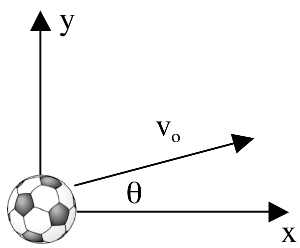
Name:___________________________________

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A soccer ball is kick with a speed of 18.0m/s at a 30.0° angle above the horizontal. Find (a)the time it is in the air and (b)the distance it travels before it hits the ground.



$x_o = 0$	$y_o = 0$
$x = ?$	$y = 0$
$v_{ox} = v_o\cos\theta$	$v_{oy} = v_o\sin\theta$
$v_x = v_{ox}$	$v_y = ?$
$a_x = 0$	$a_y = -9.80\text{m/s}^2$
$t = ?$	$t = ?$

(a)Using the kinematic equation along the y-axis,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2.$$

Putting in the things that are zero and solving for t,

$$0 = 0 + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow t = -\frac{2v_{oy}}{a_y} = -\frac{2v_o\sin\theta}{a_y}.$$

Plugging in the numbers,

$$t = \frac{-2(18.0)\sin 30.0^\circ}{-9.80} \Rightarrow \boxed{t = 1.84s}.$$

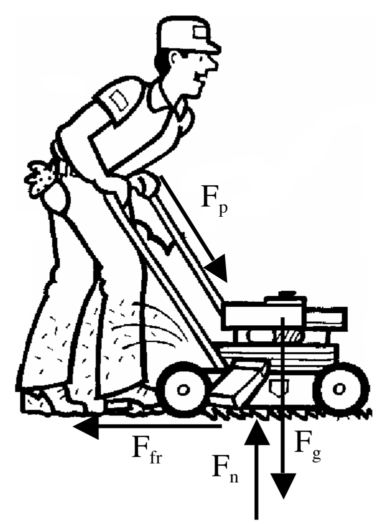
(b)Using the kinematic equation along the x-axis,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = v_o t\cos\theta.$$

Plugging in the numbers,

$$x = (18.0)(1.84)\cos 30.0^\circ \Rightarrow \boxed{x = 28.6m}.$$

2. A gardener pushes a 7.00kg lawn mower at the constant speed of 0.800m/s across level ground by exerting a force of 60.0N directed along the handle, which makes a 50.0° angle with the horizontal. Find the size of (a)the normal force and (b)the frictional force on the mower.



Applying the Second Law for each direction remembering that the constant velocity means no acceleration,

$\Sigma F_x = ma_x \Rightarrow F_p\cos\theta - F_{fr} = 0$ and

$\Sigma F_y = ma_y \Rightarrow F_n - F_p\sin\theta - F_g = 0$.

Soling the y-equation for the normal force and the x-equation for the frictional force,

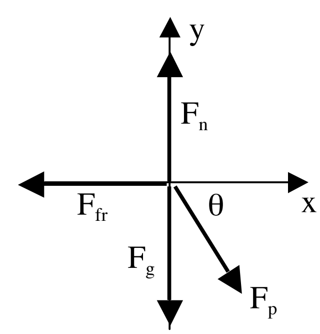
$F_{fr} = F_p\cos\theta$ and

$F_n = F_p\sin\theta + F_g$.

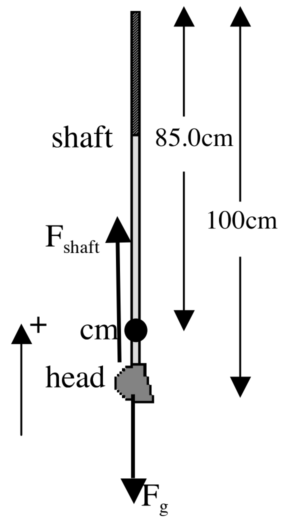
Plugging in the numbers,

$F_{fr} = 60.0\cos 50.0^\circ \Rightarrow \boxed{F_{fr} = 38.6N}$ and

$F_n = 60.0\sin 50.0^\circ + (7.00)(9.80) \Rightarrow \boxed{F_n = 115N}$.



3. The golf club shown at the right is 1.00m long. The head has a mass of 700g and the shaft has a mass of 200g. The center of mass is 85.0cm below the top of the shaft. At the bottom of the swing, the club head is moving at 9.00m/s in circular motion about the top of the shaft. Find the vertical component of the force that the shaft exerts on the head at this instant.



Applying the Second Law to the head of the club,
$\Sigma F = ma \Rightarrow F_{shaft} - F_g = ma$.

The acceleration is centripetal so,

$$F_{shaft} - F_g = m\frac{v^2}{r}.$$

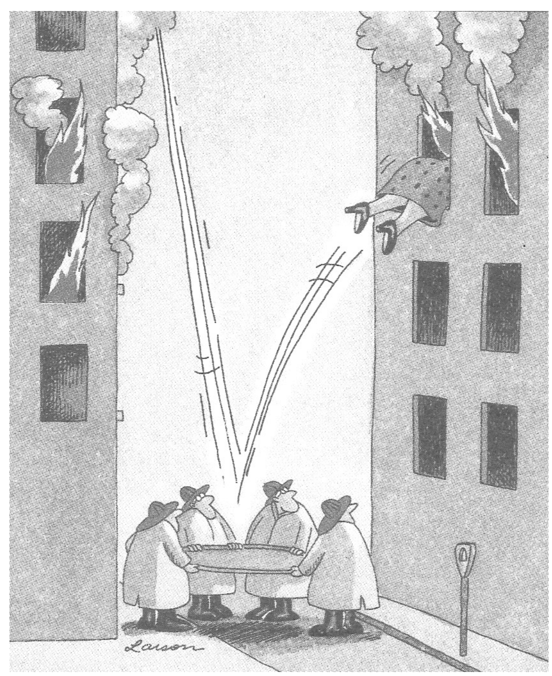
Solving for the force due to the shaft,

$$F_{shaft} = F_g + m\frac{v^2}{r} = mg + m\frac{v^2}{r} = m\left(g + \frac{v^2}{r}\right).$$

Plugging in the numbers,

$$F_{shaft} = (0.700)\left(9.80 + \frac{9.00^2}{1.00}\right) \Rightarrow \boxed{F_{shaft} = 63.6N}.$$

4. The 52.0kg woman pictured at the right falls into the safety net and stretches it 0.500m. The net springs back and he flies through the third story window 9.00m above the net at a speed of 13.0m/s. Estimate the height of the window she fell from.



As she falls out of the first window,
$U_o = mgh_o$ and $K_o = 0$.

As she flies into the second window,
$U = mgh$ and $K = \frac{1}{2}mv^2$.

Applying the Law of Conservation of Energy,
$\Delta U + \Delta K = 0 \Rightarrow (mgh - mgh_o) + (\frac{1}{2}mv^2 - 0) = 0$.

Solving for the initial height,

$$mgh_o = mgh + \tfrac{1}{2}mv^2 \Rightarrow h_o = h + \frac{v^2}{2g}.$$

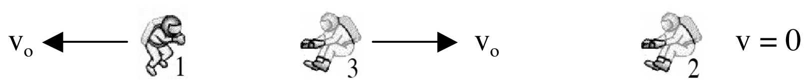
Plugging in the numbers,

$$h_o = 9.00 + \frac{(13.0)^2}{2(9.80)} \Rightarrow \boxed{h_o = 17.6m}.$$

5. Three equally massive and equally strong astronauts are outside their ship in outer space. Two of them get the bright idea to play a game of catch by throwing the third one back and forth. Suppose the game begins with the first astronaut throwing the third astronaut toward the second astronaut at a speed v_o. Describe the rest of the game. This means that you must find the velocity of each astronaut after each catch and after each throw. Sketches of the game at each stage might be the best way to explain your answer. Be sure to state the principle or principles you use.

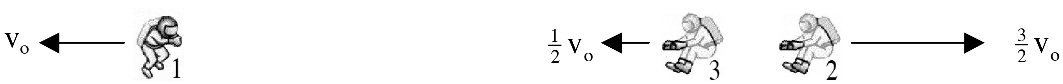
The main principle is the Law of Conservation of Linear Momentum.

The game begins with astronaut 1 pushing astronaut 3 to the right at v_o. Due to the Law of Conservation of Momentum, astronaut 1 recoils with a speed v_o to the left. Astronaut 2 is at rest.



When astronaut 3 is caught by astronaut 2, the Law of Conservation of Momentum requires that they head off to the right at one-half of v_o.

Since the astronauts are equally strong, when astronaut 2 pushes astronaut 3 back towards astronaut 1, he can only change astronaut 3's velocity by v_o. As a result, astronaut 3 heads back to the left at one-half v_o. According to the Law of Conservation of Momentum, astronaut 2 will move to the right at three-halves v_o.



At this point the game ends because astronaut3 can never catch up to astronaut 1.

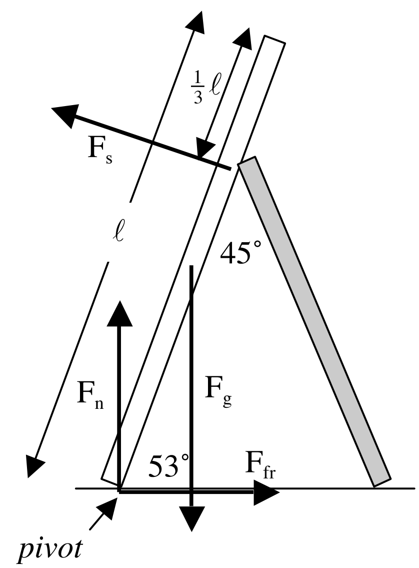
6. A circular saw blade has a mass of 200g and a radius of 9.20cm. It starts from rest and 2.00s later is had a rotation rate of 210rad/s. Find (a)the torque exerted by the motor and (b)the energy the motor supplied to the blade.

(a)Using the Second Law for Rotation, $\Sigma\tau = I\alpha \Rightarrow \tau_m = I\alpha$.
The saw blade is like a solid disk so, $I = \frac{1}{2}mr^2$.
The definition of angular acceleration is, $\alpha \equiv \frac{d\omega}{dt} \Rightarrow \alpha = \frac{\omega - \omega_o}{t} = \frac{\omega}{t}$.
Putting it all together, $\tau_m = \frac{1}{2}mr^2\frac{\omega}{t} = \frac{1}{2}(0.200)(0.0920)^2\frac{210}{2} \Rightarrow \boxed{\tau_m = 0.0889N \cdot m}$.

(b)The energy supplied by the motor is converted into the rotational kinetic energy of the blade,

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{4}mr^2\omega^2 = \tfrac{1}{4}(0.200)(0.0920)^2(210)^2 \Rightarrow \boxed{K = 18.7J}.$$

7. A 500g picture frame is propped up at a 53.0° degree angle by a stick (shown in gray) that contacts the frame one-third of the way down. The stick makes a 45.0° angle with the frame. Assuming that the stick only exerts force perpendicular to the frame, find the size of the force it exerts.



Using the Second Law for Rotation,

$$\Sigma\tau = I\alpha = 0,$$

since the frame is motionless. About the pivot indicated there are only two torques,

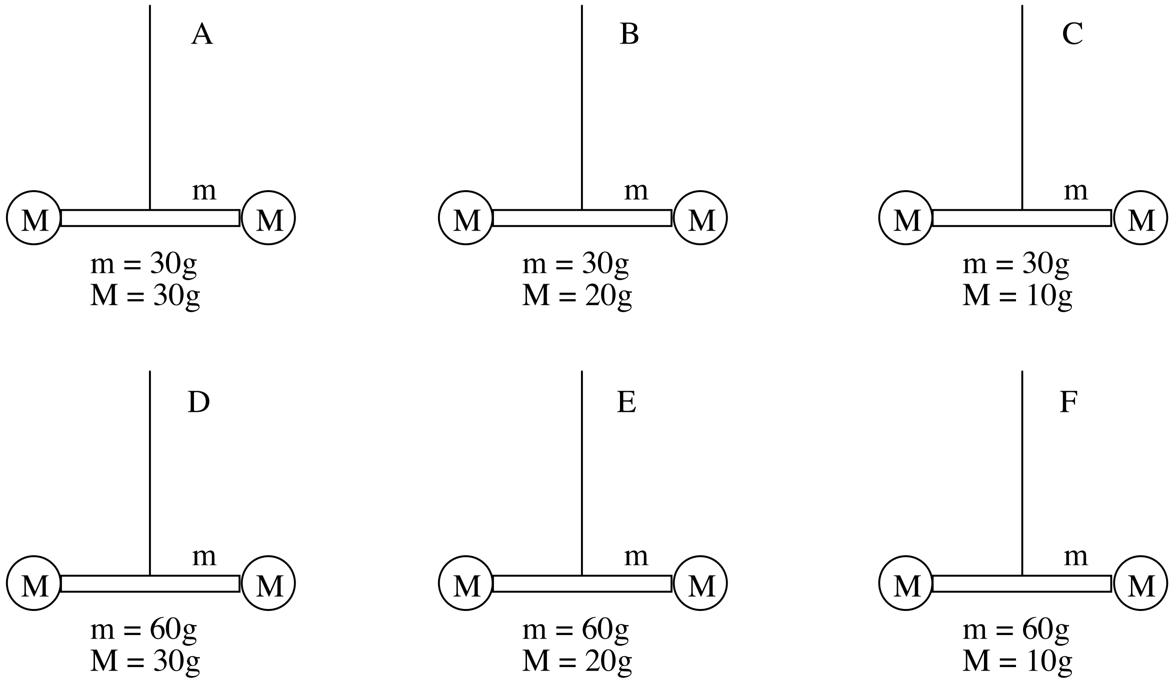
$$\tau_g = -mg\tfrac{\ell}{2}\cos 53^\circ \quad \text{and} \quad \tau_s = F_s\tfrac{2\ell}{3}.$$

Substituting into the Second law,

$$-mg\tfrac{\ell}{2}\cos 53^\circ + F_s\tfrac{2\ell}{3} = 0 \Rightarrow F_s = \tfrac{3}{4}mg\cos 53^\circ.$$

Plugging in the numbers, $F_s = \frac{3}{4}(0.500)(9.80)\cos 53^\circ \Rightarrow \boxed{F_s = 2.21N}$.

8. Shown below is a stick of mass, m, hung by a thread from its center. At the end of each stick are two equal masses, M. The threads are identical. The arrangement is set into oscillatory motion about the axis defined by the thread. In other words, the stick and masses move in a horizontal plane. Rank from greatest to least based on the period of oscillation. That is, rank the system with the longest period first and the shortest period last. Be sure to explain your reasoning.



These are torsional pendulums. The angular frequency of a torsional pendulum is $\omega = \sqrt{\frac{\kappa}{I}}$.

The frequency is related to the period $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$.

Since the threads are identical, the torsion constant is the same in all cases so the period just grows as the rotational inertia grows.

The rotational inertia for a stick about the center is $I_1 = \frac{1}{12}m\ell^2$, while the rotational inertia due to the other masses is $I_2 = 2M(\frac{\ell}{2})^2 = \frac{1}{2}M\ell^2$. So the total rotational inertia is,

$I = I_1 + I_2 = \frac{1}{12}m\ell^2 + \frac{1}{2}M\ell^2 = \left(\frac{1}{12}m + \frac{1}{2}M\right)\ell^2$.

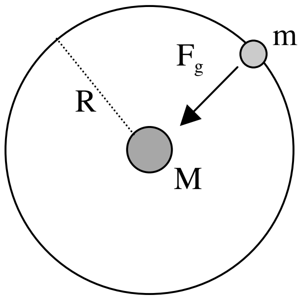
For A: $I = \left(\frac{1}{12}30 + \frac{1}{2}30\right)\ell^2 = 17.5\ell^2$ For B: $I = \left(\frac{1}{12}30 + \frac{1}{2}20\right)\ell^2 = 12.5\ell^2$

For C: $I = \left(\frac{1}{12}30 + \frac{1}{2}10\right)\ell^2 = 7.5\ell^2$ For D: $I = \left(\frac{1}{12}60 + \frac{1}{2}30\right)\ell^2 = 20\ell^2$

For E: $I = \left(\frac{1}{12}60 + \frac{1}{2}20\right)\ell^2 = 15\ell^2$ For F: $I = \left(\frac{1}{12}60 + \frac{1}{2}10\right)\ell^2 = 10\ell^2$

The ranking is therefore, D>A>E>B>F>C.

9. Complete the physics problem alluded to in the newspaper article at the right. Assume Xena's moon completes one orbit of radius R in a time T. Find the mass of Xena.



Applying the Second Law to the moon, $\Sigma F = ma$.

Using the Law of Gravitation, $G\dfrac{Mm}{R^2} = ma \Rightarrow G\dfrac{M}{R^2} = a$.

Using the centripetal acceleration, $G\dfrac{M}{R^2} = \dfrac{v^2}{R} \Rightarrow G\dfrac{M}{R} = v^2$.

The speed is the circumference over the period,

$$G\frac{M}{R} = \left(\frac{2\pi R}{T}\right)^2.$$

Solving for the mass of the planet Xena,

$$\boxed{M = \frac{4\pi^2 R^3}{GT^2}}.$$

Scientists Discover 10th Planet's Moon
By Alicia Chang
Associated Press 01 October 2005

LOS ANGELES (AP) -- The astronomers who claim to have discovered the 10th planet in the solar system have another intriguing announcement: It has a moon.

While observing the new, so-called planet from Hawaii last month, a team of astronomers led by Michael Brown of the California Institute of Technology spotted a faint object trailing next to it. Because it was moving, astronomers ruled it was a moon and not a background star, which is stationary.

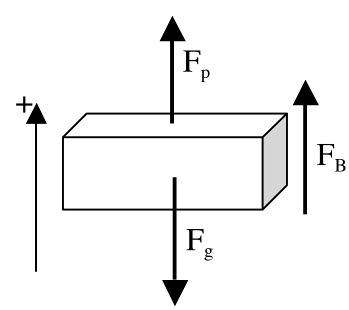
The moon discovery is important because it can help scientists determine the new planet's mass. In July, Brown announced the discovery of an icy, rocky object larger than Pluto in the Kuiper Belt, a disc of icy bodies beyond Neptune. Brown labeled the object a planet and nicknamed it Xena after the lead character in the former TV series ``Xena: Warrior Princess.'' The moon was nicknamed Gabrielle, after Xena's faithful traveling sidekick.

By determining the moon's distance and orbit around Xena, scientists can calculate how heavy Xena is. For example, the faster a moon goes around a planet, the more massive a planet is....

10. Treasure divers investigating a shipwreck find a gold bar that is 5.00cm by 10.0cm by 30.0cm. The density of gold is $19.3\text{x}10^3\text{kg/m}^3$. Find the force they must exert to lift it off the ocean floor.

Applying the Second Law,

$$\Sigma F = ma \Rightarrow F_p + F_B - F_g = 0 \Rightarrow F_p = F_g - F_B.$$



The buoyant force is equal to the weight of the water displaced,

$$F_p = mg - m_f g.$$

The mass of the displaced fluid and the mass of the gold each can be written in terms of the volume of the gold using the definition of density,

$$F_p = \rho V g - \rho_f V g = (\rho - \rho_f)Vg.$$

Plugging in the numbers,

$$F_p = (19.3x10^3 - 1.00x10^3)(0.0500)(0.100)(0.300)(9.80) \Rightarrow \boxed{F_p = 269N}.$$