

# Classical Mechanics

## Translation

The motion of objects as a whole

### Kinematics

Laws: none

Definitions:

Position  $\vec{r}$

Displacement  $d\vec{r}$

Velocity  $\vec{v} \equiv \frac{d\vec{r}}{dt}$

Acceleration

$\vec{a} \equiv \frac{d\vec{v}}{dt}$

Useful Relations:

Kinematic Equations

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$v = v_o + at$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \frac{1}{2}(v + v_o)t$$

Tangential Velocity

$$v_t = \frac{2\pi r}{T}$$

Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$

SHM Equations of Motion

$$x = A \cos(\omega t + \delta)$$

$$v = -\omega A \sin(\omega t + \delta)$$

$$a = -\omega^2 A \cos(\omega t + \delta)$$

$$v^2 = \omega^2(A^2 - x^2)$$

$$a = -\omega^2 x$$

### Force

Laws:

Newton's Laws

1. Law of Inertia
2.  $\sum \vec{F} = m\vec{a}$
3. Action/Reaction

Definitions:

Coefficient of Friction

$$\mu \equiv \frac{F_{fr}}{F_n}$$

Useful Relations:

Mass/weight Rule

$$F_g = mg$$

Air Resistance

$$F_d = bv$$

Hooke's Rule

$$F_s = kx$$

Law of Gravitation

$$F_g = G \frac{m_1 m_2}{r^2}$$

Terminal Speed

$$v_t = \frac{mg}{b}$$

$$\text{Spring } \omega = \sqrt{\frac{k}{m}}$$

$$\text{Simple Pendulum } \omega = \sqrt{\frac{g}{l}}$$

$$\text{Physical Pendulum } \omega = \sqrt{\frac{mgr}{I_p}}$$

$$\text{Torsional Pendulum } \omega = \sqrt{\frac{k}{I_p}}$$

### Momentum

Laws:

Conservation of Linear Momentum

Definitions:

Linear Momentum

$$\vec{p} \equiv m\vec{v}$$

Impulse

$$\vec{J} \equiv \int_0^t \vec{F} dt$$

Center of Mass

$$\vec{r}_{cm} \equiv \frac{1}{M} \int \vec{r} dm$$

Useful Relations:

Original 2<sup>nd</sup> Law

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

Impulse/Momentum Theorem

$$\Delta \vec{p} = \vec{J}$$

### Energy

Laws:

Conservation of Energy

$$\Delta K + \Delta U = W_{nc}$$

Definitions:

Work

$$W \equiv \int \vec{F} \bullet d\vec{s}$$

Power  $P \equiv \frac{dW}{dt}$

Kinetic Energy

$$K \equiv \frac{1}{2} mv^2$$

Potential Energy

$$\Delta U \equiv -W_c$$

Useful Relations:

Work-Energy Theorem

$$W_{net} = \Delta K$$

Gravitational Potential Energy

$$U_g = mgy$$

or

$$U_g = -G \frac{m_1 m_2}{r}$$

Spring Potential Energy

$$U_s = \frac{1}{2} kx^2$$

### Kinematics

Laws:

none

Definitions:

Angle  $\theta$

Angular Disp.  $d\theta$

Angular Velocity  $\omega \equiv \frac{d\theta}{dt}$

Angular Acc  $\alpha \equiv \frac{d\omega}{dt}$

$$\alpha \equiv \frac{d\omega}{dt}$$

Useful Relations:

Kinematic Equations (same)

Angular/Linear Rules

$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_c = \omega^2 r$$

Dot Product

$$\vec{A} \bullet \vec{B} \equiv AB \cos\theta = A_x B_x + A_y B_y + A_z B_z$$

Cross Product  $\vec{A} \times \vec{B} \equiv AB \sin\theta \hat{n}$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

## Rotation

The spinning motion of objects

### Torque

Laws:

1. Law of Rotational Inertia
2.  $\Sigma \vec{\tau} = I\vec{\alpha}$
3. Action/Reaction

Definitions:

Torque  $\vec{\tau} \equiv \vec{r} \times \vec{F}$

Rotational Inertia

$$I \equiv \int r^2 dm$$

Useful Relations:

Rotational Inertia (about cm)

$$\text{hoop } mr^2$$

$$\text{disk } \frac{1}{2} mr^2$$

solid sphere

$$\frac{2}{5} mr^2$$

hollow sphere

$$\frac{2}{3} mr^2$$

rod  $\frac{1}{12} ml^2$

rectangular plate

$$\frac{1}{12} m(a^2 + b^2)$$

### Angular Momentum

Laws:

Conservation of Angular Momentum

Definitions:

Angular Momentum

$$\vec{L} \equiv \vec{r} \times \vec{p}$$

Useful Relations:

Angular Momentum Distributed Object

$$\vec{L} = I\vec{\omega}$$

Second Law for Rotation

$$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$

### Energy

Laws:

Conservation of Energy (include rotational kinetic)

$$\Delta K + \Delta U = W_{nc}$$

Useful Relations:

Rotational Kinetic Energy

$$K = \frac{1}{2} I\omega^2$$

Acceleration due to gravity

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

Moon - mass:  $7.36 \times 10^{22} \text{ kg}$

radius:  $1.74 \times 10^6 \text{ m}$

Earth - mass:  $5.98 \times 10^{24} \text{ kg}$

radius:  $6.38 \times 10^6 \text{ m}$

Earth - moon distance:  $3.82 \times 10^8 \text{ m}$

Sun - mass:  $1.99 \times 10^{30} \text{ kg}$

radius:  $6.96 \times 10^8 \text{ m}$

Sun - Earth distance:  $1.50 \times 10^{11} \text{ m}$