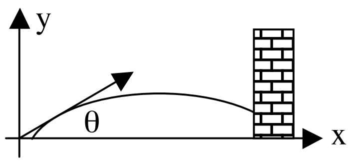
Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A firefighter 9.00m from a building directs a stream of water from a hose moving at 10.0m/s at an angle of 53.0° above the horizontal. Find (a)the time between when the water leaves the hose and it reaches the building and (b)the height at which the water strikes the building.



$x_o = 0$
$x = 9.00m$
$v_{ox} = 10.0\cos 53° = 6.00m/s$
$v_x = 6.00m/s$
$a_x = 0$
$t = ?$

$y_o = 0$
$y = ?$
$v_{oy} = 10.0\sin 53° = 8.00m/s$
$v_y = ?$

(a)Use the kinematic equation for x,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \Rightarrow x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}}$$

Putting in the numbers,

$$t = \frac{9.00}{6.00} \Rightarrow \boxed{t = 1.50s}.$$

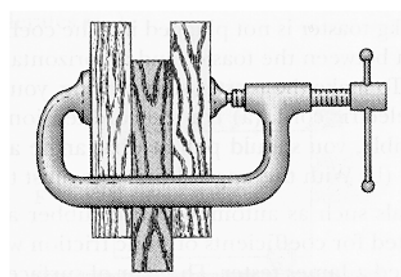
(b)Use the kinematic equation for y,

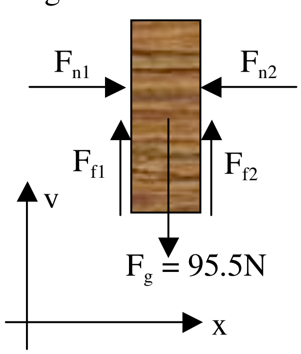
$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow y = 0 + (8.00)(1.50) - \tfrac{1}{2}(9.80)(1.50)^2$$

Finally,

$$\boxed{y = 0.975m}.$$

2. A 95.5N board is sandwiched between to other boards as shown. The coefficient of friction between the boards is 0.663. (a)Draw the forces that act on the 95.5N board. (b)Find the magnitude of each force.





Applying Newton's Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_{n1} - F_{n2} = 0 \Rightarrow F_{n1} = F_{n2}$$
$$\Sigma F_y = ma_y \Rightarrow F_{f1} + F_{f2} - F_g = 0$$
$$\Rightarrow F_{f1} + F_{f2} = F_g$$

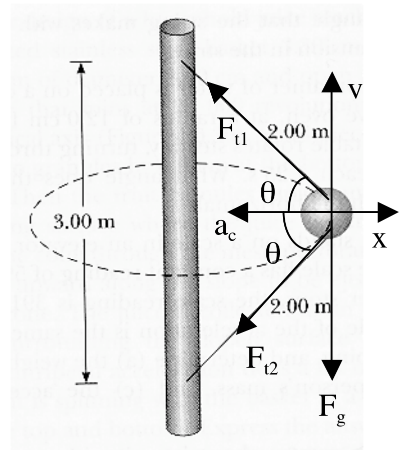
Using the definition of COF, $F_{f1} = \mu F_{n1}$ and $F_{f2} = \mu F_{n2}$.
Plugging into the equation from the y-direction,

$$\mu F_{n1} + \mu F_{n2} = F_g \Rightarrow \mu(F_{n1} + F_{n2}) = F_g .$$

Since the normal forces are equal, $2\mu F_{n1} = F_g \Rightarrow F_{n1} = \frac{F_g}{2\mu} = \frac{95.5}{2(0.663)} \Rightarrow \boxed{F_{n1} = F_{n2} = 72.0N}$.

The frictional forces must also be equal, $F_{f1} = F_{f2} = \mu F_{n1} = (0.663)(72.0) \Rightarrow \boxed{F_{f1} = F_{f2} = 47.7N}$.

3. A 400g ball is attached to a vertical rod by two strings as shown. The ball rotates in a horizontal circle at 6.00m/s. Find the tension in each string.



Applying Newton's Second Law,

$$\Sigma F_x = ma_x \Rightarrow -F_{t1}\cos\theta - F_{t2}\cos\theta = -ma_c$$
$$\Sigma F_y = ma_y \Rightarrow F_{t1}\sin\theta - F_{t2}\sin\theta - F_g = 0$$

Using the mass/weight rule and the centripetal acceleration,

$$F_{t1} + F_{t2} = \frac{mv^2}{r\cos\theta} \text{ and } F_{t1} - F_{t2} = \frac{mg}{\sin\theta}.$$

Adding these equations gives,

$$2F_{t1} = \frac{mv^2}{r\cos\theta} + \frac{mg}{\sin\theta} \Rightarrow F_{t1} = \frac{mv^2}{2r\cos\theta} + \frac{mg}{2\sin\theta}.$$

Subtracting gives,

$$2F_{t2} = \frac{mv^2}{r\cos\theta} - \frac{mg}{\sin\theta} \Rightarrow F_{t2} = \frac{mv^2}{2r\cos\theta} - \frac{mg}{2\sin\theta}.$$

Doing a bit of trigonometry,

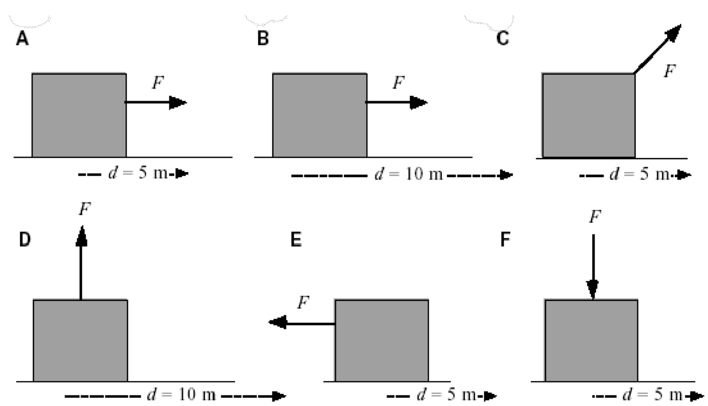
$$r = \sqrt{2.00^2 - 1.50^2} = 1.32m \quad \text{and} \quad \sin\theta = \frac{1.50}{2.00} \Rightarrow \theta = \arcsin\frac{1.50}{2.00} = 48.6^\circ.$$

Plugging in,

$$F_{t1} = \frac{(0.400)(6.00)^2}{2(1.32)\cos 48.6^\circ} + \frac{(0.400)(9.80)}{2\sin 48.6^\circ} \Rightarrow \boxed{F_{t1} = 10.9N} \text{ and}$$

$$F_{t2} = \frac{(0.400)(6.00)^2}{2(1.32)\cos 48.6^\circ} - \frac{(0.400)(9.80)}{2\sin 48.6^\circ} \Rightarrow \boxed{F_{t2} = 5.63N}.$$

4. In the figures, identical boxes of mass 10 kg are moving at the same initial velocity to the right on a flat surface. The same magnitude force, *F*, is applied to each box for the distance, *d*, indicated in the figures. Rank these situations in order of the work done on the box by *F* while the box moves the indicated distance to the right. Explain your reasoning.



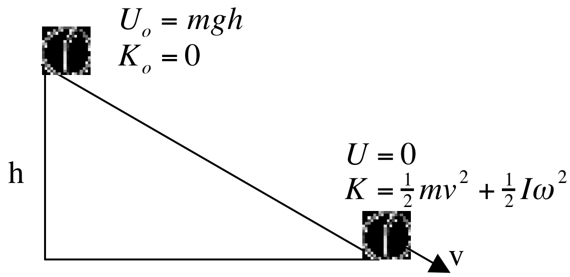
Applying the definition of work,

$$W \equiv \int \vec{F} \bullet d\vec{s}$$

to this case where the forces are constant means that the work done is just the product of the horizontal component of the force and the distance, so B is the largest because the distance is largest. Next is A then comes C because only part of the force is horizontal. D and F have no work done because the force has no horizontal component. Finally E has negative work done.

B>A>C>D=F>E

5. A 0.145kg baseball starts near rest and rolls without slipping down a 55.0cm high pitchers mound. Find the speed of the ball at the bottom.



Using the Law of Conservation of Energy,
$\Delta U + \Delta K = 0 \Rightarrow (0 - mgh) + (\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2) = 0$

Since the ball rolls without slipping, $\omega = \dfrac{v}{r}$.

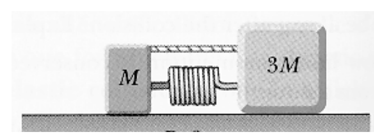
The rotational inertia of a sphere is, $I = \frac{2}{5}mr^2$.

$K_o = 0$
Substituting into the Conservation of Energy equation,

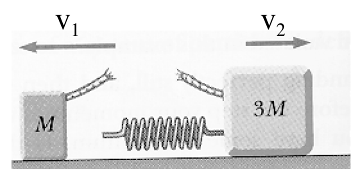
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{2}{5}mr^2)\frac{v^2}{r^2} \Rightarrow mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{5}mv^2 \Rightarrow mgh = \tfrac{7}{10}mv^2.$$

Canceling the mass and solving for the speed,
$v = \sqrt{\tfrac{10}{7}gh} = \sqrt{\tfrac{10}{7}(9.80)(0.550)} \Rightarrow \boxed{v = 2.77 m/s}$.

6. Two blocks of masses 100g and 300g are placed on a horizontal, frictionless surface. A light spring of spring constant 8.00N/m is attached to one of them and the blocks as pushed together compressing the spring 5.00cm. A cord initially holding the blocks together is burned. Find the speed of the blocks assuming the energy stored in the spring is completely transferred to the blocks motion.



before
$p_o = 0 \quad U_o = \frac{1}{2}kx^2 \quad K_o = 0$



after
$p = Mv_2 - mv_1 \quad U = 0$
$K = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$

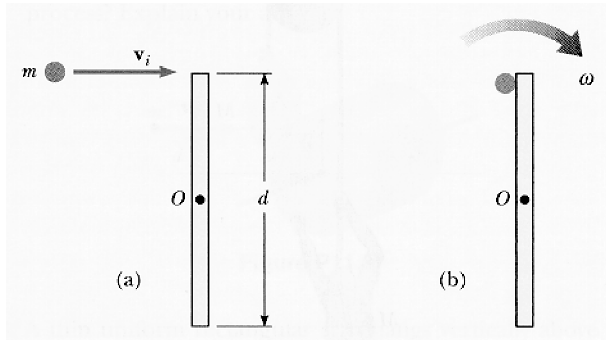
Applying the Law of Conservation of Momentum,
$0 = Mv_2 - mv_1 \Rightarrow v_2 = \frac{m}{M}v_1$.
A
pplying the Law of Conservation of Energy,
$\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \Rightarrow kx^2 = mv_1^2 + Mv_2^2$.

Substituting from the momentum equation into the energy equation,
$kx^2 = mv_1^2 + M\left(\frac{m}{M}v_1\right)^2 \Rightarrow kx^2 = \frac{m}{M}\left(m + M\right)v_1^2 \Rightarrow v_1 = \sqrt{\frac{M}{m}\cdot\frac{k}{m+M}}x$

Plugging in the numbers,
$v_1 = \sqrt{\frac{300}{100}\cdot\frac{8.00}{0.100+0.300}}(0.0500) \Rightarrow \boxed{v_1 = 0.387 m/s}$.

Using the momentum equation again,
$v_2 = \frac{m}{M}v_1 = \frac{100}{300}(0.387) \Rightarrow \boxed{v_2 = 0.129 m/s}$.

7. A 50.0g ball of clay moves to the right with a speed of 10.0m/s. It strikes and sticks to the end of a 150g meterstick pivoted about its center. Find the angular speed of the system just after the collision.



The initial angular momentum of the system is just due to the ball of clay,
$\vec{L} \equiv \vec{r} \times \vec{p} \Rightarrow L_o = \frac{d}{2} m v_i$.

The final angular momentum is due to the motion of the stick and the clay,
$L = I\omega + \frac{d}{2} m v$.

The rotational inertia of a stick pivoted about the center is, $I = \frac{1}{12} M d^2$.
The speed of the ball of clay is related to the angular speed by, $v = r\omega = \frac{d}{2}\omega$.
Applying the Law of Conservation of Angular Momentum,
$$\frac{d}{2} m v_i = I\omega + \frac{d}{2} m v = \frac{1}{12} M d^2 \omega + \frac{d}{2} m \frac{d}{2} \omega = (\frac{1}{12} M + \frac{1}{4} m) d^2 \omega \Rightarrow m v_i = (\frac{1}{6} M + \frac{1}{2} m) d\omega$$

Solving for the angular speed, $\omega = \frac{m}{\frac{1}{6}M + \frac{1}{2}m} \frac{v_i}{d} = \frac{50}{\frac{1}{6}150 + \frac{1}{2}50} \frac{10.0}{1.00} \Rightarrow \boxed{\omega = 10.0 rad/s}$.

8. A 1.00kg mass attached to a spring with a spring constant of 25.0N/m oscillates on a horizontal frictionless track. At t = 0s, the mass is released from rest at x - -3.00cm. (That is, the spring is compressed by 3.00cm.) Find (a)the period of its motion, (b)the maximum speed and (c)the maximum acceleration.

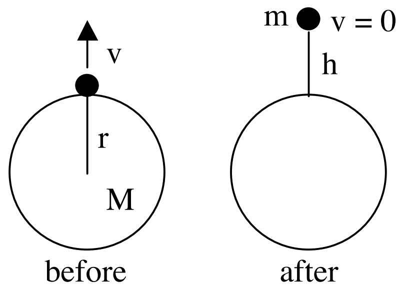
(a)The period is related to the angular frequency,
$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.00}{25.0}} \Rightarrow \boxed{T = 1.26 s}.$$

(b)The speed as a function of time is, $v = -\omega A \sin \omega t$.
The maximum speed occurs when the sine is one (minus one actually),
$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A = \sqrt{\frac{25.0}{1.00}} (0.0300) \Rightarrow \boxed{v_{\max} = 0.150 m/s}.$$

(c)The acceleration as a function of time is, $a = -\omega^2 A \cos \omega t$.
The maximum speed occurs when the cosine is one (minus one actually),

$a_{max} = \omega^2 A = \frac{k}{m} A = \frac{25.0}{1.00}(0.0300) \Rightarrow \boxed{a_{max} = 0.750\, m/s^2}$.

9. A volcano on Jupiter's moon Io spews liquid sulfur to a height of 70.0km above the surface. The mass of Io is $8.90x10^{22}$kg and the radius is 1820km. Find the speed the liquid leaves the surface.



$K_o = \frac{1}{2}mv^2$
$U_o = -G\frac{Mm}{r}$

$K = 0$
$U = -G\frac{Mm}{r+h}$

Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow (0 - \tfrac{1}{2}mv^2) + (-G\tfrac{Mm}{r+h} + G\tfrac{Mm}{r}) = 0$$

Doing some algebra,

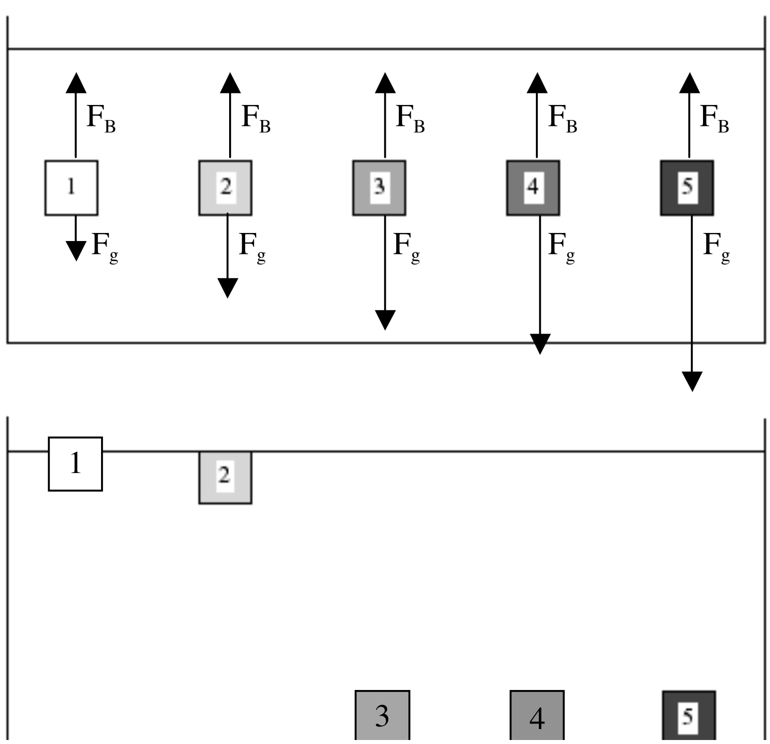
$$\tfrac{1}{2}mv^2 = -G\tfrac{Mm}{r+h} + G\tfrac{Mm}{r} \Rightarrow v^2 = 2GM\left(-\tfrac{1}{r+h} + \tfrac{1}{r}\right) = 2GM\tfrac{h}{r(r+h)}$$

Finally,

$$v = \sqrt{2GM\tfrac{h}{r(r+h)}} = \sqrt{2(6.67x10^{-11})(8.90x10^{22})\tfrac{70000}{1.82x10^6(1.89x10^6)}} \Rightarrow$$

$\boxed{v = 492 m/s}$.

10. Five blocks identical in size and shape are made of different materials with different densities. In order of mass, (1) is the lightest and (5) is heaviest. All five blocks are released from positions halfway to the bottom of a tank of water as shown in the upper picture. When block 2 comes to rest, its top surface is level with the surface of the water, as shown in the lower picture. (a)Draw free-body diagrams for each block at the moment of their release in the upper drawing. Draw the force vectors to scale. (b)The equilibrium positions of blocks 2 and 5 are shown in the lower picture. Sketch the equilibrium positions of blocks 1, 3, and 4 in the lower drawing. Explain your reasoning.



Since the blocks are all the same size, they displace the same amount of water. By Archimedes Principle, they must have the same buoyant force. Since the denser blocks have more mass, they have a greater gravitational force.

We are shown that block 2 has a buoyant force equal to the gravitational force. Therefore, blocks 3, 4, and 5 must sink to the bottom. Also, block 1 must rise to the surface and float such that the volume below the surface displaces enough water to balance the gravitational force.