Section 22 – Force, Work, and Potential Energy

Why do objects do what they do? One explanation is they do what they do because of the forces that act on them. Another answer is they do what they do because of the Law of Conservation of Linear Momentum. Another powerful idea is the Law of Conservation of Energy, which we will develop in the next few sections.

Section Outline
1. Conservative and Non-conservative Forces
2. Work and Potential Energy
3. Two Types of Potential Energy

1. Conservative and Non-conservative Forces

Imagine giving a toy car a push causing it to roll up an incline. The Work-Energy Theorem explains the stopping of the car because the initial kinetic energy is removed by the negative work done by gravity on the car. When the car comes to rest, it begins to roll back down the hill because gravity now begins doing positive work to increase the kinetic energy. The work done on the way up is equal and opposite to the work done on the way down, so the kinetic energy at the bottom is equal to the kinetic energy it had when it started. The gravitational force has the ability to return any kinetic energy that it does work against.

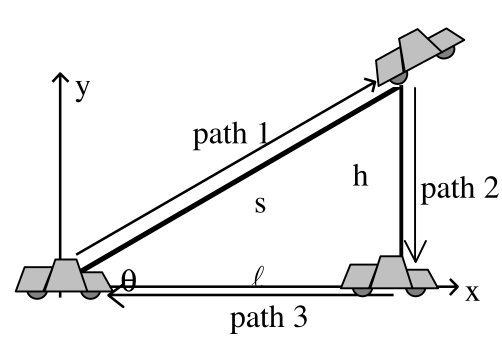
Now imagine giving a stapler a push along a desk. Again, the Work-Energy Theorem explains the stopping of the stapler, because the initial kinetic energy is removed by the negative work done by friction. However, unlike the case with gravity, the frictional force is unable to do positive work to return the kinetic energy of the stapler.

Why does gravity return the kinetic energy, but friction doesn't? Before we can understand why, we need a name to describe this property of some forces. A force that always returns the kinetic energy is said to be a "conservative force" while a force that does not return the kinetic energy is called a "non-conservative force.

Naming the effect is great, but why is gravity a conservative force and friction a non-conservative force? The physicists answer is, "The work done by gravity around a closed loop is always zero, while the work done by friction around a closed loop is not zero." Wasn't that helpful? Let's see what we can do about building some understanding of this idea.

Example 22.1: A 100g car is taken from the base to the top of a 10.0cm high ramp. Find the total work done by gravity as the car is taken up the ramp, vertically down the side, and back horizontally to its starting point.

Given: m = 0.100kg and h = 0.100m
Find: W = ?



The first thing to do is find the work done along each distinct path. Using the definition of work, along path 1,

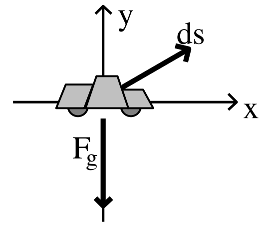
$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_1 = \int \vec{F}_g \bullet d\vec{s}\,.$$

Using the mass/weight rule and the fact that the angle between $\vec{F}_g$ and $d\vec{s}$ is $\frac{\pi}{2}+\theta$,

$$W_1 = \int mg\cos\left(\tfrac{\pi}{2}+\theta\right)ds = -\int mg\sin\theta ds\,.$$

Since m, g and θ are constant,

$$W_1 = -mg\sin\theta \int ds = -mgs\sin\theta\,.$$



Note that

$$h = s\sin\theta \Rightarrow W_1 = -mgh = -(0.100)(9.80)(0.100) \Rightarrow W_1 = -0.098J\,.$$

Along path 2 the force and motion are parallel. Using the definition of work,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_2 = \int \vec{F}_g \bullet d\vec{s} = \int mgds = mgh \Rightarrow W_2 = +0.098J\,.$$

Along path 3, no work is done by gravity because the force is perpendicular to the motion.
The total work done around the loop is,

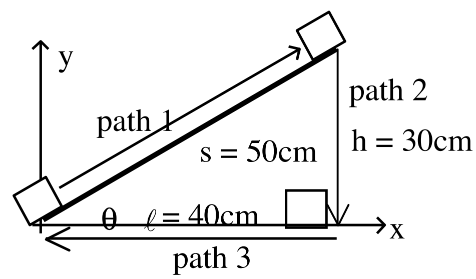
$$W = W_1 + W_2 + W_3 = -0.0980 + 0.0980 + 0 \Rightarrow \boxed{W = 0}\,.$$

Since the work done by the gravitational force around the closed path is zero, the force of gravity is a conservative force.

Now, let's repeat this process for the frictional force.

Example 22.2: A 100g stapler moves along a tabletop from the point (0,0) to the point (40.0cm, 30.0cm) to the point (40.0cm, 0) then back to the origin. Find the work done by friction along around this loop. The coefficient of friction is 0.200.

Given: m = 0.100kg, ℓ = 0.400m, h = 0.300m, s = 0.500m, and μ = 0.200.
Find: W = ?



Using the Second Law, the normal force can be shown to be equal to the weight, so by the definition of coefficient of friction,

$$\mu \equiv \frac{F_{fr}}{F_n} \Rightarrow F_{fr} = \mu F_n = \mu mg\,.$$

Using the definition of work and the facts that the frictional force is constant and opposite to the motion,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_{fr} = \int \vec{F}_{fr} \bullet d\vec{s} = -F_{fr}\int ds = -\mu mg\int ds\,.$$

Along path 1,

$$W_1 = -\mu mg \int ds = -\mu mgs = -(0.200)(0.100)(9.80)(0.500) \Rightarrow W_1 = -0.0980J\,.$$

Along path 2,

$$W_2 = -\mu mg \int ds = -\mu mgh = -(0.200)(0.100)(9.80)(0.300) \Rightarrow W_2 = -0.0588J\,.$$

Along path 2,

$$W_3 = -\mu mg \int ds = -\mu mg\ell = -(0.200)(0.100)(9.80)(0.400) \Rightarrow W_3 = -0.0784J\,.$$

The total work done around the loop is,

$$W = W_1 + W_2 + W_3 = -0.0980 - 0.0588 - 0.0784 \Rightarrow \boxed{W = -0.2352J}\,.$$

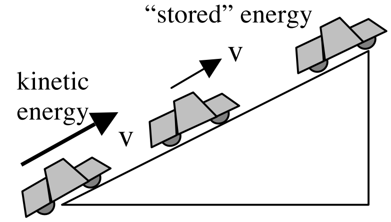
Since the work done by the frictional force around the closed path is not zero, the force of friction is a non-conservative force.

In summary, if the work done by a force around a closed loop is zero, it is a conservative force and can "store" the kinetic energy to be returned later. If the work done around a closed loop is not zero, the force is non-conservative and it cannot "store" the kinetic energy for later use.

Another way of interpreting the closed loop argument is that the work done by conservative forces only depends upon the starting and ending points where the work done by non-conservative forces depends upon the path taken between the starting and ending points. For the gravitational force, the work done to go up the incline is equal to the work to go from the bottom to the top along paths 2 and 3. For friction, the work to move along the hypotenuse is less than the work two move along the other two sides.

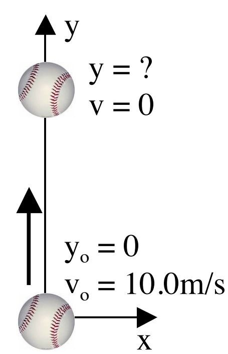
2. Work and Potential Energy

When a conservative force acts over a distance, work is done, and energy is stored. This stored energy is called "potential energy." When the car goes up the ramp, gravity does negative work, so we need to the define the potential energy as the negative of the work done by a conservative force,



The Definition of Potential Energy $\Delta U \equiv -W_c$.

Example 22.3: A 200g ball is thrown upward with a speed of 10.0m/s. Consider only the upward flight of the ball and find (a)the change in kinetic energy of the ball, (b)the maximum height of the ball, (c)the work done by gravity on the ball, and (d)the change potential energy of the ball.



Given: m = 0.200kg, $v_o = 10.0$m/s, $y_o = 0$, v = 0, and a = -9.80m/s^2.
Find: K_o = ?, y = ?, W_g = ?, and U = ?

(a)Using the definition of kinetic energy,

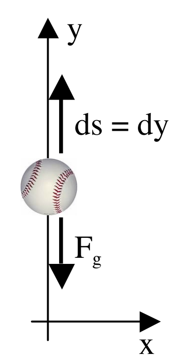
$$\Delta K = K - K_o = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_o^2 = 0 - \tfrac{1}{2}mv_o^2 = -\tfrac{1}{2}(0.200)(10.0)^2 \Rightarrow \boxed{\Delta K = -10.0J}.$$

(b)Using the kinematic equation without the time,

$$v^2 = v_o^2 + 2a(y - y_o) \Rightarrow y = -\tfrac{v_o^2}{2a} = -\tfrac{(10.0)^2}{2(-9.80)} \Rightarrow \boxed{y = 5.10m}.$$

(c)Using the definition of work,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_g = \int \vec{F}_g \bullet d\vec{s}.$$



The motion is opposite the force, so the dot product gives a minus sign. Since the motion is along the y-axis, we'll us dy instead of ds,

$$W_g = -\int_0^y F_g dy = -\int_0^y mgdy = -mg\int_0^y dy = -mgy \Rightarrow$$

$$W_g = -(0.200)(9.80)(5.10) \Rightarrow \boxed{W_g = -10.0J}.$$

(d)Using the definition of potential energy,

$$\Delta U \equiv -W_c \Rightarrow \Delta U_g = -W_g = -(-10.0J) \Rightarrow \boxed{\Delta U_g = 10.0J}.$$

This example brings up two questions. First, why is the decrease in kinetic energy exactly equal to increase in potential energy? Second, why is only the change in potential energy defined and not the potential energy itself? To answer the first question, let's start with the work-energy theorem,

$$W_{net} = \Delta K .$$

The total work done on an object can be separated into the work done by conservative forces and the work done by non-conservative forces,

$$W_c + W_{nc} = \Delta K .$$

Using the definition of potential energy, we can replace the work done by conservative force with the potential energy,

$$\Delta U = -W_c \Rightarrow -\Delta U + W_{nc} = \Delta K \Rightarrow \Delta K + \Delta U = W_{nc} .$$

For the thrown ball, there are no non-conservative forces acting so they do no work and we have,

$$\Delta K + \Delta U = 0 \Rightarrow \Delta U = -\Delta K ,$$

just as the numbers in the previous example had shown. So, we have a preliminary statement of a Law of Conservation of Energy when we only conservative forces are present,

$$\Delta K + \Delta U = 0 .$$

The change in the kinetic energy will be equal and opposite to the change in potential energy. We can now think about the upward motion of the ball as it is giving up its initial kinetic energy in exchange for increasing potential energy.

To summarize our energy ideas to this point:

The Work-Energy Theorem	$W_{net} = \Delta K$
The Definition of Potential Energy	$\Delta U \equiv -W_c$
The Law of Conservation of Energy	$\Delta K + \Delta U = 0$ (Conservative forces only)

3. Two Types of Potential Energy

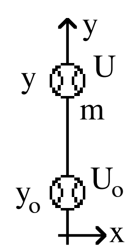
Now to answer the second question, why did we defined only the change in potential energy is and not the potential energy itself? As you can see, the Law of Conservation of Energy only depends upon the change in potential energy, so only changes matter. That said, we can establish values for the potential energy if we establish where the potential energy is zero. We do this by definition as will be seen in the discussions below.

Only conservative forces have a potential energy associated with them. The two conservative forces we will most often deal with are gravity and springs. So, let take a detailed look each one to decide where it makes sense to establish the zero for potential energy.

Gravitational Potential Energy:

Applying the definition of potential energy to a ball and using the definition of work,

$$\Delta U \equiv -W_c \Rightarrow \Delta U = -W_g = -\int_{y_o}^{y} \vec{F}_g \bullet d\vec{s}$$



Since the change in potential energy only depends upon the starting and ending point, the path doesn't matter let's go straight upward. In this case the weight and the displacement are opposite. Using the mass/weight rule,

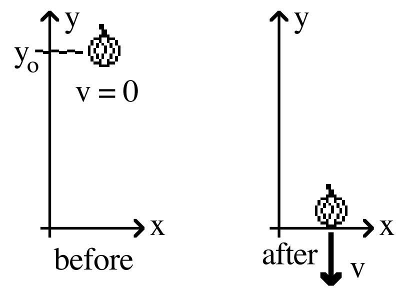
$$\Delta U = -\int_{y_o}^{y} (-mg)ds = +mg\int_{y_o}^{y} ds = mg(y - y_o) \Rightarrow U - U_o = mgy - mgy_o .$$

We can now make the association,

Gravitational Potential Energy $U_g = mgy$

Example 22.4: A 1.50kg pumpkin falls 62.0m to the ground. Find (a)the initial potential energy of the pumpkin, (b)the final potential energy of the pumpkin, and (c)the speed of the pumpkin just before hitting the ground.

Given: m = 1.50kg and y_o = 62.0m
Find: U_o = ?, U = ?, and v = ?



$U_o = mgy_o$ $\quad$ $U = 0$

$K_o = 0$ $\quad$ $K = \frac{1}{2}mv^2$

Choose the coordinates so that y=0 at the ground.
(a)Using the gravitational potential energy,
$U = mgy \Rightarrow U_o = mgy_o = (1.50)(9.80)(62.0) \Rightarrow \boxed{U_o = 911J}$.

(b)Because of the choice of coordinates,
$U = mgy = (1.50)(9.80)(0) \Rightarrow \boxed{U = 0}$.

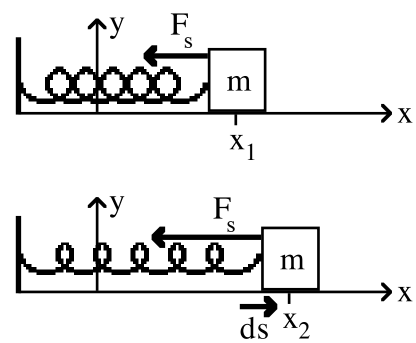
(c)We could have solved this problem using the kinematic equations, but the point here is to use energy methods. Using the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow (\tfrac{1}{2}mv^2 - 0) + (0 - mgy_o) = 0 \Rightarrow \tfrac{1}{2}mv^2 = mgy_o \Rightarrow$$
$$v = \sqrt{2gy_o} = \sqrt{2(9.80)(62.0)} \Rightarrow \boxed{v = 34.9m/s}.$$

This is a bit easier than using the kinematic equatrions.

COMMENT ON PROBLEM SOLVING:
When a problem involves the Law of Conservation of Energy, the best approach is the same as for the Law of Conservation of Linear Momentum. Draw two pictures, the "before" picture, and the "after" picture. Then figure out the kinetic and potential energy in each picture. Finally, substitute in the Law of Conservation of Energy.

Spring Potential Energy:



Applying the definition of potential energy to the mass moving to the right and using the definition of work,

$$\Delta U \equiv -W_c \Rightarrow \Delta U = -W_s = -\int_{x_1}^{x_2} \vec{F}_s \bullet d\vec{s}$$

The spring force and the displacement are opposite. Using Hooke's Rule ($F_s = -kx$),

$$\Delta U = -\int_{x_1}^{x_2} (-kx)dx = k\int_{x_1}^{x_2} xdx = \tfrac{1}{2}k(x_2^2 - x_1^2)$$
$$\Rightarrow U - U_o = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2.$$

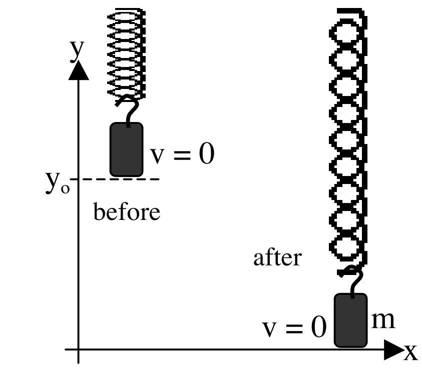
We can now make the association,

Spring Potential Energy $U_s = \frac{1}{2}kx^2$

For spring potential energy, the zero is set to be where the spring is unstretched. This makes sense because the spring is storing no energy at this point.

Example 22.5: A 5.00N weight is hung on a spring that drops a maximum of 1.02m. Find the spring constant.

Given: mg = 5.00N and y_o = 1.02m
Find: k = ?



$K_o = 0$ $\quad K = 0$

$U_o = mgy_o + 0$ $\quad U = 0 + \frac{1}{2}ky_o^2$

We have chosen coordinates so that y=0 corresponds to the place where the spring is fully extended. The location where the spring has no potential energy is at y_o. The kinetic at both the top and bottom of the motion is zero because the mass is at rest. The initial potential energy is due to gravity, while the final potential energy is due to the stretched spring. Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow (0-0) + (\tfrac{1}{2}ky_o^2 - mgy_o) = 0 \Rightarrow k = \frac{2mg}{y_o}.$$

Putting the numbers in,

$$k = \frac{2(5.00N)}{1.02m} \Rightarrow \boxed{k = 9.80N/m}.$$

This probably would be very difficult to do using forces. Energy is definitely the way to go.

<u>Section Summary</u>

Why do objects do what they do? We are adding a third answer to this question. In addition to forces and linear momentum, we are developing an understanding of energy and building the Law of Conservation of Energy. We began by distinguishing between conservative forces and non-conservative forces. Conservative forces do no work around a closed loop so the work they do depends only on the initial and final position. This fact allowed us to establish the

Definition of Potential Energy $\Delta U \equiv -W_c$

We applied this definition to two conservative forces to get specific expressions for

Gravitational Potential Energy $U_g = mgy$

and

Spring Potential Energy $U_s = \frac{1}{2}kx^2$

For systems that have only conservative forces acting we have established a preliminary statement of a Law of Conservation of Energy,

$$\Delta K + \Delta U = 0.$$

Stated in plain English,

The Law of Conservation of Energy

"Energy may be transformed from one type to another, but the total energy always remains constant."