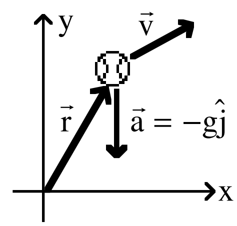
Section 7 - Projectile Motion

Section Outline
1. Freefall in Two Dimensions
2. Projectile Motion Examples

A baseball soaring toward the fence is an example of motion in more than one dimension. Lets spend a bit of time looking at the two dimensional motion of objects moving near the surface of Earth. We'll need to take the general ideas from last time and apply them to the specific case of two dimensional motion obeying the Rule of Falling Bodies.

<u>1. Freefall in Two Dimensions</u>

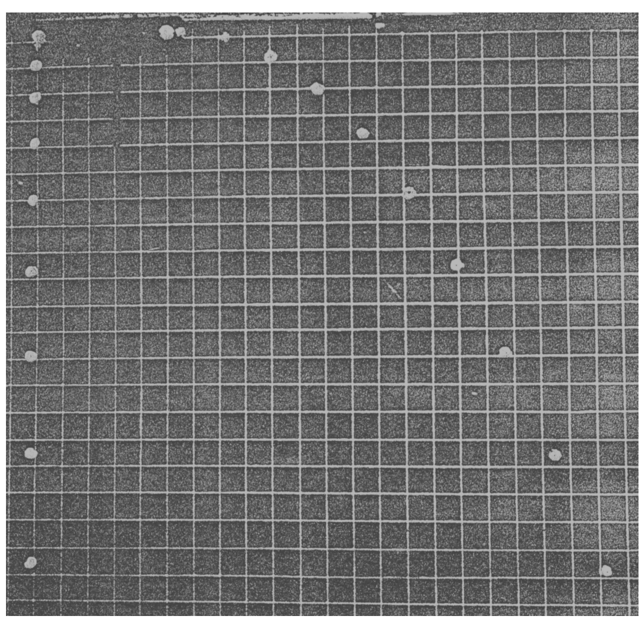
The Rule of Falling Bodies probably applies to the motion of a projectile in two dimensions. If so, the motion can be described by an acceleration, $\vec{a} = -g\hat{j}$, where the coordinates shown at the right are assumed. Using the definitions of velocity and acceleration we can completely predict this motion.



Using the definition of acceleration, $\vec{a} \equiv \frac{d\vec{v}}{dt} \Rightarrow -g\hat{j} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$.
In any vector equation the vector components on each side must match, so this is really two equations,

$$\frac{dv_y}{dt} = -g \text{ and } \frac{dv_x}{dt} = 0 .$$

We could use calculus to solve these differential equations, but let's see if we can understand what they are trying to tell us before we bring out the big mathematical tools. Below is a photo taken of two balls; one dropped and the other thrown horizontally at the same instant. The photo has nine images of each ball taken at equal time intervals during the fall.



We have studied the motion of the falling ball. It falls with a downward velocity that changes at a constant rate of -g. Since the acceleration is a constant, the kinematic equations apply and we have solved many of these one-dimensional motion problems already.

Here's an amazing feature of the vertical motion of the horizontally thrown ball. It drops the exact same amount in each image as the dropped ball. So, the vertical motion of the thrown ball is just the constant acceleration –g. This is what the equation for the y-direction was trying to tell us,

$$\frac{dv_y}{dt} = -g \ .$$

Now, let's look at the horizontal motion of the ball. It moves the same amount horizontally between each frame – a little more than one and one-half of the distance between the vertical lines. So, horizontally the ball is moving at a constant speed. In other words, it has zero acceleration just like the equation for the x-direction was trying to tell us,

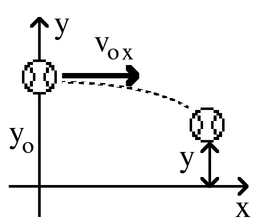
$$\frac{dv_x}{dt} = 0 \ .$$

So, instead of pounding out some awful calculus to describe the motion of a projectile, we can see that the motion of a projectile is just the combination of constant speed along the horizontal axis and freefall along the vertical axis. Furthermore, the kinematic equations should apply separately to each direction of motion. Along the x-axis the acceleration is zero and along the y-axis the acceleration is g.

This is another example of the unity of physics. Instead of having a separate theory for a thrown object as opposed to a dropped object, we can use the same kinematic equations for both. We just have the slight complication of keeping track of each direction of motion separately (We're too stupid to develop a whole new theory....it's easier to use the same old theory if we can get away with it.)

2. Projectile Motion Examples

Example 7.1: A ball is thrown horizontally at a speed of 95.0mph (42.5m/s) by a pitcher. It is released at a height of 2.00m above the ground. Find (a)the time it takes to reach home plate 60.5ft (18.5m) away and (b)the height above the ground when it gets there.



$x_o = 0$	$y_o = 2.00m$
$x = 18.5m$	$y = ?$
$v_{ox} = 42.5m/s$	$v_{oy} = 0$
$v_x = 42.5m/s$	$v_y = ?$
$a_x = 0$	$a_y = -9.80m/s^2$
$t = ?$	

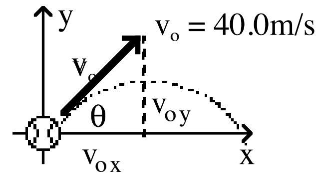
(a)Using the kinematic equation for the x-direction,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{18.5}{42.5} \Rightarrow \boxed{t = 0.435s}.$$

(b)Using the kinematic equation without the final speed in the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 = 2 + 0 - \tfrac{1}{2}(9.8)(0.435)^2 \Rightarrow \boxed{y = 1.07m}.$$

Example 7.2: The distance from third to first is 121ft (36.9m). A third baseman can throw a ball 40.0m/s. Find the angle above the horizontal that she must release it so that it gets to first base at the same height she released it.



$x_o = 0$	$y_o = 0$
$x = 36.9m$	$y = 0$
$v_{ox} = v_o \cos\theta$	$v_{oy} = v_o \sin\theta$
$t = ?$	$v_y = ?$
	$a_y = -9.80 m/s^2$

Using the kinematic equation for the x-direction,

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{x}{v_o \cos\theta}.$$

Using the kinematic equation without the final speed in the y-direction,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow 0 = (v_o \sin\theta)t + \tfrac{1}{2}a_y t^2$$

$$\Rightarrow v_o \sin\theta = -\tfrac{1}{2}a_y t.$$

Substituting for t,

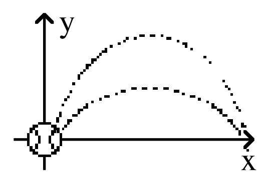
$$v_o \sin\theta = -\tfrac{1}{2}a_y \frac{x}{v_o \cos\theta} \Rightarrow 2\sin\theta\cos\theta = -\frac{a_y x}{v_o^2}.$$

Using the trig identity,

$$\sin 2\theta = -\frac{a_y x}{v_o^2} \Rightarrow \theta = \frac{1}{2}\arcsin\left(-\frac{a_y x}{v_o^2}\right).$$

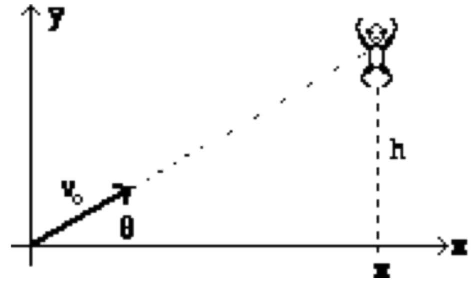
Plugging in the numbers,

$$\theta = \frac{1}{2}\arcsin\left(\frac{(9.8)(36.9)}{(40)^2}\right) \Rightarrow \boxed{\theta = 6.53^\circ \text{ or } 83.5^\circ}.$$



Two answers! You can throw the ball at a large angle so it spends a lot of time in the air with a small horizontal speed or you can throw it at a small angle so it spends less time in the air with a larger horizontal speed. Which one would a third baseman choose?

Example 7.3: A monkey hunter aims directly at a monkey hanging from a branch. The sound of the shot scares the monkey and it releases the branch and falls. Show that the monkey will get hit regardless of the speed of the bullet.



The position of the monkey as a function of time is given by the kinematic equation,

$$y = y_o + v_o t + \tfrac{1}{2}at^2 \Rightarrow y_m = h - \tfrac{1}{2}gt^2.$$

The position of the bullet as a function of time can be found using the kinematic equation for the x-direction,

$$x = x_o + v_{ox}t = v_{ox}t \Rightarrow t = \frac{x}{v_{ox}} = \frac{x}{v_o \cos\theta}.$$

Using the kinematic equation for the y position,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow y_b = (v_o \sin\theta)t - \tfrac{1}{2}gt^2.$$

Substituting for the first t only,

$$y_b = (v_o \sin\theta)\frac{x}{v_o \cos\theta} - \tfrac{1}{2}gt^2 = x\tan\theta - \tfrac{1}{2}gt^2.$$

Note that $h = x\tan\theta$. So the height of the bullet is,

$$\boxed{y_b = h - \tfrac{1}{2}gt^2 = y_m}.$$

No matter how long it takes the bullet to get there, the monkey gets it! This is because without gravity it is clear that the monkey is dead because neither the bullet nor the monkey will fall. Since gravity affects the bullet and the monkey the same way, the monkey still gets blasted!

For the monkey,

$y_o = h$	$y = y_m$
$v_o = 0$	$v = ?$
$a = -g$	$t = ?$

For the bullet,

$x_o = 0$	$y_o = 0$
$x = x$	$y = y_b$
$v_{ox} = v_o \cos\theta$	$v_{oy} = v_o \sin\theta$
$t = ?$	$v_y = ?$
	$a_y = -g$

<u>Section 7 - Summary</u>

Projectile Motion is just the combination of constant speed along the horizontal axis and freefall along the vertical axis. That is, along the x-axis the acceleration is zero and along the y-axis the acceleration is g. Since both accelerations are constant, the kinematic equations should apply separately to each direction of motion.