Section 23 – The Law of Conservation of Energy

Why do objects do what they do? We have been building the concept of energy and developing the Law of Conservation of Energy. In this section we'll continue our progress by addressing the inclusion of non-conservative forces.

Section Outline
1. A Quick Review
2. Including Non-conservative Forces

1. A Quick Review

Let's take a moment to review our progress to understanding energy and it's conservation. We began by developing the work-energy theorem,

$$W_{net} = \Delta K \ .$$

If there are only non-conservative forces,

$$W_c = \Delta K \ .$$

We have associated potential energy with conservative forces because they are able to return any energy that they collect. Using the definition of potential energy, we can replace the work done by conservative forces with the appropriate potential energy,

$$\Delta U = -W_c \Rightarrow -\Delta U = \Delta K \Rightarrow \Delta K + \Delta U = 0 \ .$$

This is the mathematical expression of the Law of Conservation of energy excluding non-conservative forces.

Example 23.1: A 0.500kg pinecone falls 15.0m to the ground. Find speed with which it hits the ground assuming no air resistance.

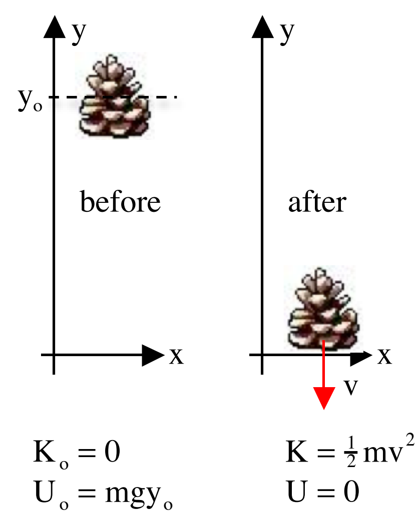
Given: m = 0.500kg, y_o = 15.0m, v_o = 0, and y = 0.
Find: v = ?

Initially, there is no kinetic energy, just gravitational potential energy. As the pinecone falls, the potential energy is converted into kinetic according to the Law of Conservation of Energy,

$$\Delta K + \Delta U = 0 \Rightarrow (\tfrac{1}{2}mv^2 - 0) + (0 - mgy_o) = 0 \ .$$

Solving for the final speed,

$$v = \sqrt{2gy_o} = \sqrt{2(9.8)(15)} \Rightarrow \boxed{v = 17.1\tfrac{m}{s}}.$$



2. Including Non-conservative Forces

Before we can formalize the Law of Conservation of Energy, we have to come to terms with non-conservative forces. Imagine giving a box a push along a horizontal surface. Friction will do work on the box to bring it to rest. According to the Work-Energy Theorem, this work done by friction will equal the change in the kinetic energy. If you try to think about this in terms of energy conservation, it appears as though the initial kinetic energy just disappears.

You must have rubbed your hands together on a cold winter day. Where does the kinetic energy of your hands go? You can feel the warmth it generates! Work done by friction generates heat. Heat is the random motion of atoms and molecules. So the kinetic energy that is "lost" to friction is really not gone at all. It appears as the kinetic energy of the atoms and molecules in the object. We will learn how to treat heat as a form of energy much later, but for know we can at least be satisfied that energy is not lost to non-conservative forces, it is just changed into a different form.

With this new acknowledgement of additional forms of energy we can now establish the Law of Conservation of Energy without excluding non-conservative forces,

The Law of Conservation of Energy

"Energy may be transformed from one type to another, but the total energy always remains constant."

We can state this law mathematically by going back to the key ideas,

The Work-Energy Theorem $W_{net} = \Delta K$

The Definition of Potential Energy $\Delta U \equiv -W_c$

Including non-conservative forces in the net work done on a system can be broken down into two parts,

$$W_{net} = W_c + W_{nc}\,.$$

Using the work-energy theorem to write the net work in terms of the change in kinetic energy and using the definition of potential energy to replace the work done by conservative forces,

$$\Delta K = -\Delta U + W_{nc} \Rightarrow \Delta K + \Delta U = W_{nc}$$

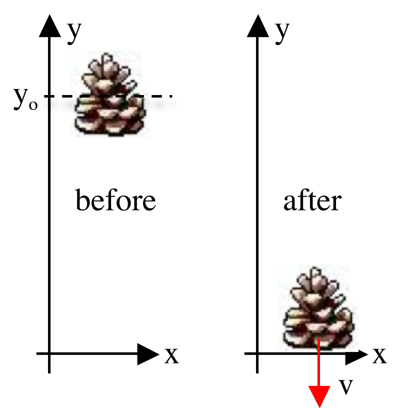
This is the mathematical statement of the Law of Conservation of Energy. It includes non-conservative forces.

The Law of Conservation of Energy $\Delta K + \Delta U = W_{nc}$

Example 23.2: The pinecone in the last example actually strikes the ground with a speed of 15.0m/s. Find the average force of air resistance.

Given: $m = 0.500\text{kg}$, $y_o = 15.0\text{m}$, $y = 0$, $v_o = 0$, and $v = 15.0\text{m/s}$.

Find: $F_{air} = ?$



$K_o = 0$ $\quad$ $K = \frac{1}{2}mv^2$

$U_o = mgy_o$ $\quad$ $U = 0$

The actual speed is less than we calculated in the last example due to the air resistance. The work done by this non-conservative force is found from the definition of work,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_{air} = \int \vec{F}_{air} \bullet d\vec{s} = -F_{air} y_o\,,$$

where we have assumed a constant force of air resistance. The minus sign is due to the fact that the force and motion are opposite.

Applying the Law of Conservation of Energy,

$$\Delta K + \Delta U = W_{nc} \Rightarrow (\tfrac{1}{2}mv^2 - 0) + (0 - mgy_o) = -F_{air} y_o\,.$$

Solving for the force,

$$\tfrac{1}{2}mv^2 - mgy_o = -F_{air} y_o \Rightarrow F_{air} = mg - \frac{mv^2}{2y_o}$$

Putting the numbers in,

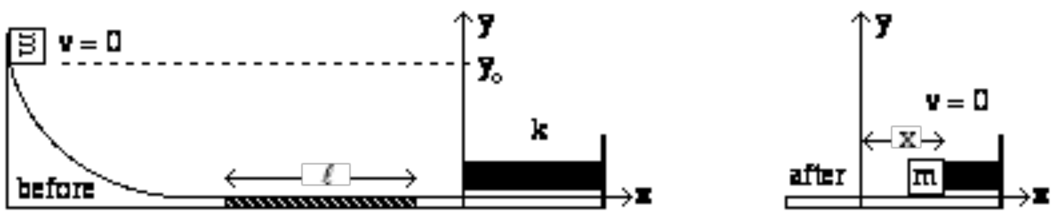
$$F_{air} = (0.5)(9.80) - \frac{(0.5)(15.0)^2}{2(15.0)} \Rightarrow \boxed{F_{air} = 1.15N}.$$

You should notice that the initial potential energy is greater than the final kinetic energy because of the work done by air resistance. This energy is not lost. It is converted into heat. You may know that this heat produced by air resistance is a major challenge for NASA as they try to bring spacecrafts back into the atmosphere. Here's a video that describes how NASA solves this problem http://www.nasa.gov/multimedia/videogallery/index.html?media_id=148281791.

Here's another example including work done by none conservative forces.

Example 23.3: A 10.0kg block slides down a 3.00m high frictionless ramp. Then it skids 6.00m along a rough surface before returning to a smooth surface and colliding with a spring of spring constant 2250N/m. The block comes to rest when the spring is compressed 30.0cm. Find the coefficient of friction between the block and the rough surface.

Given: $m = 10.0kg$, $y_o = 3.00m$, $\ell = 6.00m$, $k = 2250N/m$, and $x = 0.300m$.
Find: $\mu = ?$



$K_o = 0$ $\qquad K = 0$

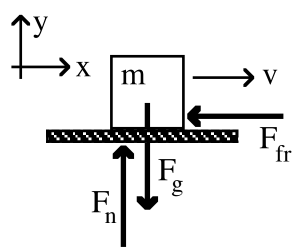
$U_o = mgy_o + 0$ $\qquad U = 0 + \frac{1}{2}kx^2$

Using the Law of Conservation of Energy,

$$\Delta K + \Delta U = W_{nc} \Rightarrow (0 - 0) + (\tfrac{1}{2}kx^2 - mgy_o) = W_{nc}$$

The work done by the frictional force can be found by starting with the Second Law to find the size of the normal force,

$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g.$$



Using the definition of coefficient of friction and the mass/weight rule,

$$\mu \equiv \frac{F_{fr}}{F_n} \Rightarrow F_{fr} = \mu F_n = \mu mg.$$

The work done can be found from the definition,

$$W \equiv \int \vec{F} \bullet d\vec{s} \Rightarrow W_{fr} = \int \vec{F}_{fr} \bullet d\vec{s} = -F_{fr}\ell = -\mu mg\ell.$$

Substituting back into the equation from Conservation of Energy,

$$\tfrac{1}{2}kx^2 - mgy_o = -\mu mg\ell \Rightarrow \mu = \frac{mgy_o - \frac{1}{2}kx^2}{mg\ell} = \frac{y_o}{\ell} - \frac{kx^2}{2mg\ell}.$$

Plugging in the numbers,

$$\mu = \frac{3.00}{6.00} - \frac{(2250)(0.300)^2}{2(10.0)(9.80)(6.00)} \Rightarrow \boxed{\mu = 0.328}.$$

<u>Section Summary</u>

Why do objects do what they do? We have now added a third answer to this question. In addition to forces and linear momentum, we now have energy and the Law of Conservation of Energy.

The Law of Conservation of Energy

"Energy may be transformed from one type to another, but the total energy always remains constant."

We wrote the Law of Conservation of Energy mathematically as,

The Law of Conservation $\Delta K + \Delta U = W_{nc}$