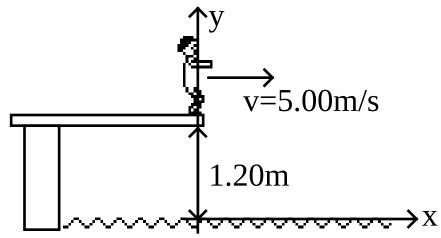
Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A child runs horizontally of the end of a diving board at a speed of 5.00m/s. The diving board is 1.20m above the water. Find (a)the time that the child is in the air and (b)the horizontal distance the child travels before hitting the water.



$x_o = 0$	$y_o = 1.20m$
$x = ?$	$y = 0$
$v_{ox} = 5.00m/s$	$v_{oy} = 0$
$v_x = 5.00m/s$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	

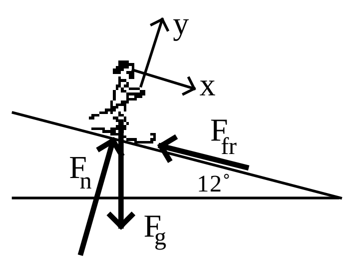
(a)Use the kinematic equation without the final speed for the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \qquad 0 = y_o + 0 + \tfrac{1}{2}a_y t^2 \qquad t = \sqrt{\frac{2y_o}{-a_y}} = \sqrt{\frac{2(1.20)}{-(-9.80)}} \qquad \boxed{t = 0.495s}.$$

(b) Use the kinematic equation without the final speed for the x-direction,

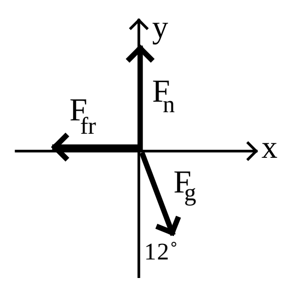
$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad x = 0 + v_{ox}t + 0 \qquad x = (5.00)(0.495) \qquad \boxed{x = 2.48m}.$$

2. A 70.0kg skier heads down a 12.0˚ incline with an acceleration of 1.00m/s². Find the coefficient of kinetic friction between the skis and the snow.



The forces that act on the skier are the weight, normal force and friction. After drawing the free body diagram, apply the Second Law to each direction separately,

$$\Sigma F_x = ma_x \qquad F_g \sin 12^\circ - F_{fr} = ma \qquad F_{fr} = F_g \sin 12^\circ - ma$$
$$\Sigma F_y = ma_y \qquad F_n - F_g \cos 12^\circ = 0 \qquad F_n = F_g \cos 12^\circ.$$



Using the definition of coefficient of friction,

$$\mu_k \equiv \frac{F_{fr}}{F_n} \qquad \mu_k = \frac{F_g \sin 12^\circ - ma}{F_g \cos 12^\circ}.$$

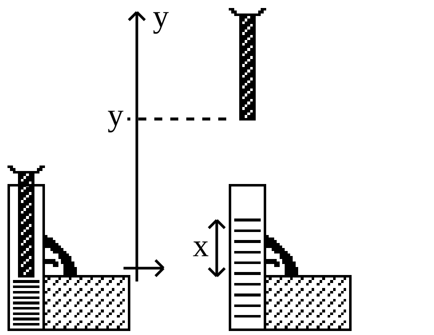
Using the mass/weight rule,

$$\mu_k = \frac{mg\sin 12^\circ - ma}{mg\cos 12^\circ} = \frac{g\sin 12^\circ - a}{g\cos 12^\circ}.$$

Plugging in the numbers,

$$\mu_k = \frac{(9.80)\sin 12^\circ - 1.00}{(9.80)\cos 12^\circ} \qquad \boxed{\mu_k = 0.108}.$$

3. The spring in a dart gun is compressed 6.00cm. When the gun is fired vertically, the 30.0g dart rises to a height of 14.0m. Find the spring constant of the spring in the gun.



before	after
$K_o = 0$	$K = 0$
$U_o = 0 + \frac{1}{2}kx^2$	$U_o = mgy + 0$

Using the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \qquad (0 - 0) + (mgy - \tfrac{1}{2}kx^2) = 0.$$

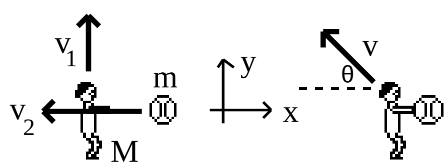
Solving for the spring constant,

$$mgy = \tfrac{1}{2}kx^2 \qquad k = \frac{2mgy}{x^2}.$$

Plugging in the values,

$$k = \frac{2(0.0300)(9.80)(14.0)}{(0.0600)^2} \qquad \boxed{k = 2290\text{N/m}}.$$

4. An 85.0kg baseball player jumps vertically upward to catch a 150g baseball traveling horizontally at 60.0m/s. The instant before the catch, the player is moving upward at 0.250m/s. Find the velocity (magnitude and direction) of the player and ball system just after the catch.



before	after
$p_{ox} = -mv_2$	$p_x = -(m + M)v\cos\theta$
$p_{oy} = +Mv_1$	$p_y = +(m + M)v\sin\theta$

Applying the Law of Conservation of Linear Momentum to each direction separately,

$$-mv_2 = -(m + M)v\cos\theta \qquad v\cos\theta = \frac{m}{m + M}v_2$$

$$Mv_1 = (m + M)v\sin\theta \qquad v\sin\theta = \frac{M}{m + M}v_1$$

Dividing the y equation by the x equation,

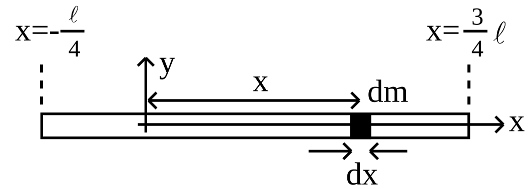
$$\frac{v\sin\theta}{v\cos\theta} = \frac{\frac{M}{m + M}v_1}{\frac{m}{m + M}v_2} \qquad \tan\theta = \frac{Mv_1}{mv_2} \qquad \theta = \arctan\frac{Mv_1}{mv_2} = \arctan\frac{(85)(0.25)}{(0.15)(60)} \qquad \boxed{\theta = 67.0^\circ}.$$

Solving the x equation for the final speed,

$$v = \frac{mv_2}{(m + M)\cos\theta} = \frac{(0.15)(60)}{(0.15 + 85)\cos 67^\circ} \qquad \boxed{v = 0.271\text{m/s}}.$$

5. Find the rotational inertia of a 300g meterstick about the 25.0cm mark.

Using the definition of rotational inertia,



$$I \equiv \int r^2 dm \qquad I = \int x^2 dm.$$

Since the stick is uniform,

$$\frac{dm}{m} = \frac{dx}{\ell} \qquad dm = \frac{m}{\ell}dx.$$

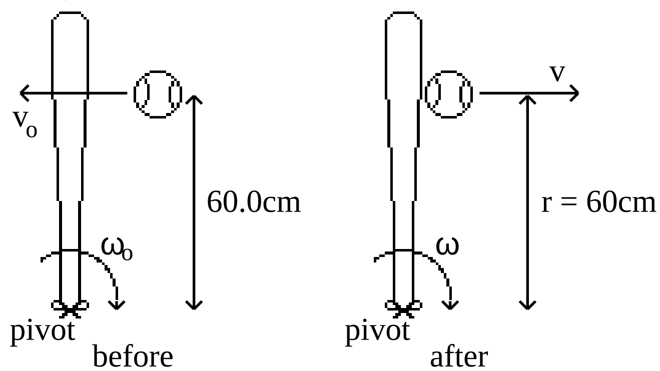
Substituting an performing the integration,

$$I = \int x^2 \frac{m}{\ell}dx = \frac{m}{\ell}\int_{-\ell/4}^{3\ell/4} x^2 dx = \frac{m}{\ell}\left[\frac{x^3}{3}\right]_{-\ell/4}^{3\ell/4} = \frac{m}{3\ell}\left[\left(3\ell/4\right)^3 - \left(-\ell/4\right)^3\right] = \frac{m\ell^3}{3\ell}\left[\frac{27}{64} + \frac{1}{64}\right] = \tfrac{28}{192} m\ell^2.$$

Plugging in the numbers,

$I = \tfrac{28}{192}(0.300)(1.00)^2$ $\boxed{I = 0.0438\text{kg m}^2}$.

6. The physics of the collision between a baseball and a bat can be modeled as shown. The bat can be treated as if it is rotating about a fixed pivot at the end and the collision time is so short that no torques have time to act during the collision. Assume the mass of the bat is 1.00kg, its rotational inertia about the pivot is $0.350\text{kg}\cdot\text{m}^2$ and its initial rotation rate is 600rpm. The mass of the ball is 150g and its initial speed is 40.0m/s. Find the speed of the ball after the collision if the final rotation rate of the bat is 300rpm and the ball strike the bat 60.0cm from the pivot while moving perpendicular to the bat at the instant of collision.



$L_o = mv_o r - I\omega_o$ $\qquad L = -mvr - I\omega$

Using the Law of Conservation of Angular Momentum,

$$mv_o r - I\omega_o = -mvr - I\omega.$$

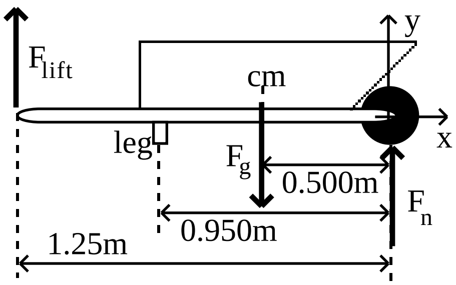
Solving for the final speed of the ball,

$$v = \frac{I}{mr}\left(\omega_o - \omega\right) - v_o.$$

Plugging in the numbers,

$v = \dfrac{0.350}{(0.150)(0.600)}\left(\dfrac{2\pi(600)}{60} - \dfrac{2\pi(300)}{60}\right) - 40.0$ $\qquad \boxed{v = 82.2\text{m/s}}$.

7. A wheel barrow full of gravel is shown. It has a mass of 100kg. Find the minimum force needed to lift the rear legs off the ground and the normal force on the front wheel under these conditions.



This minimum force must be exerted vertically at the end of the handle. There is no normal force at the leg since it is about to come off the ground.

Applying the Second Laws for translation and rotation,

$$\Sigma F_y = ma_y \qquad F_{lift} + F_n - F_g = 0 \qquad F_n = F_g - F_{lift} = mg - F_{lift} \quad \text{and}$$

$$\Sigma\tau_o = I_o\alpha_o \qquad (0.500)F_g - (1.25)F_{lift} = 0 \qquad F_{lift} = \frac{0.500}{1.25}F_g = \frac{0.500}{1.25}mg.$$

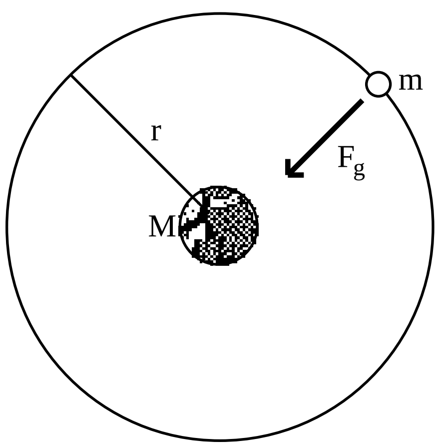
Plugging the numbers into the torque equation,

$$F_{lift} = \frac{0.500}{1.25}(100)(9.80) \qquad \boxed{F_{lift} = 392\text{N}}.$$

Now putting values into the force equation,

$$F_n = mg - F_{lift} = (100)(9.80) - 392 \qquad \boxed{F_n = 588\text{N}}.$$

8. The moon of Jupiter called Io has an orbital period of 1.77days and an orbital radius of $4.22\text{x}10^8$m. Use this information to find the mass of Jupiter.



Applying Newton's Second Law to Io,

$$\Sigma F = ma \qquad F_g = ma_c.$$

Using the Universal Law of Gravitation and the centripetal acceleration,

$$G\frac{mM}{r^2} = m\frac{v^2}{r} \qquad M = \frac{rv^2}{G}.$$

The speed is related to the period by the definition of speed,

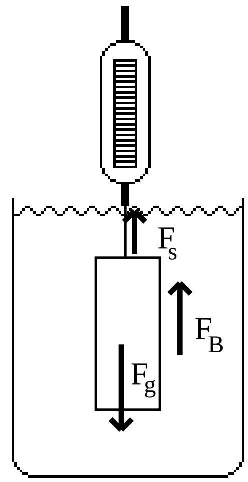
$$v \equiv \frac{\Delta s}{\Delta t} \qquad v = \frac{2\pi r}{T}.$$

Substituting,

$$M = \frac{r}{G}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r^3}{GT^2}.$$

Plugging in the data,

$$M = \frac{4\pi^2(4.22\text{x}10^8)^3}{(6.67\text{x}10^{-11})[(1.77)(24)(3600)]^2} \qquad \boxed{M = 1.90\text{x}10^{27}\text{kg}}.$$



9. A 1.00kg aluminum cylinder has a radius of 3.00cm and a height of 13.1cm. The cylinder is hung from a spring scale and suspended in water. Find the reading on the scale in kilograms.

Applying the Second Law to the cylinder,

$$\Sigma F = ma \qquad F_s + F_B - F_g = 0 \qquad F_s = F_g - F_B = mg - F_B\,.$$

Using Archimedes Principle and the definition of density,

$$F_B = m_f g = \rho_w V_{al} g = \rho_w \pi r^2 hg\,.$$

The force exerted by the scale is the scale reading in kilograms times g,

$$F_s = m_s g\,.$$

Substituting into the equation from the Second Law,

$$F_s = mg - F_B \qquad m_s g = mg - \rho_w \pi r^2 hg \qquad m_s = m - \rho_w \pi r^2 h\,.$$

Plugging in the numbers,

$$m_s = 1.00 - (1000)\pi(0.0300)^2(0.131) \qquad \boxed{m_s = 0.630\text{kg}}.$$

10. Find the period of oscillation of a meterstick held at one end and allowed to swing freely.

This is a physical pendulum. The period of a physical pendulum is,

$$\omega = \sqrt{\frac{mgr}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgr}}\,.$$

The rotational inertia about on end is,

$$I = \tfrac{1}{3} m\ell^2 \qquad T = 2\pi\sqrt{\frac{\frac{1}{3} m\ell^2}{mgr}} = 2\pi\sqrt{\frac{\ell^2}{3gr}}\,.$$

The distance from the pivot at the end to the center of mass is half the length,

$$r = \tfrac{1}{2}\ell \qquad T = 2\pi\sqrt{\frac{\ell^2}{3g\frac{1}{2}\ell}} = 2\pi\sqrt{\frac{2\ell}{3g}}\,.$$

Plugging in the values,

$$T = 2\pi\sqrt{\frac{2(1.00)}{3(9.80)}} \qquad \boxed{T = 1.64\text{s}}.$$