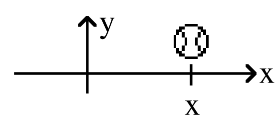
Section 2 – Motion In One Dimension

Section Outline
1. The Definition of Position
2. The Definition of Displacement
3. The Definition of Average Velocity
4. The Definition of Speed
5. The Definition of Average Acceleration

Let's start to answer the question, "What do objects do?" Whether the object is an atom, a baseball, or a star, they all move. So, we need to be able to thoroughly describe motion. We will not worry about the cause of the motion – that will come later. The description of motion is called "Kinematics." We'll begin by very carefully defining the words and ideas we'll use.

1. The Definition of Position

The first step is to describe the location of the object is to know where it is.



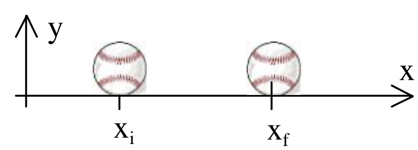
The Definition of Position:
The location of an object with respect to a coordinate system.

The location of an object is meaningless unless it is referenced to something. In physics, that something is called a "coordinate system" or "reference frame." You can choose any coordinate system that you would like, however, you must always indicate which one you have chosen. We'll discover that there are distinct advantages to using certain coordinate systems as we go further. For now, we'll usually just the describe the position as the location along the x-axis as shown above.

Rewritten mathematically,
The Definition of Position: x

2. The Definition of Displacement

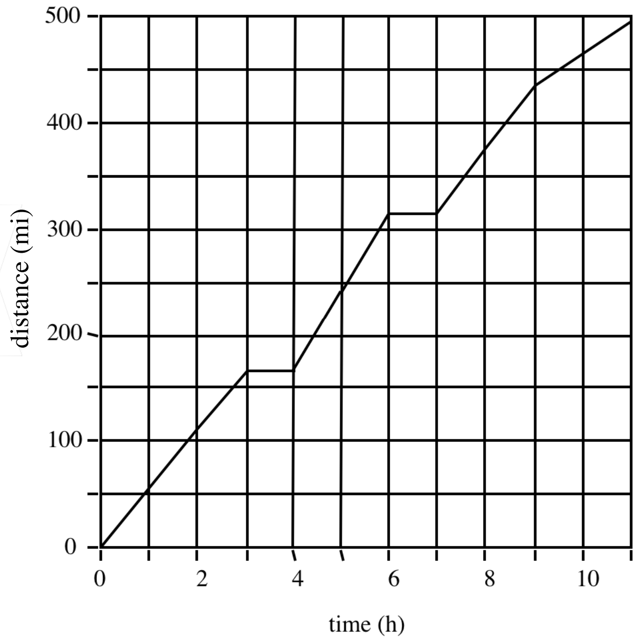
Physics wouldn't be very interesting if objects stayed in one place. So the next thing we need to define are changes in location.



The Definition of Displacement: A change in position.

Rewritten mathematically,
The Definition of Displacement: $\Delta x \equiv x_f - x_i$

Example 2.1: At the right is a graph of my last trip to Los Angeles. The origin is at Chico. Find (a)my position at t=4h, (b) my position at t=6h, (c) my displacement during these 2h and (d)my total displacement.



Given: the graph of x vs. t.
Find: x_i = ?, x_f = ?, Δx = ?, and Δx_{tot} = ?

In this example, the coordinate system has been chosen for us. The origin is at Chico and the x-axis points toward Los Angeles.

(a)We can get position values by reading the graph, $\boxed{x_i = 165\text{mi}}$.

(b)Again, reading the graph, $\boxed{x_f = 315\text{mi}}$.

(c)Using the definition of displacement,

$\Delta x \equiv x_f - x_i = 315 - 165 \Rightarrow$

$\boxed{\Delta x = 150\text{mi}}$.

(d)Estimating from the graph, using the definition of displacement,

$\Delta x \equiv x_f - x_i = 495 - 0 \Rightarrow \boxed{\Delta x_{ot} = 495\text{mi}}$.

3. The Definition of Average Velocity

The next step is to describe how quickly the location is changing (How fast is it going?)

The Definition of Average Velocity: The average rate of displacement.

Mathematically,

The Definition of Average Velocity: $\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

This might look like and equation, but it is really a mathematical description of a sophisticated idea. Every "equation" in physics has a "back story." If you don't fully understand the back story, then you don't understand the equation. Let's start exploring some of the back story.

Example 2.2: Find my average velocity (a)during the interval from t = 4h to t = 6h and (b)for the entire trip.

Given: the graph of x vs. t.
Find: $\bar{v} = ?$
Using the definition of average velocity and the results of example 2.1,

(a) $\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{150}{6-4} \Rightarrow \boxed{\bar{v} = 75.0mph}$.

You should notice that this is equal to the slope of the graph between 4h and 6h.

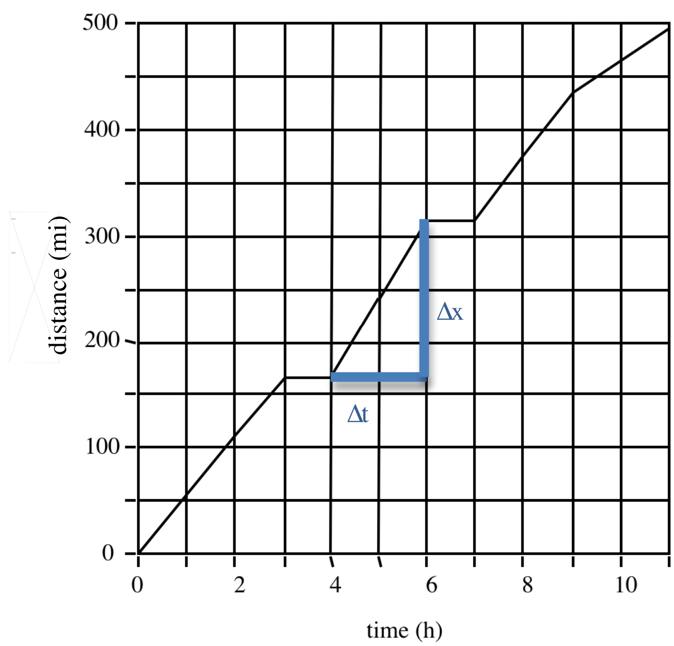
(b) $\bar{\mathrm{v}} \equiv \frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} = \frac{495}{11} \Rightarrow \boxed{\bar{\mathrm{v}} = 45.0\mathrm{mph}}$.

The answer to part a is pretty reasonable, but the answer to part b is very low. This is due to the two one-hour long stops from t = 3h to t = 4h and from t = 6h to t = 7h. So, the average velocity doesn't tell the whole story.

Delving deeper into the back story, let's look at the result of example 2.2a in a different light. The Δx (150mi) and the Δt (2h) are drawn in the graph as the blue lines. Looking at the definition of the average velocity,

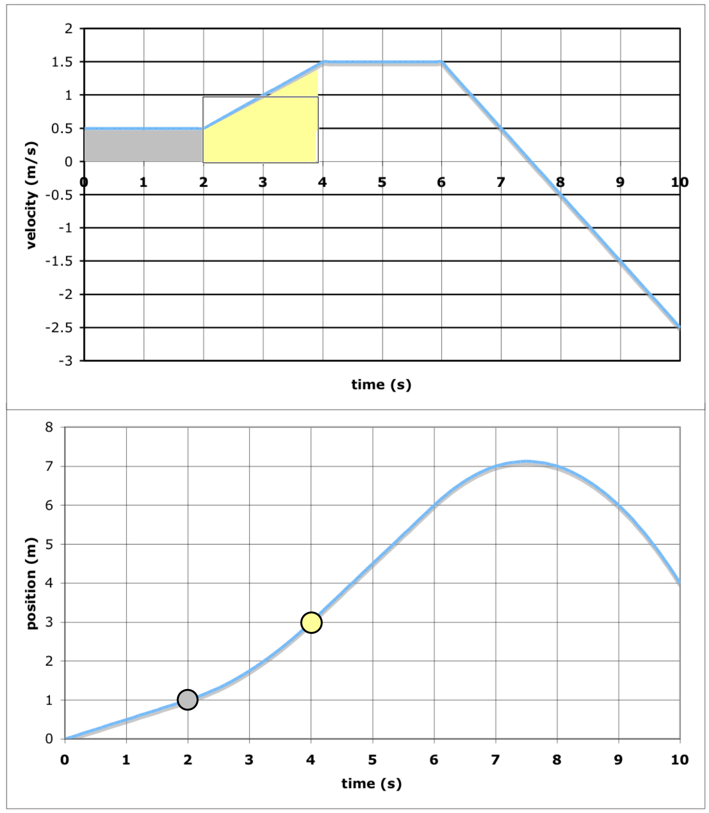
$$\bar{v} \equiv \frac{\Delta x}{\Delta t},$$

we can interpret it to be the "rise over the run," or in other words the slope of the graph.



Another piece of the back story to the definition of average velocity is to think about how it could be used to find the displacement during some time interval.

Example 2.3: The graph velocity versus time of a person walking down a street is shown at the right. Assuming they start at x =0, Find (a)the displacement during the first two seconds and (b)the displacement during the next two seconds. (c)Sketch the position as a function of time.



(a)Starting with the definition of average velocity and solving for Δx,

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v}\Delta t .$$

We can find the distance covered during the first two seconds because the velocity is constant, so the average velocity is 0.5m/s.

$$\Delta x = \bar{v}\Delta t = (0.5)(2) = 1.0m .$$

Notice that this is the width times the height of the gray rectangle. In other words, the displacement is the area under the velocity time graph.

(b)Between two seconds and four seconds the velocity is not constant. The velocity goes from 0.5 to 1.5m/s, so the average is about 1.0m/s. Using the definition of average velocity again,

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v}\Delta t = (1.0)(2) = 2.0m .$$

This is the area represented by the black rectangle. Notice that it is equal to the area under the curve shown in yellow.

(c)To sketch the position, the first displacement is plotted at two seconds. The second interval adds 2.0m to the first displacement, so the total of 3.0m is the next data point. Continuing this process will complete the position versus time graph. Do you understand why the position peaks around 7.5s and then goes negative?

Let's summarize our understanding of the definition of average velocity, $\bar{v} \equiv \frac{\Delta x}{\Delta t}$:

1. Algebraically, it can be thought of as an equation, $\bar{v} = \frac{x_f - x_i}{t_f - t_i}$.
2. Graphically, it can be thought of as the slope of the position versus time graph, $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{rise}{run} = slope$.
3. It can be rearranged and thought about in terms of the area under the velocity versus time graph, $\bar{v} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v}\,\Delta t = (height)(wide) = area$.

This is the back story of the definition of average velocity.

4. The Definition of Speed

The definition of speed is the magnitude of the velocity. Speed is always positive, while velocity can be positive or negative. The distinction between speed and velocity will become more important when we let velocity become a vector when we study two-dimensional motion.

5. The Definition of Average Acceleration

The final step in describing what objects do is to describe how quickly their velocity changes. This is called the acceleration. In analogy to the way velocity is defined in terms of position, acceleration is defined in terms of velocity.

The Definition of Acceleration: The rate of change of velocity.

Mathematically,

The Definition of Average Acceleration $\bar{a} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$

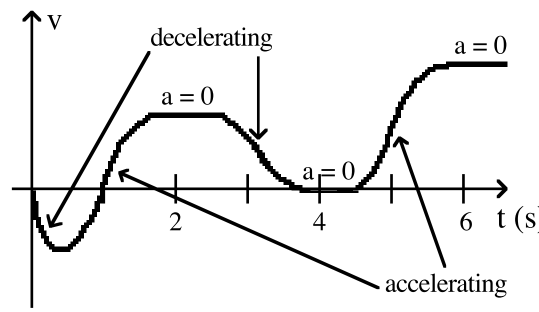
Since, the relationship between average acceleration and velocity is mathematically the same as the relationship between average velocity and position, the back story is very similar. To summarize,

1. Algebraically, it can be thought of as an equation, $\bar{a} = \dfrac{v_f - v_i}{t_f - t_i}$.
2. Graphically, it can be thought of as the slope of the velocity versus time graph, $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{rise}{run} = slope$.
3. It can be rearranged and thought about in terms of the area under the acceleration versus time graph, $\bar{a} = \dfrac{\Delta v}{\Delta t} \Rightarrow \Delta v = \bar{a}\Delta t = (height)(wide) = area$.

Example 2.4: An advertisement claims a car can go from zero to 60mph (26.8m/s) in 6.00s. Find the average acceleration of the car.

Given: $v_i = 0$, $v_f = 26.8$m/s, and $\Delta t = 6.00$s.
Find: $\bar{a} = ?$
Using the definition of average acceleration, $\bar{a} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i} = \dfrac{26.8 - 0}{6} \Rightarrow \boxed{4.47 m/s^2}$.

Example 2.5: The graph of the velocity of a car versus time is shown at the right. Indicate on the graph when the car (a)is accelerating, (b)is decelerating and (c)has zero acceleration. (d)When is the car in reverse?



Given: the graph of v vs. t.
Find: the sign of a and when v<0

Using the definition of instantaneous acceleration, $a \equiv \frac{dv}{dt}$, the acceleration should be the slope of the curve.
(a)the slope is positive from about t = 0.5s to 1.5s and 4.5s to 6s.
(b)the slope is negative from about t = 0s to 0.5s and 2.5s to 4s.
(c)the slope is zero at about 0.5s, from 1.5s to 2.5s, 4s to 4.5s, and after 6s.
(d)The car is in reverse when the velocity is negative. This is from t = 0 to t = 1s.

Section 2 - Summary

Our goal is to understand what objects do and why they do it. The point of this section is that we can now describe in detail what objects do. That is, we can describe their motion in terms of position, displacement, velocity and acceleration. We have carefully defined these ideas and built a solid back story to understand the ideas in various ways.

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system.	x
Displacement	A change in position.	$\Delta x \equiv x_f - x_i$
Average Velocity	The average rate of displacement.	$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$
Speed	The magnitude of the velocity.	v = |v|
Average Acceleration	The rate of change of velocity.	$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$