Section 8 - Projectile Motion in 3D: PitchFX

Section Outline
1. Pitch Tracking Technology
2. The Flight of the Ball
3. Fooling the Batter

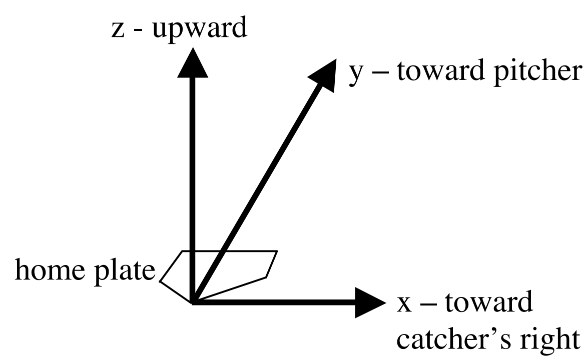
What do objects do? They move in three dimensions. In this section we'll examine data from a real life three-dimensional problem.

On August 7th, 2007 Barry Bonds of the San Francisco Giants was at bat waiting for a 3-2 pitch from Mike Bacsik of the Washington Nationals. The ball left the pitcher's hand at 84.7 mph and arrived at home plate traveling 77.2 mph. It was within 0.2 inches of the center of home plate and about 3.2 feet above the ground when Bonds swung and hit his 756th home run making him the all-time leader for homers in a career. Thanks to a company called Sportsvision (www.sportvision.com) and Major League Baseball you can get kinematic data on any pitch thrown at http://gd2.mlb.com/components/game/mlb/. So, let's look at the details of the motion of this particular pitch.

1. Pitch Tracking Technology

Video technology to track pitches started to be installed in major league ballparks in 2005. The project was completed in 2007 and the system is called PITCHf/x. PITCHf/x calculates the position versus time for each pitch using input from dedicated digital cameras. Sportvision claims to be able to measure the position of the baseball to within an inch. The system uses this trajectory data to extract the initial position, initial velocity, and acceleration along the x, y, and z-axes. Note that the acceleration is assumed constant over the entire flight of the pitch. This assumption leads to equations that claim to reproduce the actual trajectory. The system has the processing power to generate this data in real time during the game. Information on each pitch is available in a fraction of a second and is displayed in the ballpark, MLB GameDay, and often in the right hand corner of the TV broadcast.

The sketch at the right illustrates the coordinate system used. The origin is at the back point of home plate on the ground. The x-axis points to the catcher's right when he is facing the pitcher. The y-axis points directly toward the pitcher so the y-component of the pitched ball's velocity is always negative. The z-axis is oriented upward.



The table on the next page contains the data from MLB for the pitch that Bonds hit. You can view this homer at the Major League Baseball web site:

http://mlb.mlb.com/video/play.jsp?content_id=7143409

Quantity	**Value**	**Units**	**Description**
start_speed	84.1	mph	Speed at y_o=50ft
end_speed	77.2	mph	Speed at the front of home plate y=1.417ft
px	-0.012	ft	x-position at the front of home plate (y=1.417ft)
pz	2.743	ft	z-position at the front of home plate (y=1.417ft)
x_0	1.664	ft	x-position at y_o=50ft
y_0	50.0	ft	Arbitrary fixed initial y-value
z_0	6.597	ft	z-position at y_o=50ft
v_{ox}	-6.791	ft/s	x-velocity at y_o=50ft
v_{oy}	-123.055	ft/s	y-velocity at y_o=50ft
v_{oz}	-5.721	ft/s	z-velocity at y_o=50ft
a_x	13.233	ft/s/s	x-acceleration assumed constant throughout the pitch.
a_y	25.802	ft/s/s	y-acceleration assumed constant throughout the pitch.
a_z	-17.540	ft/s/s	z-acceleration assumed constant throughout the pitch.

Note that these numbers have way too many significant figures. In the example problems below, we'll keep all the significant figures even though it is pretty clear that most of them aren't really significant. Also, since these values are all in English units, we'll stay with these units for this section.

2. The Flight of the Ball

Using the values in the table above, we can apply the kinematic equations along each of the three axes separately to answer questions about the motion of the ball. This is the key piece of physics – since the acceleration along each axis is constant, the motion along each direction in independent.

> *Example 8.1: Compare the z-component of the acceleration of the ball with the acceleration due to gravity.*
>
> Given: $a_z = -17.54 \text{ft/s}^2$ and $g = 9.80 \text{m/s}^2$.
> Find: g in ft/s^2
>
> Converting units,
> $$g = 9.80\tfrac{m}{s^2}\left(\frac{1ft}{0.3048m}\right) \Rightarrow \boxed{g = 32.15\tfrac{ft}{s^2}}.$$
> So, the vertical motion of this pitch is not just due to gravity. The ball must have been thrown with backspin causing the air to exert an upward force on the ball. This force is due to the "Magnus Effect."

Example 8.2: Use the components of the initial velocity to verify the initial speed (y_o = 50.0ft).

Given: v_{ox} = -6.791ft/s, v_{oy} = -123.055ft/s, and v_{oz} = -5.721ft/s.
Find: v_o = ?

In 3-dimensions the initial speed is the magnitude of the initial velocity vector. Since all three components are listed in the table we just need to take the square root of the sum of their squares,

$$v_o = \sqrt{v_{ox}^2 + v_{oy}^2 + v_{oz}^2} = \sqrt{(6.791)^2 + (-123.055)^2 + (-5.721)^2} = 123.4\,\tfrac{ft}{s}\,.$$

Converting to mph,

$$v_o = 123.4\,\tfrac{ft}{s}\left(\frac{1mi}{5280\,ft}\right)\left(\frac{3600s}{h}\right) \Rightarrow \boxed{v_o = 84.1mph}.$$

This agrees precisely with the PITCHf/x value listed in the first row of the table.

Example 8.3: Find (a)y-component of the final velocity of the pitch when it reaches the front of home plate (y=1.417 ft) and (b)the time to get there.

Given: y_o = 50.0ft, y = 1.417ft, v_{oy} = -123.055ft/s and a_y = 25.802ft/s^2.
Find: v_y = ? and t = ?

(a)Since the acceleration is constant, the kinematic equations describe the motion. Given the initial and final y-values we can get the y-component of the velocity using the kinematic equation,

$$v_y^2 = v_{oy}^2 + 2a_y(y - y_o) \Rightarrow v_y = -\sqrt{v_{oy}^2 + 2a_y(y - y_o)} = -\sqrt{(-123.055)^2 + 2(25.802)(1.417 - 50)} \Rightarrow$$

$$\boxed{v_y = -112.4\,\tfrac{ft}{s}}.$$

Note that we want the negative value of the root so as to agree with the coordinate system.
(b)The time can be found from another kinematic equation,

$$v_y = v_{oy} + a_y t \Rightarrow t = \tfrac{v_y - v_{oy}}{a_y} = \tfrac{-112.4-(-123.055)}{25.802} \Rightarrow t = 0.4127s\,.$$

Example 8.4: Find (a)x-component and (b)the z-component of the final velocity of the pitch when it reaches the front of home plate (y=1.417 ft).

Given: v_{ox} = -6.791ft/s, a_x = 13.233ft/s^2, v_{oz} = -5.721ft/s, a_z = -17.54ft/s^2, and t = 0.4127s.
Find: v_x = ? and v_z = ?

(a)Using the kinematic equations for the x-axis,

$$v_x = v_{ox} + a_x t = -6.791 + (13.233)(0.4127) \Rightarrow \boxed{v_x = -1.330\,\tfrac{ft}{s}}.$$

(b)Repeating for the z-axis,

$$v_z = v_{oz} + a_z t = -5.721 + (-17.54)(0.4127) \Rightarrow \boxed{v_z = -12.960\,\tfrac{ft}{s}}.$$

Example 8.5: (a)Write the final velocity in unit vector notation and (b)find the final speed.

Given: v_x = -1.330ft/s, v_y = -112.4ft/s, and v_z = -12.960ft/s.
Find: $\vec{v} = ?$ and v = ?

(a)In unit vector notation, $\boxed{\vec{v} = (-1.330\,\tfrac{ft}{s})\hat{i} + (-112.4\,\tfrac{ft}{s})\hat{j} + (-12.960\,\tfrac{ft}{s})\hat{k}}$.
(b)The speed is the magnitude of the velocity vector. So,
$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(-1.330)^2 + (-112.4)^2 + (-12.960)^2} \Rightarrow \boxed{v = 113.16\,\tfrac{ft}{s} = 77.2mph}$$
This also agrees with the PITCHf/x value listed in the second row of the table.

Notice that over the flight of the ball, the speed has dropped from 84.1mph to 77.2mph. This is typical of all pitches moving at about this speed. The slowing is caused by "air resistance." This is the same thing you feel when you stick your hand out of the window of a car moving at highway speeds. Air at this speed feels rather thick and gooey as opposed to still air.

3. Fooling the Batter

A typical batter might not get a sense of the motion of a pitch until the ball is about 40ft away from home plate. This gives the batter some time to track the pitch so as to estimate where it will be when it crosses home plate. The batter can then begin to execute an appropriate swing of the bat.

Example 8.6 (a)Find the time it takes the ball to get from y = 50ft to y = 40ft and (b)the x and z components of the position and velocity when it gets there.

Given: x_o = 1.664ft, y_o = 50.0ft, y = 1.417ft, z_o = 6.597ft, v_{ox} = -6.791ft/s, v_{oy} = -123.055ft/s, v_{oz} = -5.721ft/s, a_x = 13.233ft/s^2, a_y = 25.802ft/s^2, and a_z = -17.54ft/s^2.
Find: t_{40} = ?, x_{40} = ?, v_{x40} = ?, z_{40} = ?, and v_{z40} = ?

(a)Using the kinematic equation and solving the resulting quadratic,
$$y = y_o + v_{oy}t_{40} + \tfrac{1}{2}a_y t_{40}^2 \Rightarrow t_{40} = \frac{-v_{oy} \pm \sqrt{v_{oy}^2 - 2a_y(y_o - y)}}{a_y}.$$
We'll need to use the minus sign to get a positive time,
$$t_{40} = \frac{-(-123.055) \pm \sqrt{(-123.055)^2 - 2(25.802)(50 - 1.417)}}{25.802} \Rightarrow \boxed{t_{40} = 0.08197s}.$$
(b)The x-position and velocity can now be found using the other kinematic equations,
$$x_{40} = x_o + v_{ox}t_{40} + \tfrac{1}{2}a_x t_{40}^2 = 1.664 + (-6.791)(0.08197) + \tfrac{1}{2}(13.233)(0.08197)^2 \Rightarrow \boxed{x_{40} = 1.152ft},$$
$$v_{x40} = v_{ox} + a_x t_{40} = -6.791 + (13.233)(0.08197) \Rightarrow \boxed{v_{x40} = -5.706 ft/s},$$
as can the z-position and velocity,
$$z_{40} = z_o + v_{oz}t_{40} + \tfrac{1}{2}a_z t_{40}^2 = 6.597 + (-5.721)(0.08197) + \tfrac{1}{2}(-17.54)(0.08197)^2 \Rightarrow \boxed{z_{40} = 6.069ft},$$
$$v_{z40} = v_{oz} + a_z t_{40} = -5.721 + (-17.54)(0.08197) \Rightarrow \boxed{v_{z40} = -7.159 ft/s}.$$

Now that the batter has a sense of the position and velocity of the ball, he can begin to plan his swing. The batter has plenty of experience dealing with the force that the air exerts on the ball due to the drag of air resistance. Therefore, the batter might expect the ball to arrive at home plate at the usual rate determined by the actual y-component of the velocity he can estimate at y=40 ft and typical air resistance. Therefore, time of flight from y=40 ft can be found from by subtracting the total time from the time to get to y=40 ft,

$$t_h = t - t_{40} = 0.4127 - 0.08197 = 0.3307s.$$

Example 8.7: Assuming the air had no effect on the motion of the ball in the x and z directions starting at the point y=40 ft, (a)find the x and (b)z-positions of the ball when it gets to the front of home plate.

Given: $x_{40} = 1.152$ft, $z_{40} = 6.069$ft, $v_{x40} = -5.706$ft/s, $v_{z40} = -7.159$ft/s, $a_x = 0$, $a_z = -32.15$ft/s^2, and $t_h = 0.3307$s.
Find: x_{noair} = ? and z_{noair} = ?

(a)Along the x-direction there would be no acceleration. Using the kinematic equation,

$$x_{noair} = x_{40} + v_{x40}t_h + \tfrac{1}{2}a_x t_h^2 = 1.152 + (-5.706)(0.3307) + 0 \Rightarrow \boxed{x_{noair} = -0.735\,ft}.$$

(b)Along the z-axis there would only be gravity,

$$z_{noair} = z_{40} + v_{z40}t_h + \tfrac{1}{2}a_z t_h^2 = 6.069 + (-7.159)(0.3307) + \tfrac{1}{2}(-32.15)(0.3307)^2 \Rightarrow \boxed{z_{noair} = 1.942\,ft}.$$

This is where an inexperienced batter might think the pitch will be when it reaches home plate. However, the pitcher has put a lot of spin on the ball. The spin interacting with the air creates additional acceleration on the ball due to the Magnus Effect. Batters describe the effect of spin on the ball as the "break." One way to analytically define the break is the difference between where the ball actually arrives and where is would have arrived without any spin.

Example 8.8: Using the final position of the ball given in the table, find the break of the pitch in the x and z directions in inches.

Given: x = -0.012ft, x_{noair} = -0.735ft, z = 2.743ft, and z_{noair} = 1.942ft.
Find: x_{break} = ? and z_{break} = ?

The break is just the difference between the actual position and the position predicted with no interaction with the air,

$$x_{break} = x - x_{noair} = -0.012 - (-0.735) = 0.723\,ft\,\tfrac{12in}{ft} \Rightarrow \boxed{x_{break} = 8.68\,in} \text{ and}$$

$$z_{break} = z - z_{noair} = 2.743 - 1.942 = 0.801\,ft\,\tfrac{12in}{ft} \Rightarrow \boxed{z_{break} = 9.61\,in}.$$

So, you can see how pitchers fool batters. They put spin on the ball causing the ball to move many inches away from the trajectory the ball would have if only gravity caused the acceleration. Since the ball and bat are each just a few inches in diameter, this break can often fool the batter. Strike three!

Section 8 - Summary

As we saw with two-dimensional motion, constant acceleration means that the kinematic equations apply separately to each direction. In this section we extended this idea to three dimensions using real kinematic data from a famous pitch in Major League Baseball.