Section 16 – Momentum & Impulse

Outline

1. The Definition of Momentum
2. The Impulse-Linear Momentum Theorem

We are trying to understand why objects do what they do. The idea of force, summarized by Newton's Laws of Motion, have done a pretty good job of answering that question. However, it turns out the Newton never actually wrote F = ma. Instead, he defined a quantity which we now call "linear momentum" and expressed the Second Law in terms of it. Linear Momentum or just momentum, for short, turns out to be a very powerful idea for extending our understanding of why objects do what they do.

1. The Definition of Momentum

Newton actually wrote his Second Law in a different form than we have been using,

$$\text{The Original Second Law } \Sigma\vec{F} = \frac{d\vec{p}}{dt},$$

where we establish,

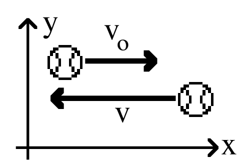
$$\text{The Definition of Linear Momentum } \vec{p} \equiv m\vec{v}.$$

For an object that can be treated like a single particle, such as a baseball of mass, m, moving at a velocity, $\vec{v}$, the original Second Law doesn't seem to tell us anything new,

$$\Sigma\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d(\vec{v})}{dt} \Rightarrow \Sigma\vec{F} = m\vec{a}.$$

However, it at least is consistent with the Second Law we have been using.

Example 16.1: An 0.150kg baseball pitched at 30.0m/s is hit straight back at the pitcher at 50.0m/s. The ball is in contact with the bat for 1.00ms. Find (a)the initial momentum, (b)the final momentum and (c)the average force on the ball.



(a)Using the definition of linear momentum,

$$\vec{p} \equiv m\vec{v} \Rightarrow p_o = mv_o = (0.150)(30.0) \Rightarrow \boxed{p_o = 4.50 kg \cdot m/s}.$$

(b)Using the definition of linear momentum again,

$$\vec{p} \equiv m\vec{v} \Rightarrow p = mv = (0.150)(-50.0) \Rightarrow \boxed{p = -7.50 kg \cdot m/s}.$$

Note that momentum is a vector quantity and we have just found the x-component.

(c)We can now apply the Second Law changing the d's to Δ's because we want the average force,

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow F = \frac{\Delta p}{\Delta t} = \frac{p - p_o}{\Delta t} = \frac{-7.50 - (4.50)}{1.00x10^{-3}} \Rightarrow \boxed{F = -12.0\text{kN}}.$$

The force is to the left, in the negative x direction.

Notice that we could have solved this problem using

$$\Sigma\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = m\frac{\vec{v} - \vec{v}_o}{\Delta t}$$

and gotten the same result.

Given:
m = 0.150kg
v_o = 30.0m/s
v = -50.0m/s
$\Delta t = 1.00x10^{-3}$s

Find:
p_o = ?, p = ?,
and F = ?

2. The Impulse-Linear Momentum Theorem

Even though it appears that we haven't gained anything by developing the concept of momentum, we have. If you take the original Second Law and solve for the change in momentum.

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt \Rightarrow \int_{\vec{p}_o}^{\vec{p}} d\vec{p} = \int_{t_o}^{t} \vec{F}dt \Rightarrow \vec{p} - \vec{p}_o = \int_{t_o}^{t} \vec{F}dt$$

The quantity on the right hand side is defined to be the "impulse."

$$\text{The Definition of Impulse } \vec{J} \equiv \int_{t_o}^{t} \vec{F}dt$$

The relationship becomes,

$$\vec{p} - \vec{p}_o = \vec{J},$$

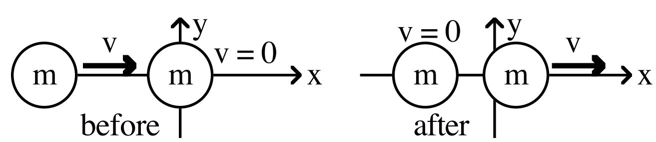
which is known as the "Impulse-Linear Momentum Theorem."

$$\text{The Impulse-Linear Momentum Theorem } \Delta\vec{p} = \vec{J}$$

When a force acts on an object over a period of time, it changes the momentum of an object. Think about dropping a ball of soft clay. When is hits the ground, it comes to rest regardless of whether it hits a hard concrete floor or lands on carpet. Therefore, it has the same change in momentum in either case. By the Impulse-Momentum Theorem, it must also experience the same impulse either way. However, we can use the definition of impulse to understand the difference in the way the force acts in the two different cases. In one case there is a large force over a short time while in the other there is a smaller force over a longer time.

Example 16.2: A 50.0g ball moving at 6.00m/s collides with a second ball of equal mass. The first ball comes to rest in 1.00ms. Find (a)the impulse imparted to the first ball, (b)average force acting on the first ball during the collision, (c)the average force acting on the second ball during the collision, (d)the impulse imparted to the second ball and (e)the speed of the second ball after the collision.

Given: $m = 0.0500\text{kg}$, $v = 6.00\text{m/s}$, and $\Delta t = 1.00\text{x}10^{-3}\text{s}$
Find: $J_1 = ?$, $F_1 = ?$, $F_2 = ?$, $J_2 = ?$, and $v_2 = ?$



(a)Using the Impulse-Momentum Theorem,

$$\Delta\vec{p} = \vec{J} \Rightarrow J_1 = p - p_o = 0 - mv = -mv$$

Plugging in the values,

$$J_1 = -(0.0500)(6.00) \Rightarrow \boxed{J_1 = -0.300\text{N}\cdot\text{s}}.$$

(b)Using the definition of impulse,

$$\vec{J} \equiv \int_{t_o}^{t} \vec{F}dt \Rightarrow J_1 = \overline{F}_1\Delta t \Rightarrow \overline{F}_1 = \frac{J_1}{\Delta t} = \frac{-0.300}{0.00100} \Rightarrow \boxed{\overline{F}_1 = -300\text{N}}.$$

(c)Using Newton's Third Law, the force that the first ball exerts on the second ball must be equal and opposite to the force that the second ball exerted on the first.

$$\boxed{\overline{F}_2 = 300\,\text{N}}.$$

(d) Using the definition of impulse for the second ball,

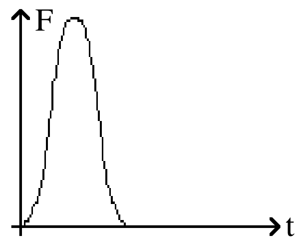
$$\vec{J} \equiv \int_{t_o}^{t} \vec{F}dt \Rightarrow J_2 = \overline{F}_2 \Delta t \Rightarrow \overline{F}_2 = \frac{J_2}{\Delta t} = \frac{+0.300}{0.00100} \Rightarrow \boxed{J_2 = +0.300\text{N}\cdot\text{s}}.$$

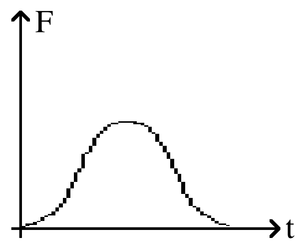
(e) Using the Impulse-Momentum Theorem,

$$\Delta\vec{p} = \vec{J} \Rightarrow J_2 = p - p_o = mv_2 - 0 = mv_2 \Rightarrow v_2 = \frac{J_2}{m} = \frac{0.300}{0.0500} \Rightarrow \boxed{v_2 = 6.00 m/s}.$$

Note that the second ball heads off at the same speed the first ball came in and that the size of the force is bigger if the collision time is shorter.

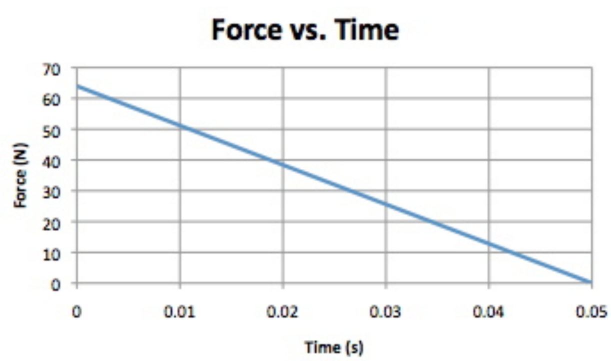
The definition of impulse has to do with the integral of force over time. In terms of a graph of force versus time, impulse is the area under the curve. The graph of force vs. time during the collision between the clay ball and the concrete floor might look like the upper graph on the right. If the ball hits a carpet, the graph might look like the lower one. Since the impulse is the same and the impulse is essentially the area under the force vs. time curve, the area remains the same. With the carpet, a lower force acts for a longer time to stop the ball.





This explains why you bend your knees when you land from a jump, how to catch an egg gently in an egg-toss game and why padded dashboards are safer. In any collision, you can trade off the size of the force for the duration of the collision.

Example 16.3: A 0.800kg pinecone falls onto the ground below and comes to rest. The force exerted on the pinecone versus time is shown at the right. Find (a)the impulse imparted to the pinecone by the ground and (b)the speed of the pinecone just before is hit the ground.



Given: m = 0.800kg and the graph
Find: J =? and v = ?

(a)The definition of impulse, $\vec{J} \equiv \int_{t_o}^{t} \vec{F}\,dt$ tells us that the impulse will equal the area under the force versus time curve. We can find this area two ways. First, it is just the area of the triangle,

$$A = \tfrac{1}{2}(base)(height) \Rightarrow J = \tfrac{1}{2}(0.05)(64) \Rightarrow \boxed{J = 1.6 N{\cdot}s}.$$

Or we could use the equation for the line,

$$y = mx + b \Rightarrow F = \tfrac{-64}{.05}t + 64 \Rightarrow F = -1280t + 64\,,$$

and do the integration,

$$J = \int_0^{0.05} (-1280t + 64)dt = \left[-\tfrac{1280}{2}t^2 + 64t\right]_0^{0.05} = -\tfrac{1280}{2}(0.05)^2 + 64(0.05) \Rightarrow \boxed{J = 1.6 N{\cdot}s}.$$

The result is the same.

(b)Using the Impulse-Momentum Theorem,

$$\Delta\vec{p} = \vec{J} \Rightarrow p - p_o = J \Rightarrow 0 - mv = J \Rightarrow v = -\frac{J}{m} = -\frac{1.6}{0.8} \Rightarrow \boxed{v = -2.0 m/s}.$$

Section Summary

The central idea in the section is Newton's Original Second Law,

$$\text{The Original Second Law } \Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

that we used to get a deeper understanding of why objects do what they do. The Original Second Law requires a new concept called "linear momentum."

$$\text{The Definition of Linear Momentum } \vec{p} \equiv m\vec{v}$$

Rearranging the Original Second Law led us to define the concept of impulse,

$$\text{The Definition of Impulse } \vec{J} \equiv \int_{t_o}^{t} \vec{F}dt \text{ ,}$$

to establish the Impulse-Linear Momentum Theorem,

$$\text{The Impulse-Linear Momentum Theorem } \Delta\vec{p} = \vec{J} \text{ .}$$

The idea is that momentum is changed by a force acting over a time. A large force acting over a short time can produce the same effect as a small force acting over a larger time.