Section 13 – Tension in Ropes with Pulleys

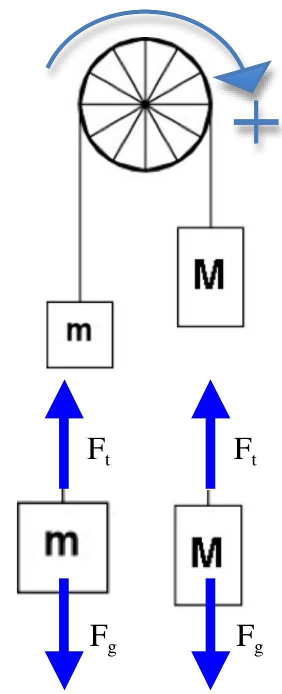
Outline
1. Examples with Ropes and Pulleys

We are still building our understanding of why do objects do what they do – in terms of forces. In this section we will look at the behavior of systems that involve ropes and pulleys. Note that many of these examples involve multiple systems.

<u>1. Examples with Ropes and Pulleys</u>

For a light string or rope, we have assumed that the tension is transmitted undiminished throughout the rope. We will continue to assume this to be true even when the rope changes direction due to a light frictionless pulley. When a pulley changes the direction of motion some complications need to be addressed.

Example 13.1: A 0.500kg mass is connected by a string to a second mass as shown at the right. The 500g mass accelerates downward at 1.20m/s^2. Find (a)the other mass and (b)the tension in the string.



Given: M = 0.500kg and a = 1.20m/s^2.
Find: m = ? and F_t = ?

This device is called an "Atwood Machine." The easiest way to deal with pulleys is to choose (and stick with) a consistent coordinate system. In this problem, let's chose clockwise motion as positive. Now, we'll look separately at the forces on each mass. Applying the Second Law to the 500g mass,

$$\Sigma F = ma \Rightarrow F_g - F_t = Ma \Rightarrow Mg - F_t = Ma \Rightarrow F_t = Mg - Ma .$$

Note that the force of gravity is positive and the tension is negative using the chosen coordinates. Plugging in to find the tension,

$$F_t = (0.5)(9.8) - (0.5)(1.2) \Rightarrow \boxed{F_t = 4.3N}.$$

Now, looking at the other mass,

$$\Sigma F = ma \Rightarrow F_t - F_g = ma \Rightarrow F_t - mg = ma \Rightarrow F_t = mg + ma .$$

Note that both masses feel the same tension because it is undiminished throughout the light string. The acceleration is the same because the string doesn't stretch so both masses must move together. Setting the two equations equal to each other,

$$Mg - Ma = mg + ma \Rightarrow (M - m)g = (M + m)a .$$

Notice that the left side is just the net for on the system as a whole and the right side is the total mass times the acceleration. Solving for the second mass,

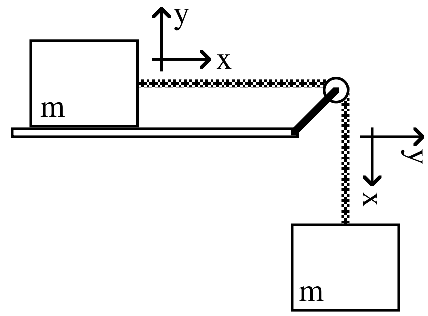
$$Mg - Ma = mg + ma \Rightarrow M(g - a) = m(g + a) \Rightarrow m = M\frac{g-a}{g+a} .$$

Plugging in the numbers,

$$m = M\frac{g-a}{g+a} = (500)\frac{9.8-1.2}{9.8+1.2} \Rightarrow \boxed{m = 391g}.$$

Example 13.2: Two 0.500kg masses are connected by a string as shown at the right. The hanging mass pulls the second mass along a smooth horizontal surface. Find the acceleration of the system and the tension in the string.

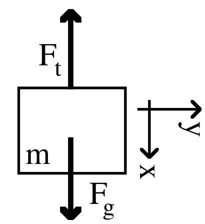
Given: $m = 0.500\text{kg}$ Find: $a = ?$ and $F_t = ?$



Note that the coordinate system must be consistent. If motion to the right is positive for the mass on the horizontal surface, then the downward motion of the hanging mass must be positive as well.

Examine the forces on the hanging block and apply the Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_g - F_t = ma \,.$$



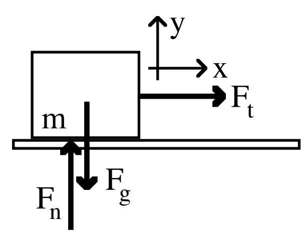
Using the mass/weight rule,

$$mg - F_t = ma \quad (1).$$

This is one equation for the two unknowns.

Looking at the forces that act on the other block and applying the Second Law,

$$\Sigma F_x = ma_x \Rightarrow F_t = ma \quad (2).$$



Note that the acceleration must be the same for both blocks. Now we have two equations and two unknowns. Substitute eq. 2 into eq. 1 and solve for the acceleration.

$$mg - ma = ma \Rightarrow mg = 2ma \Rightarrow a = \frac{g}{2} \Rightarrow \boxed{a = 4.90 \,\text{m/s}^2}.$$

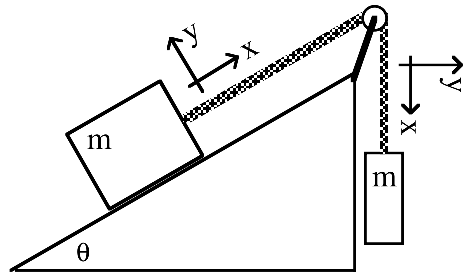
Plugging this back into eq. 2,

$$F_t = (0.500)(4.90) \Rightarrow \boxed{F_t = 2.45\text{N}}.$$

Notice that another way to look at the problem is to realize the weight of the hanging block must accelerate both masses so,

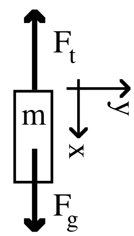
$$\Sigma F_x = ma_x \Rightarrow F_g = 2ma \Rightarrow mg = 2ma \Rightarrow a = \frac{g}{2}.$$

Example 13.3: Two 5.00kg masses are connected by a string as shown at the right. The hanging mass pulls the second mass up a 37.0° incline. The coefficient of friction is 0.150. Find the acceleration of the system and the tension in the string.



Given: $m = 5.00\text{kg}$, $\theta = 37.0°$, and $\mu = 0.150$
Find: $a = ?$ and $F_t = ?$

Note that the coordinate systems must be consistent. If motion up the incline is positive then the downward motion of the hanging mass is positive as well. Examine the forces on the hanging block and apply the Second Law,



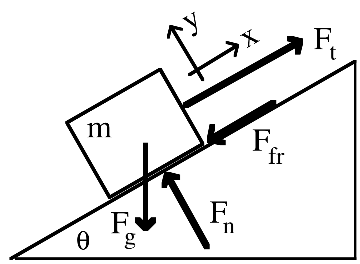
$$\Sigma F_x = ma_x \Rightarrow F_g - F_t = ma .$$

Using the mass/weight rule,

$$mg - F_t = ma \quad (1).$$

This is one equation for the two unknowns.

Looking at the forces that act on the block along the incline and applying the Second Law,



$$\Sigma F_x = ma_x \Rightarrow F_t - F_{fr} - F_g \sin\theta = ma .$$

$$\Sigma F_y = ma_y \Rightarrow F_n - F_g \cos\theta = 0 \Rightarrow F_n = F_g \cos\theta .$$

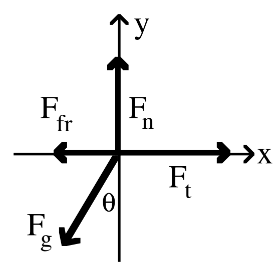
Using the mass/weight rule,

$$F_t - F_{fr} - mg\sin\theta = ma \quad (2).$$

$$F_n = mg\cos\theta \quad (3).$$

Now we have three equations, but four unknowns.

Free Body Diagram



Using the definition of coefficient of friction and eq. 3,

$$\mu \equiv \frac{F_{fr}}{F_n} \Rightarrow F_{fr} = \mu F_n \Rightarrow F_{fr} = \mu mg\cos\theta \quad (4).$$

Finally, we have four equations and four unknowns.
Substituting eq. 4 into eq. 2,

$$F_t - \mu mg\cos\theta - mg\sin\theta = ma .$$

Equating this with the left side of eq. 1,

$$F_t - \mu mg\cos\theta - mg\sin\theta = mg - F_t \Rightarrow$$

$$2F_t = mg(1 + \mu\cos\theta + \sin\theta) \Rightarrow F_t = \tfrac{1}{2}mg(1 + \mu\cos\theta + \sin\theta)$$

Plugging in the numbers,

$$F_t = \tfrac{1}{2}(5.00)(9.80)[1 + (0.150)\cos 37° + \sin 37°] \Rightarrow \boxed{F_t = 42.2\text{N}}.$$

Solving eq. 1 for the acceleration,

$$mg - F_t = ma \Rightarrow a = g - \frac{F_t}{m} = 9.80 - \frac{42.2}{5.00} \Rightarrow \boxed{a = 1.36\,\text{m/s}^2}.$$

Section Summary

We examined systems with pulleys and ropes, which change the direction of motion. The power of treating separate objects as distinct systems was shown. By applying the Second Law to each system we were able to combine the resulting equations to solve the problems.

In summary, when solving a problem involving ropes and pulleys, you must be sure that the coordinate system is consistent:

- Choose a positive direction.
- All forces causing motion that way are now positive.
- Acceleration in that direction is also positive.

Objects tied together with a rope move together, so they have the same acceleration. In addition, since the pulleys are considered massless and the ropes are light, the tension is transmitted undiminished throughout the rope.