal acceleration has been used.
The definition of speed involves the period,

$$v = \frac{2\pi r}{T} \qquad G\frac{M}{r} = \left(\frac{2\pi r}{T}\right)^2 \qquad T = \sqrt{\frac{4\pi^2 r^3}{GM}}.$$

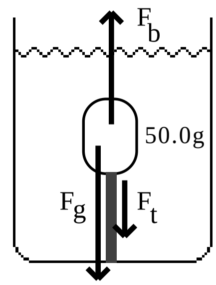
The radius of orbit is the radius of Earth plus the altitude,

$$r = R + h = 6.37\text{x}10^6 + 350\text{x}10^3 = 6.72\text{x}10^6\,\text{m}.$$

Calculating the period,

$$T = \sqrt{\frac{4\pi^2\left(6.72\text{x}10^6\right)^3}{(6.67\text{x}10^{-11})(5.98\text{x}10^{24})}} \qquad \boxed{T = 5480\text{s} = 91.3\text{min}}.$$

9. A 50.0g cork is held underwater by a string as shown at the right. Cork has a density of 560kg/m^3 and water has a density of 1000kg/m^3. Find the tension in the string.



Applying the Second Law,

$$\Sigma F_x = ma_x \qquad F_b - F_t - F_g = 0 \qquad F_t = F_b - F_g = F_b - mg.$$

According to Archimedes' Principle, the buoyant force is equal to the weight of the displaced fluid,

$$F_t = m_f g - mg.$$

The mass of the displaced fluid is related to the density of the fluid. Using the definition of density,

$$\rho \equiv \frac{m}{V} \qquad m_f = \rho_f V \qquad F_t = \rho_f V g - mg,$$

where V is the volume of the cork which can also be found from the definition of density,

$$\rho \equiv \frac{m}{V} \qquad V = \frac{m}{\rho} \qquad F_t = \rho_f \frac{m}{\rho} g - mg \qquad F_t = \left(\frac{\rho_f}{\rho} - 1\right) mg.$$

Putting in the numbers,

$$F_t = \left(\frac{1000}{560} - 1\right)(0.0500)(9.80) \qquad \boxed{F_t = 0.385\text{N}}.$$

10. The golf club designer wants to test the club she has designed and built. She holds it at the end of the shaft and lets it swing. Find (a)the center of mass of the club and (b)the predicted period of oscillation.

(a)Assuming the cm of the shaft is in the middle and using the definition of the cm,

$$r_{cm} = \frac{m\frac{\ell}{2} + M\ell}{m + M} = \frac{(300)(0.600) + (699)(1.20)}{300 + 699} \qquad \boxed{r_{cm} = 1.02\text{m}}.$$

(b)The angular frequency of a physical pendulum is,

$$\omega = \sqrt{\frac{mgr}{I_p}} = \sqrt{\frac{(m + M)gr_{cm}}{I}}.$$

The angular frequency is related to the period,

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{(m + M)gr_{cm}}}.$$

Putting in the values,

$$T = 2\pi\sqrt{\frac{1.15}{(0.300 + 0.699)(9.80)(1.02)}} \qquad \boxed{T = 2.13\text{s}}.$$