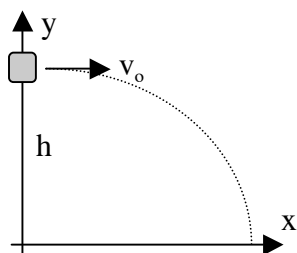


Name: _____

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(only if you want your grade posted on the web.)

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles shown on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A helicopter flying horizontally at 30.0m/s is 80.0m above the ground and plans to drop supplies to tsunami victims below. Find (a) the time before they are directly overhead that they must release the supplies so that they land near these victims and (b) the horizontal distance from where they release them to the point where they land.



$x_o = 0$	$y_o = 80.0\text{m}$
$x = ?$	$y = 0$
$v_{ox} = 30.0\text{m/s}$	$v_{oy} = 0$
$v_x = 30.0\text{m/s}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80\text{m/s}^2$
$t = ?$	$t = ?$

(a) Using the kinematic equation along the y-direction,

$$y = y_o + v_{oy}t + \frac{1}{2}a_yt^2$$

Plugging in the numbers,

$$0 = y_o + \frac{1}{2}a_yt^2 \Rightarrow t = \sqrt{\frac{-2y_o}{a_y}} = \sqrt{\frac{-2(80.0)}{-9.80}} \Rightarrow \boxed{t = 4.04\text{m/s}}$$

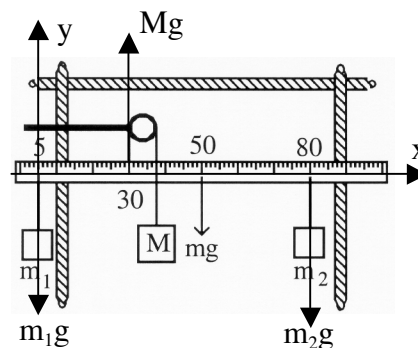
(a) Using the kinematic equation along the x-direction,

$$x = x_o + v_{ox}t + \frac{1}{2}a_xt^2$$

Plugging in the numbers,

$$x = v_{ox}t \Rightarrow x = (30.0)(4.04) \Rightarrow \boxed{x = 121\text{m}}$$

2. A 150g meterstick is suspended horizontally from a string at the 30cm mark that is wrapped over a pulley supporting a mass M. Two weights hang from the stick. One has a mass of 100g at the 80cm mark and the other is at the 5cm mark. Find the mass at the 5cm mark and the mass M needed to keep the meterstick in equilibrium.



Applying the Second Law for Rotation about the origin,

$$\Sigma\tau = I\alpha \Rightarrow 25Mg - 45mg - 75m_2g = 0$$

Solving for M,

$$M = \frac{45m + 75m_2}{25} = \frac{45(150) + 75(100)}{25} \Rightarrow \boxed{M = 570\text{g}}$$

Applying the Second Law along the y-direction,

$$\Sigma F_y = ma_y \Rightarrow Mg - mg - m_1g - m_2g = 0$$

Solving for m_1 ,

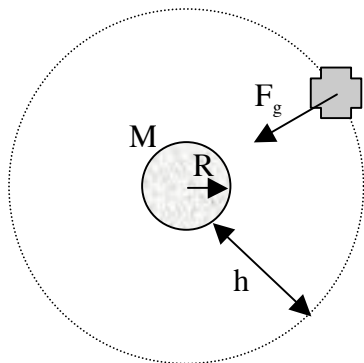
$$m_1 = M - m - m_2 = 570 - 150 - 100 \Rightarrow \boxed{m_1 = 320\text{g}}$$

3. The US military has a 200kg satellite that orbits Earth at an altitude of 300km. Earth exerts a gravitational force on the satellite to keep it in orbit. You claim that this means that the satellite exerts a gravitational force on Earth. Your friend says that this is ridiculous because the satellite is so small compared to Earth that it can't affect Earth at all. Explain as clearly as you can to your friend why you are correct and be sure to compare the size of the force exerted on the satellite with the size of the force exerted on Earth.

According to Newton's Third Law, if one object exerts a force on a second object, the second object exerts an equal force back on the first. Since Earth exerts a force on the satellite, the satellite must exert an equal force back on Earth.

Earth is not affected much by this force because of Newton's Second Law, which says that the acceleration of an object is inversely proportional to the mass of the object. Since the mass of Earth is so large compared with the mass of the satellite, it is the satellite that does all the accelerating. Earth's motion is barely changed by this force.

4. Find the period of orbit of the satellite in the previous problem.



Applying Newton's Second Law to the satellite,

$$\Sigma F = ma \Rightarrow F_g = ma$$

Using the Universal Law of Gravitation,

$$G \frac{Mm}{r^2} = ma \Rightarrow a = G \frac{M}{r^2}$$

and the centripetal acceleration,

$$\frac{v^2}{r} = G \frac{M}{r^2} \Rightarrow v^2 = G \frac{M}{r}$$

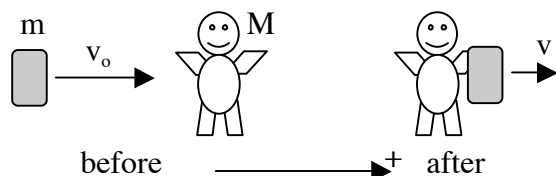
The speed is the circumference over the period,

$$\left(\frac{2\pi r}{T} \right)^2 = G \frac{M}{r} \Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

Note that $r = R + h$. Plugging in the numbers,

$$T = \sqrt{\frac{4\pi^2 (6.38 \times 10^6 + 3.00 \times 10^5)^3}{(6.67 \times 10^{-11})(5.97 \times 10^{24})}} \Rightarrow \boxed{T = 5440 \text{ s}}$$

5. A 60.0kg astronaut is on a space walk to repair a communications satellite. She realizes that she needs to consult the repair manual, which her colleague tosses toward her at 4.00m/s relative to the spaceship. If she was at rest relative to the ship before she catches the 3.00kg manual. Find her speed after she catches it.



The initial linear momentum is,

$$p_o = mv_o.$$

The final momentum is,

$$p = (m + M)v.$$

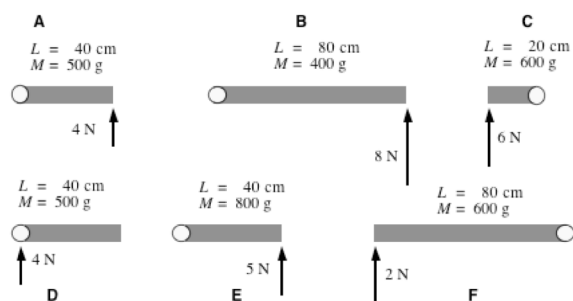
Applying the Law of Conservation of Linear Momentum,

$$p = p_o \Rightarrow mv_o = (m + M)v \Rightarrow v = \frac{m}{m+M} v_o.$$

Plugging in the values,

$$v = \frac{3.00}{3.00+60.0} (4.00) \Rightarrow \boxed{v = 0.190 \text{ m/s}}.$$

6. Shown at the right in a top view are six uniform rods that vary in mass (M) and length (L). Also shown are circles representing a vertical axis around which the rods are going to be rotated in a horizontal plane and arrows representing forces acting to rotate the rods. The forces change direction in order to always act perpendicular to the rods. Specific values for the lengths and masses of the rods and the magnitudes of the forces are given in each figure. Rank these rods from greatest to least, on the basis of (a) their rotational inertia, (b) the torque they feel, and (c) their change in angular momentum in 1.00s. Explain your reasoning for full credit.



(a) The rotational inertia for a rod pivoted about its end is $I = \frac{1}{3} ML^2$ so the ranking is,

$$\boxed{F > B > E > A = D > C}.$$

(b) The definition of torque is, $\vec{\tau} \equiv \vec{r} \times \vec{F} \Rightarrow \tau = LF$ because the forces are perpendicular to the rods. Keeping in mind that counterclockwise rotations are due to positive torques the ranking is,

$$\boxed{B > E > A > D > C > F}.$$

(c) Since the Second Law states that the torques is equal to the rate of change of angular momentum, the answer is the same as part (b), $\boxed{B > E > A > D > C > F}.$

Here are the numbers if you care....

Rod	A	B	C	D	E	F
$I(\text{kg}\cdot\text{m}^2)$	0.027	0.085	0.008	0.027	0.043	0.128
$\tau(\text{N}\cdot\text{m})$	1.6	6.4	-1.2	0	2.0	-1.6

7. A comet of mass $3.50 \times 10^5 \text{ kg}$ orbits the sun. At closest approach (A) it has a speed of 75.0 km/s . The distance from the sun to point C is four times the distance from the sun to point A. Find its speed at point C.

The angular momentum at the point A is,

$$L_a = mv_a r_a.$$

The angular momentum at the point C is,

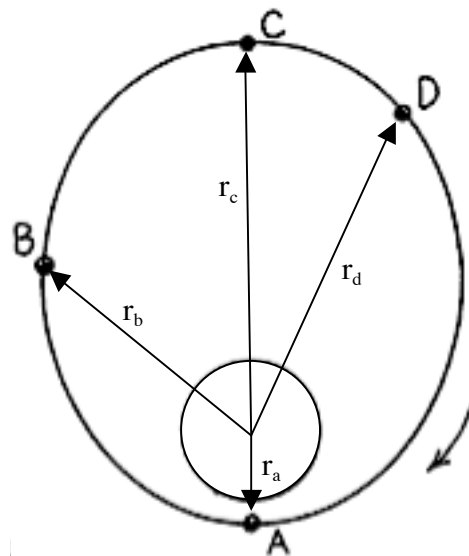
$$L_c = mv_c r_c.$$

Applying the Law of Conservation of Angular Momentum,

$$L_a = L_c \Rightarrow mv_a r_a = mv_c r_c \Rightarrow v_c = v_a \frac{r_a}{r_c}.$$

Since $r_c = 4r_a$,

$$v_c = v_a \frac{r_a}{4r_a} = \frac{1}{4} v_a = \frac{1}{4} (75.0) \Rightarrow \boxed{v_c = 18.8 \text{ km/s}}.$$



8. As the comet in problem 7 goes from point A to point C, (a) find the change in kinetic energy, (b) the change in potential energy, and (c) the radii r_a and r_c .

(a) Using the definition of kinetic energy,

$$\Delta K = \frac{1}{2} mv_c^2 - \frac{1}{2} mv_a^2 = \frac{1}{2} m (v_c^2 - v_a^2).$$

Putting in the numbers,

$$\Delta K = \frac{1}{2} (3.50 \times 10^5) [(18.8 \times 10^3)^2 - (75.0 \times 10^3)^2] \Rightarrow \boxed{\Delta K = -9.23 \times 10^{14} \text{ J}}.$$

(b) Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow \Delta U = -\Delta K \Rightarrow \boxed{\Delta U = +9.23 \times 10^{14} \text{ J}}.$$

(c) Using the Gravitational Potential Energy,

$$\Delta U = -G \frac{Mm}{r_c} + G \frac{Mm}{r_a} = GMm \left(\frac{1}{r_a} - \frac{1}{r_c} \right).$$

Since $r_c = 4r_a$,

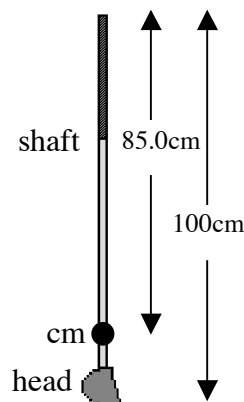
$$\Delta U = GMm \left(\frac{1}{r_a} - \frac{1}{4r_a} \right) = G \frac{Mm}{r_a} \left(1 - \frac{1}{4} \right) = G \frac{3Mm}{4r_a}.$$

Solving for the radius,

$$r_a = G \frac{3Mm}{4\Delta U} = (6.67 \times 10^{-11}) \frac{3(1.99 \times 10^{30})(3.50 \times 10^5)}{4(9.23 \times 10^{14})} \Rightarrow \boxed{r_a = 3.77 \times 10^{10} \text{ m}}.$$

Since $r_c = 4r_a \Rightarrow \boxed{r_c = 1.51 \times 10^{11} \text{ m}}.$

9. The golf club shown at the right is 1.00m long. The head has a mass of 700g and the shaft has a mass of 200g. The center of mass is 85.0cm below the top of the shaft. A bored golfer decides to let the club gently swing back and forth while she chats with her partners. The period of oscillation is 2.00s. Find the rotational inertia of the club about the end of the shaft.



For a physical pendulum,

$$\omega = \sqrt{\frac{mgr}{I_p}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I_p}{mgr}}.$$

Solving for the rotational inertia,

$$I_p = mgr \frac{T^2}{4\pi^2}.$$

The r is from the pivot to the center of mass while m is the total mass,

$$I_p = (0.700 + 0.200)(9.80)(0.850) \frac{(2.00)^2}{4\pi^2} \Rightarrow \boxed{I_p = 0.760 \text{ kg} \cdot \text{m}^2}.$$

10. A block of metal has a mass of 200g in air, but the same scale only reads 175g when the block is submerged in water. Find the density of the metal.

Applying the Second Law to the block in water,

$$\Sigma F = ma \Rightarrow F_{s,\text{water}} - mg + F_B = 0 \Rightarrow m_w g - mg + F_B = 0,$$

where m_w is the mass reading in water (175g).

The buoyant force is given by Archimedes Principle as,

$$F_B = \rho_w g V,$$

where V is the volume of the block. The 2nd Law equation becomes,

$$m_w g - mg + \rho_w g V = 0 \Rightarrow m_w - m + \rho_w V = 0 \Rightarrow \rho_w V = m - m_w.$$

Using the definition of density the volume can be rewritten as,

$$\rho \equiv \frac{m}{V} \Rightarrow V = \frac{m}{\rho}.$$

Substituting into the 2nd Law equation,

$$\rho_w \frac{m}{\rho} = m - m_w \Rightarrow \rho = \rho_w \frac{m}{m - m_w}$$

Putting in the numerical values,

$$\rho = (1000) \frac{200}{200 - 175} \Rightarrow \boxed{\rho = 8000 \text{ kg/m}^3}.$$

