Section 24 – Collisions and Conservation of Energy

Why do objects do what they do? We now have three powerful explanations; forces and Newton's Laws of Motion, linear momentum and the Law of Conservation of Linear Momentum, and energy and the Law of Conservation of Energy. In this section we'll look again at collisions.

Section Outline

1. Elastic Collisions
2. Inelastic Collisions

A collision is a short, strong interaction where objects exert equal and forces on each other according to the Newton's Third Law. As a result, we showed that the Law of Conservation of Linear Momentum always holds for collisions. Since the Law of Conservation of Energy holds for all interactions everywhere, all the time, it is also valid for all collisions. However, when non-conservative forces are involved in the collision, the Law of Conservation of Energy is often not very helpful because we can't calculate how much work is done by these forces.

For example, think about ball colliding with a concrete wall. If the ball is a superball, the ball will rebound with just about the same kinetic energy it had before the collision. However, if the ball is made of clay, it might just stick to the wall and all the kinetic energy would be spent on work done by non-conservative forces. So for collisions, it is not a question of whether the Law of Conservation of Energy applies because it does, it is a question of whether that knowledge is useful. The key distinction is whether the kinetic energy alone is conserved. When the kinetic energy is conserved, we say the collision is "elastic" and when it is not, we say the collision is inelastic.

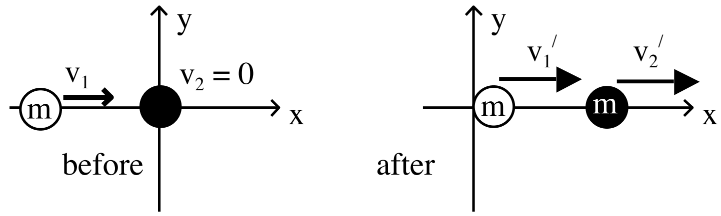
In summary for collisions,

Collisions	**Conserved?**
Linear Momentum	yes – as long as external force can be neglected
Total Energy	yes - always but often not helpful
Kinetic Energy	sometimes • if yes the collision is elastic • if no the collision is inelastic

Below are several example problems first elastic collision, then inelastic.

1. Elastic Collisions

Example 24.1: The cue ball is traveling at 4.00m/s when it hits the waiting eight ball head on. Find the speed of both balls after collision assuming the collision is elastic and both balls have equal mass.



Given: v_1 = 4.00m/s, v_2 = 0, and $m_1 = m_2$.
Find: v_1' = ? and v_2' = ?

$$p_o = mv_1 \qquad p = mv_1' + mv_2'$$

$$K_o = \tfrac{1}{2}mv_1^2 \qquad K = \tfrac{1}{2}m(v_1')^2 + \tfrac{1}{2}m(v_2')^2$$

The Law of Conservation of Linear Momentum requires,

$$p_o = p \Rightarrow$$

$$mv_1 = mv_1' + mv_2' \Rightarrow v_1 = v_1' + v_2'.$$

The fact that the collision is elastic requires,

$$K_o = K \Rightarrow \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m(v_1')^2 + \tfrac{1}{2}m(v_2')^2 \Rightarrow v_1^2 = (v_1')^2 + (v_2')^2.$$

Solving the momentum equation for v_2',

$$v_2' = v_1 - v_1',$$

and substituting into the elasticity equation,

$$v_1^2 = (v_1')^2 + (v_1 - v_1')^2 = (v_1')^2 + v_1^2 - 2v_1v_1' + (v_1')^2 \Rightarrow 0 = 2(v_1')^2 - 2v_1v_1' \Rightarrow v_1' = v_1 \text{ or } \boxed{v_1' = 0}.$$

There are two solutions that conserve both momentum and kinetic energy. The first one is where the cue ball completely misses the eight ball (what would happen if I played). The second solution is the one that interests us. The cue ball stops dead. Using the previous equation for the speed of the second ball,

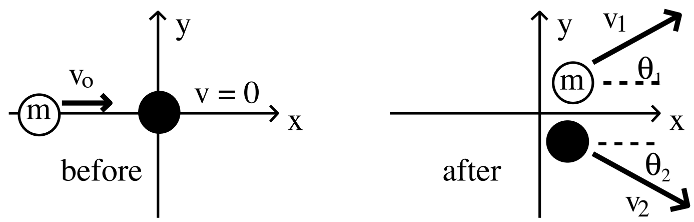
$$v_2' = v_1 - v_1' = 4.00 - 0 \Rightarrow \boxed{v_2' = 4.00 m/s}.$$

If you have played this game you know that in a ahead on collision (with no spin) the cue ball comes to rest and the other ball heads off with the same speed the cue ball had before the collision.

Now, let's look at a two dimensional collision.

Example 24.2: The cue ball is traveling at 4.00m/s when it collides with the eight ball. The cue heads off at 40.0° to its original direction. Again assume the collision is elastic and both balls have equal mass. Find the velocity of the eight ball after the collision.

Given: $v_o = 4.00$m/s and
$\theta_1 = 40.0°$
Find: v_1 = ?, v_2 = ?, and θ_2 = ?



Recall that momentum is a vector quantity, so each component must be kept track of separately.

The Law of Conservation of Momentum applies to each direction independently,

$$p_{ox} = mv_o \qquad p_x = mv_1\cos\theta_1 + mv_2\cos\theta_2$$
$$p_{oy} = 0 \qquad p_y = mv_1\sin\theta_1 - mv_2\sin\theta_2$$
$$K_o = \tfrac{1}{2}mv_o^2 \qquad K = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2$$

$$p_{ox} = p_x \Rightarrow mv_o = mv_1\cos\theta_1 + mv_2\cos\theta_2 \Rightarrow v_2\cos\theta_2 = v_o - v_1\cos\theta_1 \qquad (1)$$
$$p_{oy} = p_y \Rightarrow 0 = mv_1\sin\theta_1 + mv_2\sin\theta_2 \Rightarrow v_2\sin\theta_2 = v_1\sin\theta_1 \qquad (2)$$

The elasticity of the collisions requires that,

$$K_o = K \Rightarrow \tfrac{1}{2}mv_o^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2 \Rightarrow v_o^2 = v_1^2 + v_2^2 \qquad (3)$$

Squaring eq. 2 and adding it to the square of eq. 1,

$$(v_2\sin\theta_2)^2 + (v_2\cos\theta_2)^2 = (v_1\sin\theta_1)^2 + (v_o - v_1\cos\theta_1)^2 \Rightarrow$$
$$v_2^2(\sin^2\theta_2 + \cos^2\theta_2) = v_o^2 + v_1^2(\sin^2\theta_1 + \cos^2\theta_1) - 2v_ov_1\cos\theta_1 \Rightarrow$$
$$v_2^2 = v_o^2 + v_1^2 - 2v_ov_1\cos\theta_1 \qquad (4)$$

Substituting eq. 4 into eq. 3,

$$v_o^2 = v_1^2 + v_o^2 + v_1^2 - 2v_ov_1\cos\theta_1 \Rightarrow 0 = 2v_1^2 - 2v_ov_1\cos\theta_1 \Rightarrow v_1 = v_o\cos\theta_1.$$

Plugging in the numbers,

$$v_1 = v_o\cos\theta_1 = 4.00\cos 40° \Rightarrow \boxed{v_1 = 3.06m/s}.$$

Putting this value into eq. 4,

$$v_2 = \sqrt{v_o^2 + v_1^2 - 2v_ov_1\cos\theta_1} = \sqrt{(4.00)^2 + (3.06)^2 - 2(4.00)(3.06)\cos 40°} \Rightarrow \boxed{v_2 = 2.58m/s}.$$

We can get θ_2 by dividing eq. 2 by eq. 1,

$$\frac{v_2\sin\theta_2}{v_2\cos\theta_2} = \frac{v_1\sin\theta_1}{v_o - v_1\cos\theta_1} \Rightarrow \tan\theta_2 = \frac{v_1\sin\theta_1}{v_o - v_1\cos\theta_1} \Rightarrow \theta_2 = \arctan\frac{v_1\sin\theta_1}{v_o - v_1\cos\theta_1}$$

Plugging in the numbers,

$$\theta_2 = \arctan\left(\frac{(3.06)\sin 40°}{4.00 - (3.06)\cos 40°}\right) \Rightarrow \boxed{\theta_2 = 50.0°}.$$

If you play this game you probably know that (with no spin) the cue ball heads off at a right angle to the ball that is struck. This is because the balls both have the same mass and the collision is very nearly elastic.

2. Inelastic Collisions

During inelastic collisions, momentum is conserved, but kinetic energy is not.

> *Example 24.3: You probably watched the collision of the Smart Car with the concrete barrier at http://www.youtube.com/watch?v=cfi4c9TuFvk. Let's simplify matters and assume all the motion happened along a line. The 820kg car is traveling at a speed of 31m/s as it collides with the 20,000kg concrete barrier. It rebounds at 3.0m/s. Find (a)the speed of the barrier just after collision and (b)the fraction of the kinetic energy that is lost in the collision.*
>
> Given: m = 820kg, M = 20,000kg, v_1 = 31m/s, v_1' = 3.0m/s, and v_2 = 0.
> Find: v_2' = ? and $\frac{K_o - K}{K_o}$ = ?
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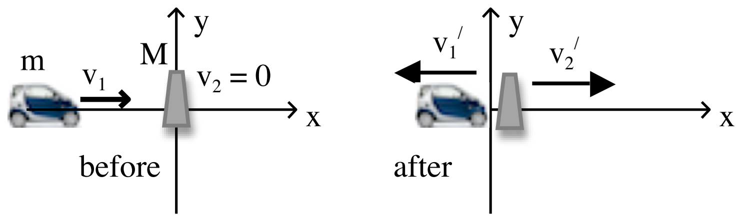
> 
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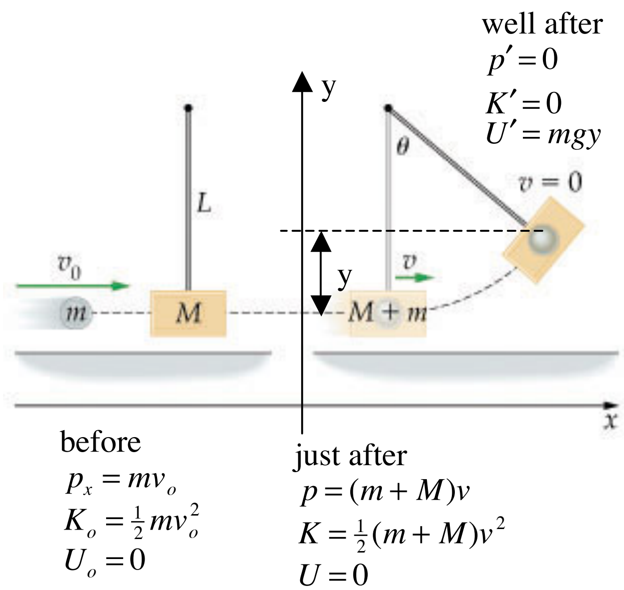
>
> $p_o = mv_1$ $\quad$ $p = Mv_2' - mv_1'$
>
> $K_o = \frac{1}{2}mv_1^2$ $\quad$ $K = \frac{1}{2}m(v_1')^2 + \frac{1}{2}M(v_2')^2$
>
> (a)The Law of Conservation of Linear Momentum requires,
>
> $p_o = p \Rightarrow mv_1 = Mv_2' - mv_1'$.
>
> Solving for the barriers speed,
>
> $$v_2' = \frac{m(v_1 + v_1')}{M} = \frac{(820)(31)}{20000} \Rightarrow \boxed{v_2' = 1.4\,\tfrac{m}{s}}.$$
>
> (b)The fraction of kinetic energy lost is,
>
> $$\frac{K_o - K}{K_o} = 1 - \frac{K}{K_o} \Rightarrow 1 - \frac{\frac{1}{2}m(v_1')^2 + \frac{1}{2}M(v_2')^2}{\frac{1}{2}mv_1^2} = 1 - \frac{m(v_1')^2 + M(v_2')^2}{mv_1^2}$$
>
> Plugging in the numbers,
>
> $$\frac{K_o - K}{K_o} = 1 - \frac{(820)(3)^2 + 20000(1.4)^2}{(820)(31)^2} \Rightarrow \boxed{\frac{K_o - K}{K_o} = 0.94 = 94\%}.$$
>
> I bet you can guess where the energy went! Just remember that after all the bending, smashing, and crunching is all wound up as heat.

Here is a second example drawn from a common lab experiment.

Example 24.4: You will use the device shown below in lab to study collisions. It is called a "ballistic pendulum." A ball of mass 50.0g is fired at a catching device of mass 300g. Once the ball is caught, it swings up through an angle of 37.0° on the end of a 25.0cm long arm. Find (a)the initial velocity of the ball, (b)the initial kinetic energy of the ball, (c)the kinetic energy of the system just after collision, and (d)explain whether the collision is elastic or not.

Given: m = 0.0500kg, M = 0.300kg, θ = 37.0°, and L = 0.250m
Find: v_o = ?, K_o = ?, and K = ?



(a)During the collision, the Law of Conservation of Linear Momentum applies,

$$p_o = p \Rightarrow mv_o = (m+M)v .$$

We can't assume the collision is elastic, but once the collision is completed and the system begins to swing upward, there are no non-conservative forces doing work, so the Law of Conservation of Energy can be useful,

$$\Delta K + \Delta U = W_{nc} \Rightarrow$$

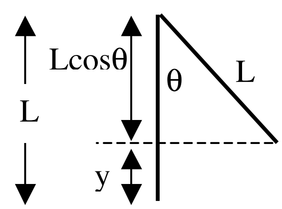
$$[0 - \tfrac{1}{2}(m+M)v^2] + [(m+M)gy - 0] = 0 \Rightarrow$$

$$\tfrac{1}{2}(m+M)v^2 = (m+M)gy \Rightarrow v = \sqrt{2gy}$$

Substituting into the momentum equation,

$$mv_o = (m+M)\sqrt{2gy} \Rightarrow v_o = \tfrac{(m+M)}{m}\sqrt{2gy} .$$

The height can be found using some trig,



$$L = L\cos\theta + y \Rightarrow y = L - L\cos\theta = L(1-\cos\theta).$$

Finally then,

$$v_o = \tfrac{(m+M)}{m}\sqrt{2gL(1-\cos\theta)} = \tfrac{(0.0500+0.300)}{0.0500}\sqrt{2(9.80)(0.250)(1-\cos 37^\circ)} \Rightarrow \boxed{v_o = 6.95 m/s}.$$

(b)Using the definition of kinetic energy,

$$K_o = \tfrac{1}{2}mv_o^2 = \tfrac{1}{2}(0.0500)(6.95)^2 \Rightarrow \boxed{K_o = 1.21J}.$$

(c)Again with the definition of kinetic energy,

$$K = \tfrac{1}{2}(m+M)v^2 = (m+M)gy = (m+M)gL(1-\cos\theta) \Rightarrow$$

$$K = (m+M)gL(1-\cos\theta) = (0.0500+0.300)(9.80)(0.250)(1-\cos 37^\circ) \Rightarrow \boxed{K = 0.173J}.$$

(d)Since the kinetic energy drops during the collision, it is inelastic.

<u>Section Summary</u>

Why do objects do what they do? We have now added a third answer to this question. In addition to forces and linear momentum, we now have energy and the Law of Conservation of Energy.

The Law of Conservation of Energy

"Energy may be transformed from one type to another, but the total energy always remains constant."

The law can be written mathematically as,

$$\text{The Law of Conservation } \Delta K + \Delta U = W_{nc}$$

We looked at several examples of the use of the law in collisions. All collisions conserve energy and momentum, but some (even most) involve non-conservative forces making the application of the Law of Conservation of Energy problematic. Collisions that only involve conservative forces are said to be "elastic" which means that the collision conserves kinetic energy. "Inelastic" collisions involve non-conservative forces and so don't conserve kinetic energy.

A Summary of the Course to Date

Now we can explain why objects do what they do using force, linear momentum, or energy depending upon which is more convenient or useful.