Section 21 – Power, Energy, and Force

Section Outline
1. The Definition of Power
2. Utility Bills

Why do objects do what they do? Now we understand work and its relationship to energy (kinetic energy at least). We'll now take moment to understand the idea of power and its connection with energy and force. We'll relate it to electrical energy bills such as from Pacific Gas and Electric.

1. The Definition of Power

Sometimes, we want information about how quickly work is done. This is called "power."

The Definition of Power: The rate at which work is done.

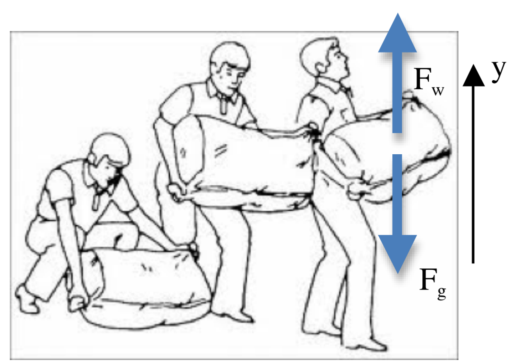
Mathematically,

$$\text{The Definition of Power } P \equiv \frac{dW}{dt}$$

The units of power are, $\frac{1\text{Joule}}{1\text{second}} \equiv 1\text{Watt} = 1\text{W}$. The horsepower is an alternate unit: 1hp=746W.

You probably weren't familiar with joules, the unit for work, but you are comfortable with the units of power. For example, a 100W light bulb or a 1200W hair dryer. A 100W bulb uses 100J every second. Similarly, a 1200W hair dryer uses 1200J every second.

Example 21.1: A worker lifts 30kg sacks of concrete from the ground to his shoulder 1.6m above. He lifts each sack in 0.80s. Find (a)the minimum work done against gravity and (b)the minimum power produced during each lift.



Given: $m = 30\text{kg}$, $y_o = 0$, $y = 1.6\text{m}$, and $\Delta t = 0.80\text{s}$
Find: $W_w = ?$ and $P = ?$

(a)At minimum there are two equal forces on each bag, Earth pulling down and the worker lifting up. At minimum there is no acceleration. Using the Second Law,

$$\Sigma F = ma \Rightarrow F_w - F_g = 0 \Rightarrow F_w = F_g = mg\,.$$

Applying the Definition of Work,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_w = F_w(y - y_o) = mg(y - y_o) = (30)(9.8)(1.6 - 0) \Rightarrow \boxed{W_w = 470J}.$$

(b)Using the definition of power,

$$P \equiv \frac{dW}{dt} \Rightarrow P = \frac{\Delta W}{\Delta t} = \frac{W_w - 0}{\Delta t} = \frac{470}{0.8} \Rightarrow \boxed{P = 588W}.$$

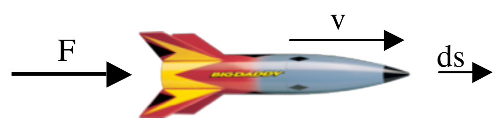
Substantially less than one horsepower.

Example 21.2: A 60kg cheetah can go from 0 to 96km/h in only three seconds. Find (a)the work done and (b)the horsepower produced.

Given: $m = 60kg$, $v_o = 0$, $v = 96km/h = 26.7m/s$, and $\Delta t = 3.0s$
Find: W = ? and P = ?

(a)Using the Work-Energy Theorem,
$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}(60)(26.7)^2 - 0 \Rightarrow \boxed{W = 2.14kJ}$.
(b)Using the definition of power,
$$P \equiv \frac{dW}{dt} \Rightarrow P = \frac{\Delta W}{\Delta t} = \frac{W}{\Delta t} = \frac{21400}{3} \Rightarrow \boxed{P = 7130W = 9.56hp}.$$

When a force is exerted on an object over a distance, work is done. Since this takes some time, the power supplied must be related to the applied force. At the right is a rocket that feels a force, F, over a distance, ds, for a time, dt. Starting with the definition of power and using the definition of work,



$$P \equiv \frac{dW}{dt} = \frac{\vec{F} \bullet d\vec{s}}{dt} = \vec{F} \bullet \frac{d\vec{s}}{dt}.$$

The velocity of the rocket is, $\vec{v} = \frac{d\vec{s}}{dt}$, so the power is related to the force by,

The Power/Force Rule $P = \vec{F} \bullet \vec{v}$.

Example 21.3: The engine of a car supplies 35.0hp as the car travels at 80.0km/h for 15.0 minutes. Find (a)the work done by the engine and (b)the size of the resistive forces on the car.

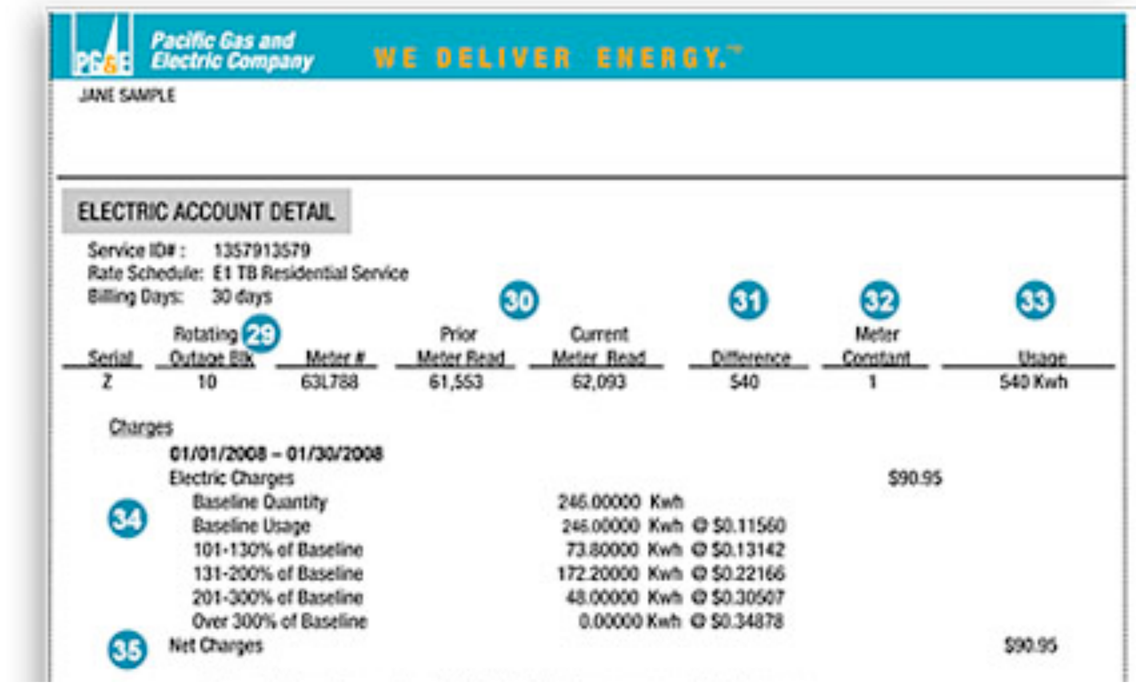
Given: P = 35.0hp = 26.1kW, v = 80.0km/h = 22.2m/s, and Δt = 15.0m = 900s.
Find: W = ? and F_r = ?

(a)Using the definition of power and the fact that the power is constant,
$$P \equiv \frac{dW}{dt} \Rightarrow \int dW = \int Pdt = P\int dt \Rightarrow W = Pt.$$
Putting in the numbers,
$$W = (26.1k)(900) \Rightarrow \boxed{W = 23.5MJ}.$$
(b)Using the power/force rule,
$$P \equiv \frac{dW}{dt} = \frac{\vec{F} \bullet d\vec{s}}{dt} = \vec{F} \bullet \frac{d\vec{s}}{dt} = \vec{F} \bullet \vec{v}$$
Since the car is traveling at a constant speed, the net force on it is zero. The resistive forces that act on the car are equal and opposed to the force that is related to the work done by engine. The power supplied by the engine is consumed by the resistive forces. So,
$$P = \vec{F} \bullet \vec{v} = F_r v \Rightarrow F_r = \frac{P}{v} = \frac{26.1k}{22.2} \Rightarrow \boxed{F_r = 1.18kN}.$$

2. Utility Bills

Its hard to think about power without have electrical energy and the associated utility bills come to mind. Let's take a short detour to look at this topic and bring some real world ideas into our discussion of energy.

Here is a portion of an electric bill:



Pacific Gas and Electric Company WE DELIVER ENERGY.™

JANE SAMPLE

ELECTRIC ACCOUNT DETAIL

Service ID#: 1357913579
Rate Schedule: E1 TB Residential Service
Billing Days: 30 days

Serial	Rotating Outage Blk (29)	Meter #	Prior Meter Read (30)	Current Meter Read	Difference (31)	Meter Constant (32)	Usage (33)
Z	10	63L788	61,553	62,093	540	1	540 Kwh

Charges
01/01/2008 – 01/30/2008

Electric Charges		$90.95
(34) Baseline Quantity	246.00000 Kwh	
Baseline Usage	246.00000 Kwh @ $0.11560	
101-130% of Baseline	73.80000 Kwh @ $0.13142	
131-200% of Baseline	172.20000 Kwh @ $0.22166	
201-300% of Baseline	48.00000 Kwh @ $0.30507	
Over 300% of Baseline	0.00000 Kwh @ $0.34878	
(35) Net Charges		$90.95

Notice that while we often talk about electrical power, PG&E states, "We deliver energy." This is the case, you are charged for energy, not for power. The unit of energy for which they charge is kWh – the kilowatt-hour. Since the kWh is a unit of energy, we should be able to convert it to Joules,

$$1kW \cdot h = (1000)(1\tfrac{J}{s})(3600s) \Rightarrow 1kW \cdot h = 3.6x10^6 J\ .$$

On the bill shown above, 3.6 million Joules costs between 11.5¢ and 34.8¢. That sure sounds like a bargain. The table below compares the cost per Joule for various forms of energy.

Source	Energy (J)	Cost ($)	Cost per J ($/J)
Gallon of gas	$1.3x10^8$	4	$3.1x10^{-8}$
kWh	$3.6x10^6$	0.12	$4.3x10^{-7}$
Candy bar	$1x10^6$	1	$1x10^{-6}$
AA battery	1000	1	$1x10^{-3}$

From this table, gasoline is by far the best bargain, while a calculator battery is the worst deal.

The example below illustrates how to estimate the costs for using an electrical appliance.

Example 21.4: Your PG&E bill charges you about $0.12 for each kilowatt-hour. Find (a)the work done by a 1200W hair dryer in 15.0 minutes, (b)the cost to dry your hair, and (c)the mass of water that must fall 100m over hydroelectric power plant to dry your hair.

Given: price = $0.12/kWh, P = 1200W, and t = 0.250h
Find: W = ?, cost = ?, and m = ?

(a)Use the definition of power and solve for the work done,

$$P \equiv \frac{dW}{dt} \Rightarrow dW = Pdt \Rightarrow \int_0^W dW = \int_0^t Pdt \Rightarrow W = Pt = (1200)(0.250) = 300W \cdot h$$

Converting the Wh and joules,

$$W = 300W \cdot h\left(\frac{1kW}{1000W}\right) \Rightarrow \boxed{W = 0.300kWh}.$$

$$W = 0.300kWh\left(\frac{3.6x10^6 J}{kWh}\right) \Rightarrow \boxed{W = 1.08x10^6 J}.$$

(b)It costs 12¢ per kWh, so,

$$\text{cost} = 0.300kWh\left(\frac{12¢}{kWh}\right) \Rightarrow \boxed{\text{cost} = 3.6¢}$$

(c)The work done by gravity on a falling object can be found from the definition of work,

$$W \equiv \int \vec{F} \bullet d\vec{s} = mgh .$$

Solving for m and putting in the work done by the hair dryer,

$$m = \frac{W}{gh} = \frac{1.08x10^6}{(9.80)(100)} \Rightarrow \boxed{m = 1200kg}.$$

Yikes! This is over a cubic meter of water pouring over a dam somewhere just to dry your hair!

<u>Section 21 - Summary</u>

The Work-Energy Theorem explains that when net work is done on an object, the object speeds up. How quickly this happens is explained by

The Definition of Power: The rate at which work is done,

summarized mathematically as,

$$\text{The Definition of Power } P \equiv \frac{dW}{dt} .$$

Power is related to the force or forces that supply it by,

$$\text{The Power/Force Rule } P = \vec{F} \bullet \vec{v} .$$

The ideas in the last three sections have set the stage for the second conservation law, the Law of Conservation of Energy, the topic of the next portion of the course. The Law of Conservation of Energy will add additional insight into why objects do what they do.