Section 31 – Planets, Moons, and Gyroscopes

What do objects do and why do they do it? We can describe and explain the rotational motion of objects with rotational kinematics, torques, rotational kinetic energy, and now angular momentum. In this section we'll look at two additional systems to build our understanding of angular momentum; planetary orbits and gyroscopes.

Section Outline

- 1. Planetary Orbits
- 2. Gyroscopic Motion

1. Planetary Orbits

The planets of our solar system have nearly circular orbits. This is no coincidence. Anything that isn't in a near circular orbits will eventually collide into something that is in a circular orbit. This happens from time to time when asteroids or meteors collide with planets. In fact, it was the collision of a large asteroid with Earth that caused the mass extinction that ended the dinosaurs sixty-six million years ago. More recently, in 2013 a small asteroid entered the atmosphere over Russia and exploded. Video can be seen at http://en.wikipedia.org/wiki/Chelyabinsk_meteor.



Halley's comet is an example of an typical object in a non-circular orbit. It was easily visible in the night sky as it passed Earth in 1986. It will return in 2061. A photograph of the comet in the night sky is pictured above as is its orbit through the solar system.

Since the gravitational force acts along the line between the orbiting body and the sun, there can be no torque exerted. Therefore, the angular momentum and the Law of Conservation of Angular Momentum can give us detailed information about orbits.

Example 31.1: Halley's comet is closest distance to the sun is 0.586AU and the farthest distance is 35.1AU. When it is farthest away it is traveling about 0.911km/s. Find the speed when it is closest to the sum.

Given: r = 0.586AU, R = 35.1AU, and v = 0.911km/s. Find: V = ?



An AU is the distance between Earth and the sun. However, we won't need to change units because we'll only need ratios. The angular momentum at closest approach is,

$$L_o = mVr$$

When the comet is farthest away,

$$L = mvR$$
.

Using the Law of Conservation of Angular Momentum,

$$L_o = L \Longrightarrow mVr = mvR \Longrightarrow V = v\frac{R}{r}.$$

The mass of the comet doesn't matter just as in most cases where gravity is the force involved in the motion. Plugging in the numbers,

$$V = v \frac{R}{r} = (0.911) \frac{35.1}{0.586} \Rightarrow V = 54.6 \frac{km}{s}$$

It is going much faster when it is closer. This makes sense not only to maintain the constant angular momentum, but also in terms of Conservation of Energy.

2. Gyroscopic Motion

You may have noticed when you ride your bike that you really don't turn the handlebars very much when you want to round a corner. You mostly just lean toward the direction you want to turn and the bike changes direction. You may have also noticed that this leaning technique is more effective when the bike is going faster.





The best way to understand what's going on is to think about a gyroscope. A gyroscope is a toy that has some fascinating properties. If it is vertical, it acts like a top. However, its most amazing behavior occurs

when it is tipped off the vertical. Instead of tipping over, it begins to rotate about the vertical axis. This motion is called "precession." The sketch at the right is of a horizontal gyroscope spinning in the vertical plane. So, it has an angular momentum toward the right. It is resting on a pivot on its left. There is a torque about the pivot point caused by the weight of the gyroscope. The magnitude and direction are given by the definition of torque,

 $\vec{\tau} \equiv \vec{r} \times \vec{F} \Rightarrow \tau = \text{rmg} = \text{mgh}$,

into the page by the right hand rule.

Notice that the torque is perpendicular to the angular momentum. When we're confused about rotational effects, it is often useful to think about the analogy to translational motion. Recall that when the force is perpendicular to the linear momentum, circular motion occurs. You might expect the same thing to happen now that the torque is perpendicular to the angular momentum.

The net torque is responsible for a change in angular momentum according to the Second Law for Rotation. Since the torque is perpendicular to the original angular momentum, L_o , the angular momentum can't change magnitude, only direction. This is shown in the top downward view at the right. The torque causes the angular momentum to change direction by the angle, $d\phi$.

To find rate at which the gyroscope rotates apply the Second Law for Rotation,

$$\Sigma \vec{\tau} = \frac{dL}{dt} \Longrightarrow d\vec{L} = \vec{\tau} dt \Longrightarrow dL = \tau dt$$

From the geometry of the vector diagram at the right,

$$dL = Ld\phi \Rightarrow \tau dt = Ld\phi \Rightarrow \frac{d\phi}{dt} = \frac{\tau}{L}$$

The rate at which the gyroscope rotates horizontally is called the "Rate of Precession." The symbol used is typically the Greek letter Ω ,

Gyroscopic Precession Rate
$$\Omega = \frac{\tau}{L}$$





On your bike you generate a torque that points either forward or backward depending on which way you lean. The angular momentum of the wheels points horizontally to your left (assuming you are going forward). Since the torque is perpendicular to the angular momentum, the change it generates in the angular momentum is just a change in direction causing the angular momentum vector to precess, which must turn the bike.

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Example 31.2: The 3.00kg gyroscope shown below is horizontal and in the plane of the paper. It stays horizontal as it precesses. It spins at 600rpm with the part facing you moving downward and has a rotational inertia of 0.0500kg·m². Find (a)the angular momentum of the gyroscope, (b)the forces that act on the gyroscope, (c)the torque on the gyroscope about the pivot, (d)the precession rate, and (e)the time to precess once around.

Given: m = 3.00 kg, $\omega = 600 \text{rpm} = 62.8 \text{ rad/s}$, I = 0.0500 kg·m², and r = 0.200 m. Find: $\vec{L} = ?$, forces, and $\vec{\tau} = ?$.

(a)Using the definition of angular momentum,

$$L \equiv I\omega = (0.0500)(62.8) \Longrightarrow L = 3.14 kg \cdot m^2 / s.$$

The direction is given by the right hand rule and is shown in the sketch.

(b)The two forces acting on the gyroscope are gravity at the center-of-mass and the normal force at the pivot. The size of the gravitational force is given by the mass/weight rule,

$$F_g = mg = (3.00)(9.80) \Rightarrow F_g = 29.4N.$$

Since there is no vertical motion we can use the Second Law to find the normal force,

$$\Sigma F = ma \Rightarrow F_n - F_g = 0 \Rightarrow F_n = F_g \Rightarrow F_n = 29.4N.$$

The directions of the forces are shown in the sketch.

(c)The normal force creates no torque about the pivot. The torque generated by the gravitational force can be found from the definition of torque,

$$\vec{\tau} \equiv \vec{r} \times \vec{F} \Rightarrow \tau_g = rF_g = (0.200)(29.4) \Rightarrow \overline{\tau_g = 5.88N \cdot m},$$

since the force and radius vector are perpendicular. The direction of the torque is into the page as shown.

(d)The gyroscopic precession rate is,

$$\Omega = \frac{\tau}{L} = \frac{5.88}{3.14} \Rightarrow \overline{\Omega = 1.87 rad/s}.$$

(e)The time to precess once around is given by the definition of angular speed,

$$\omega \equiv \frac{d\theta}{dt} \Rightarrow \Omega = \frac{d\phi}{dt} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\Omega} = \frac{2(3.14)}{1.87} \Rightarrow \boxed{T = 3.35s}.$$





Section Summary

What do objects do and why do they do it? We have built deeper understanding of rotational motion by applying our knowledge to two systems. First, we have begun to understand planetary motion as an application of the Law of Conservation of Angular Momentum. Second, we applied the Second Law for Rotation to a gyroscope. The gyroscope is fascinating because the angular momentum and the torque are not aligned. This results in a very interesting form of rotational motion called precession.

Gyroscopic Precession Rate
$$\Omega = \frac{\tau}{L}$$
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A Summary of the Course to Date

Using a strong analogy with translational ideas we have extended the answer to the question, "What do objects do and why do they do it?" to include rotational motion. We have now completed the structure of Newtonian Mechanics!

