Section 12 – The Frictional Force

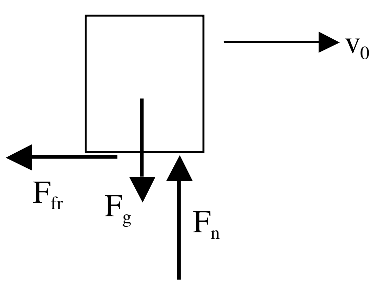
Outline
1. The Coefficient of Kinetic Friction
2. The Coefficient of Static Friction

Why do objects do what they do? Because of the forces acting on them is one answer. In the last section we introduced four common forces in our everyday world, the gravitational force, tension, the normal force and friction. For simplicity, we kept the forces acting along one coordinate axis. Now we will focus on the frictional force and expand to forces that act along more than one axis.

Remember the big picture, once we know the forces that act on an object, we can use Newton's Laws to find the acceleration of the object and from there find the complete description of the motion of the object. The general method is,
1. Find the forces acting on the object.
2. Use Newton's Laws to find the acceleration.
3. Use the definitions of velocity and acceleration as well as the initial position and initial velocity to completely describe the motion.

1. The Coefficient of Kinetic Friction

If you push on a crate to get it moving, it will quickly come to rest once you stop pushing. Newton's Laws explain that the crate comes to rest not because rest is the natural state of motion, but because frictional forces cause the crate to slow down. The detailed behavior of the frictional force is quite complex. It is due to the molecular forces between two surfaces. Let's simplify matters as much as possible by looking at the frictional force empirically. Let's look at two cases: one where the crate is moving and the frictional force is said to be "kinetic friction" and one where the crate is at rest and the frictional force is said to be "static friction."



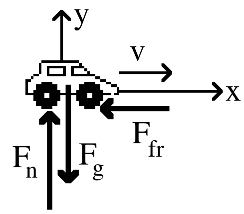
Looking at the kinetic case, suppose the crate is moving to the right at some speed v_0, but skidding along so it will soon be at rest. The forces on the crate are shown at the left. The crate will accelerate to the left due to the frictional force and soon come to rest. Suppose, the crate is moving to the right at the same speed, but someone had put a second crate on top of it. What would be different? The weight of the one crate we are looking at would still be the same, but the normal force, and the frictional force would both be larger. The point is that you probably have the sense that things that weigh more "sort of have more friction." However, from the point of view of Newton's Laws, objects that feel more normal force feel more frictional force. We can say that the frictional force is proportional to the normal force or

$$F_{kf} = \mu F_n$$

where μ is mathematically, the constant of proportionality. Physically, μ represents all the complex interactions between the molecules of the two surfaces. If the surfaces are rough, μ would be larger. If the surfaces are smooth, μ is smaller. Instead of defining μ in terms of the intermolecular forces, it is easier to define the coefficient of kinetic friction as the ration of the frictional force to the normal force.

$$\text{Definition of Coefficient of Kinetic Friction } \mu_k \equiv \frac{F_{kf}}{F_n}$$

Example 12.1: A 750kg car traveling at 90.0km/h (25.0m/s) locks its brakes and skids to rest after traveling 35.0m. Find (a)the acceleration (b)the frictional force and (c)the coefficient of kinetic friction.



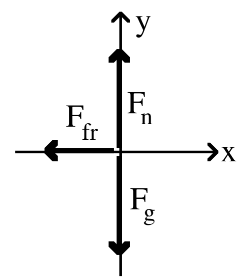
$x_o = 0$
$x = 35.0m$
$v_o = 25.0m/s$
$v = 0$
$a = ?$
$t = ?$

(a)Use the kinematic equation without the time,

$$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow 0 = v_o^2 + 2ax \Rightarrow$$

$$a = -\frac{v_o^2}{2x} = -\frac{(25)^2}{2(35)} \Rightarrow \boxed{a = -8.93 m/s^2}.$$

Free body diagram



(b)Given: m = 750kg and a = -8.93m/s^2

Find: F_{fr} = ? and μ_k = ?

Thanks to Newton, we know what causes this acceleration. It is the forces that act on the car which are drawn in the sketch above. To make applying Newton's Second Law easier, the forces are redrawn on the coordinate system with their tails at the origin. This is called the "free body diagram." Now apply the Second Law to the x-direction only,

$$\Sigma F_x = ma_x \Rightarrow -F_{fr} = ma \Rightarrow F_{fr} = -ma = -(750)(-8.93) \Rightarrow \boxed{F_{fr} = 6700N}.$$

Notice that the frictional force is in the minus x-directon.

(c)Applying the Second Law to the y-direction only,

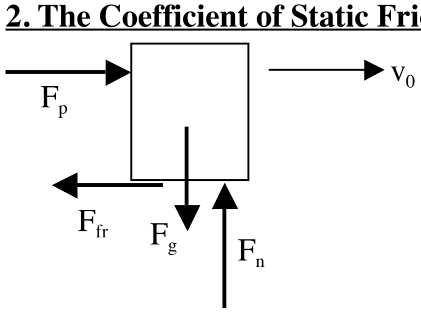
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g = mg = (750)(9.80) \Rightarrow F_n = 7350N$$

(c)Using the definition of the coefficient of kinetic friction, $\mu_k \equiv \frac{F_{kf}}{F_n} = \frac{6700}{7350} \Rightarrow \boxed{\mu_k = 0.912}$

COMMENT ON PROBLEM SOLVING:
We need to add a step when the force vectors involve more than one coordinate axis. When you want to use the Second Law to solve a problem:

1. Use your sketch to identify the object to which you want to apply the Second Law.
2. Draw just the forces that act on this object due to other objects.
3. Choose a convenient coordinate system and indicate it in the sketch.
4. Draw a free body diagram with the tails of all the force vectors at the origin
5. Use the Second Law to write an equation for each axis separately.

2. The Coefficient of Static Friction

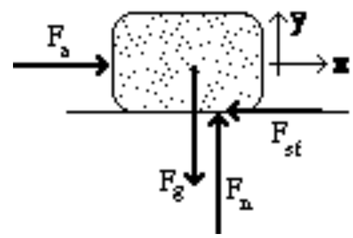


Now, on to the static friction case. Suppose you push on a very heavy crate and it doesn't move. The forces on the crate are shown at the left. Note that since the crate remains at rest, Newton's Laws require the static frictional force be exactly equal to the pushing force. If you push harder and the crate still refuses to budge, then the static frictional force must be larger as well. You have probably noticed, when you have been in this situation, that once you push hard enough to get the crate moving, you can ease up a bit to keep it moving. In technical terms, you have noticed that the maximum static friction is typically more than the kinetic friction. This is true for most surfaces.

We will define the coefficient of static friction in analogy to the coefficient of kinetic friction, but we can only define it in terms of the maximum possible static friction force.

$$\text{Definition of Coefficient of Static Friction } \mu_s \equiv \frac{F_{sf,max}}{F_n}$$

Example 12.2: A 2.00kg block on a horizontal table has a horizontal force of 10.0N applied and it still remains at rest. (a)Find the size of the frictional force on the block. (b)If the coefficient of static friction is 0.650, find the applied force needed to move the block.



(a)Given: $m = 2.00\text{kg}$, $F_a = 10.0\text{N}$, and $a = 0$

Find: $F_{sf} = ?$

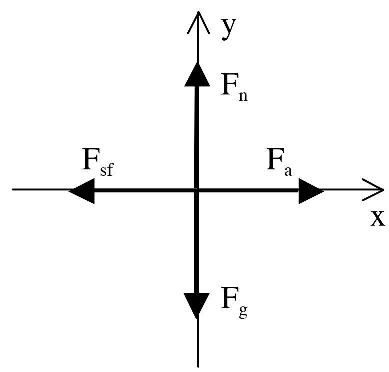
The forces that act on the block are shown at the right. Using the free body diagram we can apply the Second Law to the x-axis remembering the block at rest,

$$\Sigma F_x = ma_x \Rightarrow F_a - F_{sf} = 0 \Rightarrow F_{sf} = F_a \Rightarrow \boxed{F_{sf} = 10.0\text{N}}.$$

(b)Given: $m = 2.00\text{kg}$, $\mu_s = 0.650$, and $a \cong 0$

Find: $F_a = ?$

When the block is about to move, the frictional force is a maximum yet the acceleration is still barely zero. Applying the Second Law now,



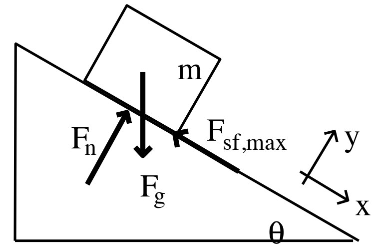
$$\Sigma F_x = ma_x \Rightarrow F_a - F_{sf,max} = 0 \Rightarrow F_{sf,max} = F_a.$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g$$

Using the definition of the coefficient of static friction and the mass/weight rule,

$$\mu_s \equiv \frac{F_{sf,max}}{F_n} = \frac{F_a}{F_g} = \frac{F_a}{mg} \Rightarrow$$
$$F_a = \mu_s mg = (0.650)(2.00)(9.80) \Rightarrow \boxed{F_a = 12.7\text{N}}.$$

Let's up the ante and look at a couple problems where the force vectors don't all lie along the axes.

Example 12.3: The block of example 2 is on a ramp where angle of incline is slowly increased until the block begins to slide. Given this angle is 52.0°, find the coefficient of static friction.

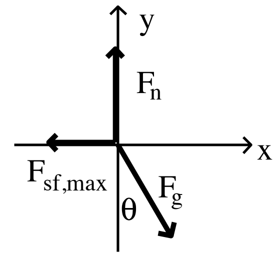


Given: m = 2.00kg, θ = 52.0°, and a ≅ 0

Find: μ_s = ?

Notice that the gravitational force points downward, but that the normal force points perpendicular to the surface. The gravity and normal forces are no longer co-parallel and therefore can't be equal.

Free Body Diagram



In this problem, two of the three forces will be along one of the axes if we choose the x-axis to be along the ramp instead of horizontal. Then we will only have to find vector components for gravitational force. A careful free body diagram is essential here. The free body diagram is always draw by rotating the coordinates and all the vectors until the y-axis is vertical. Then it is easier to wrap our brains around finding the vector components.

Note that when this rotating is done, the angle of the ramp is equal to the angle between the y-axis and the force of gravity.

Using the free body diagram, apply the Second Law to each direction,

$$\Sigma F_x = ma_x \Rightarrow F_g \sin\theta - F_{sf,max} = 0 \Rightarrow F_{sf,max} = F_g \sin\theta .$$

$$\Sigma F_y = ma_y \Rightarrow F_g \cos\theta - F_n = 0 \Rightarrow F_n = F_g \cos\theta .$$

We have assumed that the acceleration is very near zero.

Using the definition of the coefficient of static friction,

$$\mu_s \equiv \frac{F_{sf,max}}{F_n} = \frac{F_g \sin\theta}{F_g \cos\theta} = \frac{\sin\theta}{\cos\theta} \Rightarrow \mu_s = \tan\theta .$$

Plugging in the numbers,

$\mu_s = \tan 52.0° \Rightarrow \mu_s = 1.28$.

COMMENT ON PROBLEM SOLVING:

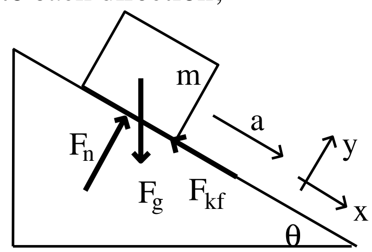
Choosing a convenient coordinate system is an interesting issue. The behavior of an object can't possibly be determined by choosing a coordinate system, because it is essentially and imaginary device. However, the mathematics of solving a problem can be greatly simplified. In the previous problem, using the traditional horizontal and vertical coordinates would have resulted in having to find components for both the frictional and normal forces. The result would have been the same, but the math far more ugly. Here are some tips for choosing a convenient coordinate system. Generally, it is best to choose coordinates where most of the vectors are along an axis. This includes, not just the force vectors, but the acceleration vector as well.

Example 12.4: Assume that the coefficient of kinetic friction is 90.0% of the coefficient of static friction. Find the acceleration of the block down the incline once it is started.

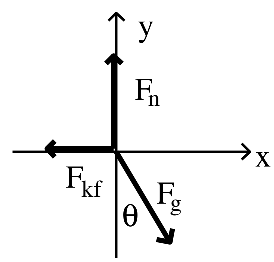
Given: m = 2.00kg, $\theta = 52.0^\circ$, and $\mu_k = 0.9\mu_s = 0.9(1.28) = 1.15$
Find: $a \cong ?$
The free body diagram is the same, but now there is acceleration, so the equations generated by the Second Law will be a bit different. Applying the Second Law to each direction,

$$\Sigma F_x = ma_x \Rightarrow F_g \sin\theta - F_{sf,max} = ma \Rightarrow a = \frac{F_g \sin\theta - F_{sf,max}}{m}.$$

$$\Sigma F_y = ma_y \Rightarrow F_g \cos\theta - F_n = 0 \Rightarrow F_n = F_g \cos\theta.$$



Free Body Diagram



Using the definition of the coefficient of kinetic friction,

$$\mu_k \equiv \frac{F_{sf,max}}{F_n} \Rightarrow F_{sf,max} = \mu_k F_n = \mu_k F_g \cos\theta.$$

Substituting into the expression for the acceleration and using the mass/weight rule,

$$a = \frac{F_g \sin\theta - \mu_k F_g \cos\theta}{m} = \frac{\not{m} g \sin\theta - \mu_k \not{m} g \cos\theta}{\not{m}} \Rightarrow$$

$$a = g(\sin\theta - \mu_k \cos\theta)$$

Plugging in the numbers,

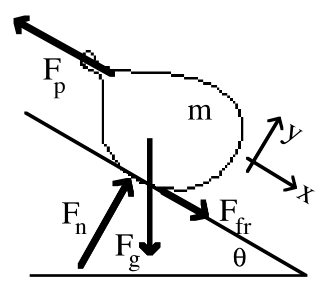
$$a = (9.80)[\sin 52.0^\circ - (1.15)\cos 52.0^\circ] \Rightarrow \boxed{a = 0.772\ \text{m/s}^2}.$$

The math is a bit messier than example 12.3, but the process is the same.

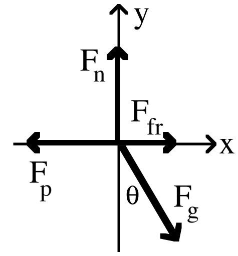
Example 12.5: A student drags a 20.0kg laundry bag up a 30.0° ramp at a constant speed by exerting a force of 150N along the ramp. Find the forces that act on the laundry bag.

Given: m = 20.0kg, $\theta = 30.0^\circ$, $F_p = 150N$, and a = 0
Find: $F_g = ?$, $F_n = ?$, $F_{fr} = ?$, and $F_p = ?$
The velocity is constant, so the acceleration is zero. The four forces on the laundry bag are the pull of the student upward along the ramp, the weight of the bag (gravity) vertically downward, normal force perpendicular to the ramp, and friction opposite the motion along the ramp. The pull of the student is given,

$$\boxed{F_p = 150N}.$$



Free Body Diagram



Gravity can be found using the mass/weight rule,

$$F_g = mg = (20.0)(9.80) \Rightarrow \boxed{F_g = 196N}.$$

To find the other forces, we must apply the Second Law. We'll choose coordinates where the x-axis is along the ramp and draw the free body diagram. Using the free body diagram to carefully apply the Second Law to each direction separately,

$$\Sigma F_x = ma_x \Rightarrow F_g \sin\theta + F_{fr} - F_p = 0 \Rightarrow F_{fr} = F_p - F_g \sin\theta.$$

Putting in the numbers,

$$F_{fr} = 150 - 196\sin 30.0^\circ \Rightarrow \boxed{F_{fr} = 52.0N}.$$

For the y-direction,
$\Sigma F_y = ma_y \Rightarrow F_n - F_g\cos\theta = 0 \Rightarrow F_n = F_g\cos\theta$
Putting in the numbers, $F_n = 196\cos 30^\circ \Rightarrow \boxed{F_n = 170\text{N}}$.
Notice that the normal force is not equal to the weight.

Section Summary

We looked at the frictional force is some detail and defined the coefficients of static and kinetic friction,

$$\text{Definition of Coefficient of Static Friction } \mu_s \equiv \frac{F_{sf,\max}}{F_n}$$

$$\text{Definition of Coefficient of Kinetic Friction } \mu_k \equiv \frac{F_{kf}}{F_n}$$

and used them to describe the following properties of the frictional force.

1. The force of static friction is always less than or equal to the coefficient of static friction times the normal force. $F_{sf} \le \mu_s F_n$
2. The maximum value of the force of static friction is equal to the coefficient of static friction times the normal force. $F_{sf,\max} = \mu_s F_n$
3. The maximum force of static friction is generally greater that the force of kinetic friction.
4. The force of kinetic friction is equal to the coefficient of kinetic friction times the normal force. $F_{kf} = \mu_k F_n$