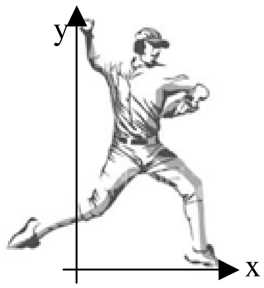
Name:___________________________________

☐ Check here to have your grade posted on the class web site.

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles. The equations you need are on the equation sheet. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The pitcher throws a pitch from the mound toward home plate 18.4m away. The ball is released horizontally from a height of 2.00m with a velocity of 42.0m/s. Find (a)the time it takes for the ball to reach home plate and (b)the height above the ground when it gets there.



(a)Use the kinematic equation along the x-direction to get the time,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2.$$

Plugging in the things that are zero and solving for t,

$$x = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{18.4}{42.0} \Rightarrow \boxed{t = 0.438s}.$$

(a)Use the kinematic equation without v_y to get the final height,

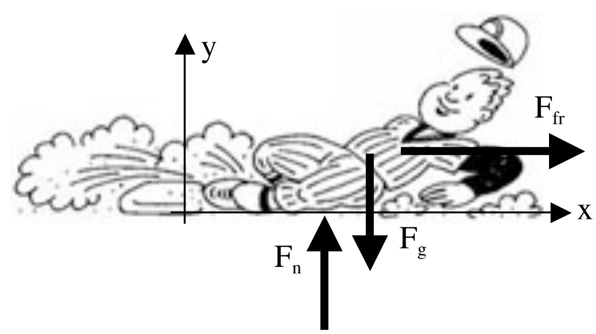
$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2.$$

The initial velocity is zero so plugging the numbers in,

$$y = y_o + \tfrac{1}{2}a_y t^2 = 2.00 - \tfrac{1}{2}(9.80)(0.438)^2 \Rightarrow \boxed{y = 1.06m}.$$

$x_o = 0$	$y_o = 2.00m$
$x = 18.4m$	$y = ?$
$v_{ox} = 42.0m/s$	$v_{oy} = 0$
$v_x = 42.0m/s$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	$t = ?$

2. An 85.0kg base runner tries to steal second base. When he is 3.00m from the base he begins his slide at a speed of 9.00m/s. He comes to rest just as he touches the base. Find (a)his acceleration, (b)the average frictional force exerted by the ground on the sliding runner, and (c)the coefficient of friction between the runner and the ground.



given:
- $x_o = 3.00m$
- $x = 0$
- $v_o = -9.00m/s$
- $v = 0$
- $a = ?$
- $m = 85.0kg$
- $F_{fr} = ?$
- $\mu = ?$

(a)Use the kinematic equation without time,

$$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow 0 = v_o^2 - 2ax_o \Rightarrow a = \frac{v_o^2}{2x_o} = \frac{(-9.00)^2}{2(3.00)} \Rightarrow \boxed{a = 13.5m/s^2}.$$

(b)Applying the Second Law along the x-direction,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} = ma = (85.0)(13.5) \Rightarrow \boxed{F_{fr} = 1150N}.$$

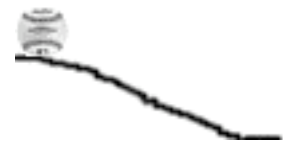
(c)Applying the Second Law along the y-direction,

$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g = mg.$$

Using the definition of COKF,

$$\mu_k \equiv \frac{F_{fr}}{F_n} = \frac{ma}{mg} = \frac{a}{g} = \frac{13.5}{9.80} \Rightarrow \boxed{\mu_k = 1.38}.$$

3. A 150g baseball starts from rest and rolls without slipping down the 42.0cm high pitcher's mound. Find its speed at the bottom.



At the top there is gravitational potential energy and no kinetic energy,

$$U_o = mgh \quad \text{and} \quad K_o = 0.$$

At the bottom there is no potential energy, but both rotational and translational kinetic energy,

$$U = 0 \quad \text{and} \quad K = \tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}I\omega^2.$$

Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow (\tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}I\omega^2 - 0) + (0 - mgh) = 0 \Rightarrow \tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}I\omega^2 = mgh.$$

Since it rolls without slipping the cm speed is related to the rotation rate, $v_{cm} = r\omega$.

The rotational inertia of a solid sphere is, $I = \tfrac{2}{5}mr^2$.

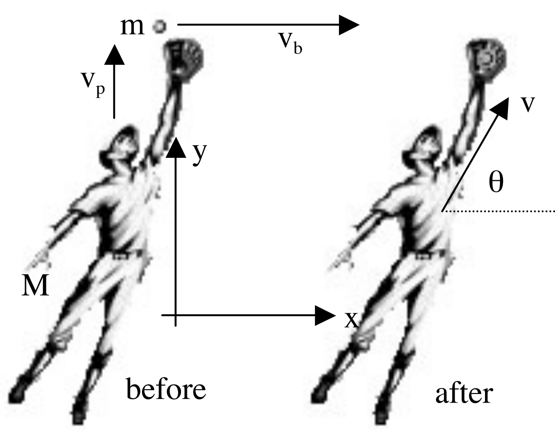
Putting these into the conservation of energy equation,

$$\tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}\tfrac{2}{5}mr^2\omega^2 = mgh \Rightarrow \tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{5}mv_{cm}^2 = mgh \Rightarrow \tfrac{7}{10}mv_{cm}^2 = mgh.$$

Solving for the speed at the bottom,

$$\tfrac{7}{10}mv_{cm}^2 = mgh \Rightarrow v_{cm} = \sqrt{\tfrac{10}{7}gh} = \sqrt{\tfrac{10}{7}(9.80)(0.420)} \Rightarrow \boxed{v_{cm} = 2.42m/s}.$$

4. A 75.0kg shortstop jumps upward to catch the 150g baseball traveling horizontally at 55.0m/s. At the instant before the catch the shortstop is traveling upward at 0.250m/s. Find the velocity (magnitude and direction) of the shortstop and ball just after the catch.



The initial components of the linear momentum are,

$$p_{ox} = mv_b \text{ and } p_{oy} = Mv_p.$$

Just after the catch the components can be written as,

$$p_x = (M+m)v\cos\theta \text{ and } p_y = (M+m)v\sin\theta.$$

Applying the Law of Conservation of Linear Momentum,

$$p_{ox} = p_x \Rightarrow mv_b = (M+m)v\cos\theta \text{ and}$$
$$p_{oy} = p_y \Rightarrow Mv_p = (M+m)v\sin\theta.$$

Dividing the y-equation by the x-equation will eliminate the final speed,

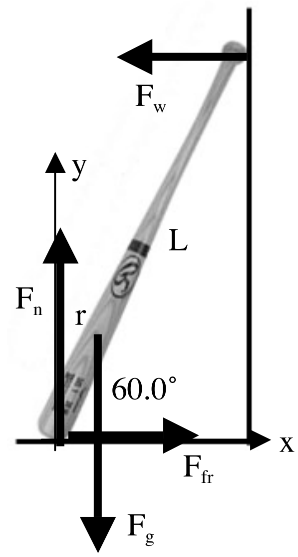
$$\frac{Mv_p}{mv_b} = \frac{(M+m)v\sin\theta}{(M+m)v\cos\theta} = \tan\theta \Rightarrow \theta = \arctan\left[\frac{Mv_p}{mv_b}\right] = \arctan\left[\frac{(75.0)(0.250)}{(0.150)(55.0)}\right] \Rightarrow \boxed{\theta = 66.3^\circ}.$$

Squaring the y-equation and adding it to the square of the x-equation will eliminate the angle,

$$\left(Mv_p\right)^2 + \left(mv_b\right)^2 = (M+m)^2v^2\sin^2\theta + (M+m)^2v^2\cos^2\theta \Rightarrow \left(Mv_p\right)^2 + \left(mv_b\right)^2 = (M+m)^2v^2$$

$$\Rightarrow v = \frac{\sqrt{\left(Mv_p\right)^2 + \left(mv_b\right)^2}}{M+m} = \frac{\sqrt{(75.0 \cdot 0.250)^2 + (0.150 \cdot 55.0)^2}}{75.0 + 0.150} \Rightarrow \boxed{v = 0.273m/s}.$$

5. A 1.00kg baseball bat 85.0cm long has a center of mass that is 20.0cm above the fat end. It leans against a wall smooth wall making a 60.0° angle with the ground. Find the magnitude of each force that acts on the bat and show its direction in the sketch below.



Applying the Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_w = 0 \Rightarrow F_{fr} = F_w$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g$$

Applying the Second Law for Rotation about the origin,

$$\Sigma \tau_o = I\alpha \Rightarrow LF_w \sin 60.0° - rF_g \cos 60.0° = 0 \Rightarrow F_w = \frac{r\cos 60.0°}{L \sin 60.0°} F_g$$

Using the mass/weight rule, $F_g = mg = (1.00)(9.80) \Rightarrow \boxed{F_g = 9.80N}$.

Using the torque equation, $F_w = \frac{(20.0)\cos 60.0°}{(85.0)\sin 60.0°}(9.80) \Rightarrow \boxed{F_w = 1.33N}$.

Using the x-equation, $F_{fr} = F_w \Rightarrow \boxed{F_{fr} = 1.33N}$.

Using the y-equation, $F_n = F_g \Rightarrow \boxed{F_n = 9.80N}$.

6. A sporting goods catalog lists the masses and lengths that are available in a given style of bat. Rank them according to their rotational inertia from smallest to largest. Be sure to explain your reasoning.

A  m = 900g L = 75cm

B  m = 1000g L = 80cm

C  m = 1100g L = 85cm

D  m = 800g L = 75cm

E  m = 900g L = 80cm

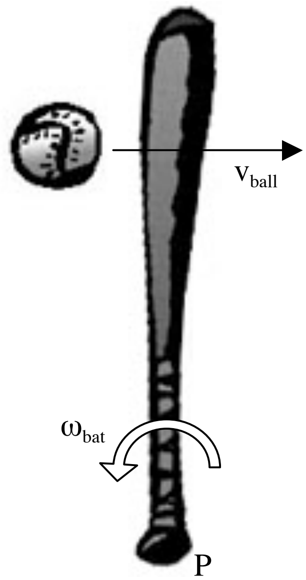
F  m = 1000g L = 85cm

The rotational inertia of an object like a bat is some multiple of the mass and length squared. Since the multiple is the same for all bats of similar shape, the ranking goes as this product. The shortest lightest bat is the smallest (D) and the longest heaviest bat is the largest (C). The rest must be calculated.

A: $mL^2 = (0.900)(0.75)^2 = 0.51 kg \cdot m^2$ B: $mL^2 = (1.000)(0.80)^2 = 0.64 kg \cdot m^2$
C: $mL^2 = (1.100)(0.85)^2 = 0.79 kg \cdot m^2$ D: $mL^2 = (0.800)(0.75)^2 = 0.45 kg \cdot m^2$
E: $mL^2 = (0.900)(0.80)^2 = 0.58 kg \cdot m^2$ F: $mL^2 = (1.000)(0.85)^2 = 0.72 kg \cdot m^2$

The ranking is, $\boxed{D<A<E<B<F<C}$.

7. A 1.00kg bat is rotating about the knob (P) at 600rpm as the 150g ball approaches with a speed of 40.0m/s. The rotational inertia of the bat about the knob end is 0.350kg·m² and the ball strikes the bat 60.0cm from the knob. Find (a)the angular momentum of the ball about P, (b)the angular momentum of the bat about P, and (c)the velocity of the ball after the collision assuming the rotation rate of the bat is 300rpm.



(a)Using the definition of angular momentum,

$$\vec{L} \equiv \vec{r} \times \vec{p} \Rightarrow L = rmv = (0.600)(0.150)(40.0) \Rightarrow \boxed{L = 3.60 kg \cdot m^2/s}.$$

The direction is into the paper by the right hand rule.

(b)First convert the units of the rotational rate,

$$\omega = (600\tfrac{rev}{\min})(\tfrac{\min}{60s})(\tfrac{2\pi rad}{rev}) = 62.8 rad/s.$$

The angular momentum of a rigid body is,

$$L = I\omega = (0.350)(62.8) \Rightarrow \boxed{L = 22.0 kg \cdot m^2/s}.$$

This is out of the paper by the right hand rule.

(c)The total angular momentum before the collision is the sum of the angular momentum ball and the angular momentum of the bat,

$$L_o = 22.0 - 3.60 = 18.4 kg \cdot m^2/s.$$

After the collision the total will be, $L = rmv + I\omega$.

Using the Law of Conservation of Angular Momentum,

$$L = L_o \Rightarrow L_o = rmv + I\omega \Rightarrow v = \frac{L_o - I\omega}{rm} = \frac{18.4 - (0.350)(31.4)}{(0.600)(0.150)} \Rightarrow \boxed{v = 82.3 m/s}.$$

8. A 0.900±0.001kg baseball bat is 85.0±0.2cm long and its center of mass is 55.0 ±0.2cm from the knob end. When held at the knob end and allowed to oscillate, it is found to have a period of 1.60±0.05s. Find the rotational inertia of the bat about the knob end and find the uncertainty in this value.

The bat acts like a physical pendulum. The oscillation frequency is, $\omega = \sqrt{\frac{mgr}{I}}$.

The period is related to the frequency by, $\omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{mgr}{I}} \Rightarrow I = \frac{mgrT^2}{4\pi^2}$.

Plugging in the numbers, $I = \frac{(0.900)(9.80)(0.550)(1.60)^2}{4\pi^2} \Rightarrow \boxed{I = 0.315 kg \cdot m^2}$.

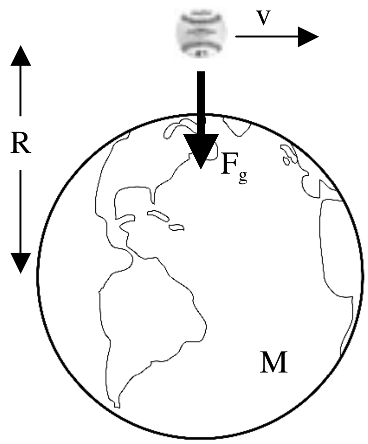
The uncertainty can be found using the multiplication rule, $\frac{\Delta I}{I} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + \left(2\frac{\Delta T}{T}\right)^2}$.

Plugging in the numbers, $\frac{\Delta I}{I} = \sqrt{\left(\frac{0.001}{0.900}\right)^2 + \left(\frac{0.2}{55.0}\right)^2 + \left(2\frac{0.05}{1.60}\right)^2} = 0.072$.

Therefore, $\Delta I = 0.072 I = (0.072)(0.315) = 0.02 kg \cdot m^2$.

Finally, $\boxed{I = 0.32 \pm 0.02 kg \cdot m^2}$.

9. A baseball announcer describing a long home run states that the batter "put that one in orbit." Find the speed that the ball would have to leave the bat in order to go into orbit just above the surface of the earth.



Applying the Second Law to the ball, $\Sigma F = ma \Rightarrow F_g = ma$.

The acceleration is centripetal, $a = \frac{v^2}{r} \Rightarrow F_g = m\frac{v^2}{R}$.

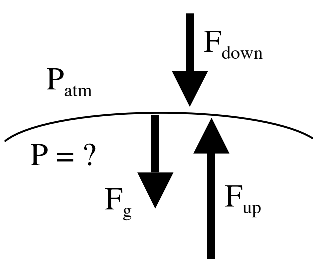
Using the Law of Universal Gravitation, $F_g = G\frac{mM}{R^2}$,

$$G\frac{mM}{R^2} = m\frac{v^2}{R} \Rightarrow G\frac{M}{R} = v^2 \Rightarrow v = \sqrt{G\frac{M}{R}}.$$

Plugging in the numbers,

$$v = \sqrt{(6.67x10^{-11})\frac{5.97x10^{24}}{6.38x10^6}} \Rightarrow \boxed{v = 7900m/s}.$$

10. The Metrodome is a domed stadium in Minneapolis where the Minnesota Twins play baseball. The air inside the stadium, which has a density of 1.29kg/m, supports its roof almost entirely[3]. The mass of the roof is 2.640x10[5]kg and is it 7.44x10[4]m[2] in area. Find (a)the pressure difference across the roof and (b)the speed that air would travel through an open door to the stadium. (c)Would the air move in or out?



(a)the pressure difference across the roof exerts the upward force to cancel the weight. Applying the Second Law,

$$\Sigma F = ma \Rightarrow F_{up} - F_g - F_{down} = 0 \Rightarrow F_{up} - F_{down} = F_g \Rightarrow F_{up} - F_{down} = mg.$$

Using the definition of pressure, $P \equiv \frac{F}{A} \Rightarrow F = PA$.

Substituting into the force equation,

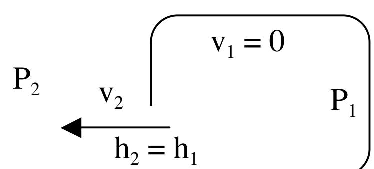
$$PA - P_{atm}A = mg \Rightarrow P - P_{atm} = \frac{mg}{A} \Rightarrow \Delta P = \frac{mg}{A}.$$

Plugging in the numbers, $\Delta P = \frac{(2.640x10^5)(9.80)}{7.44x10^4} \Rightarrow \boxed{\Delta P = 34.8N/m^2}$.

(b)Use Bernoulli's Principle, $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$.

The height terms cancel and the v_1 term is zero,

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow \frac{1}{2}\rho v_2^2 = P_1 - P = \Delta P \Rightarrow v_2 = \sqrt{\frac{2\Delta P}{\rho}}.$$



Putting in the values, $v_2 = \sqrt{\frac{2(34.8)}{1.29}} \Rightarrow \boxed{v_2 = 7.34m/s}$.

(c)Since the pressure inside is larger than outside the air moves out.