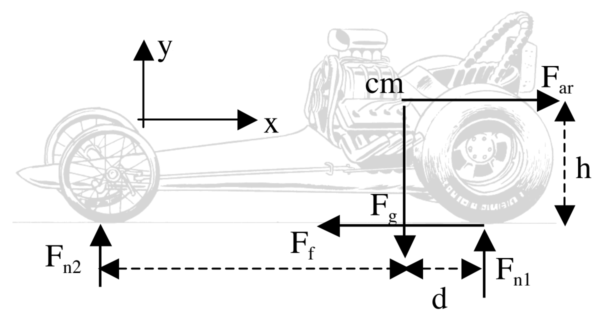
Section 33 – Equilibrium Examples

Now we have a fairly complete answer to our question, "what do objects do and why do they do it?" We will continue our look at objects in static equilibrium. Since such objects have no acceleration and no angular acceleration the Second Laws for Translation and Rotation require,

$$\Sigma\vec{F} = m\vec{a} \Rightarrow \Sigma\vec{F} = 0 \text{ and } \Sigma\vec{\tau} = m\vec{\alpha} \Rightarrow \Sigma\vec{\tau} = 0.$$

That is, the net force on the object must be zero and we have shown the net torque on the object about any point must also be zero.

Example 33.1: A 375kg drag racer is designed so that the center of mass is 1.00m above the ground and 0.800m in front of the rear tire. It is traveling at a constant speed of 150km/h. Assuming the force of air resistance acts through the center of mass, find the maximum allowed frictional force such that the front wheel just barely remains on the ground.



Given: m = 375kg, h = 1.00m, d = 0.800m, and v = 150km/h.
Find: F_{fr} = ?

Notice that the car is not in "static" equilibrium because it is moving. It does have zero acceleration however. This is called "dynamic equilibrium."

Applying the Second Law for Translation using the coordinates shown,

$$\Sigma F_x = ma_x \Rightarrow F_{ar} - F_{fr} = 0 \Rightarrow F_{fr} = F_{ar}. \quad (1)$$

$$\Sigma F_y = ma_y \Rightarrow F_{n1} + F_{n2} - F_g = 0 \Rightarrow F_{n1} + F_{n2} = F_g = mg. \quad (2)$$

Since the problem states that the frictional force is such that the front wheel is barely touching the ground, we'll assume F_{n2} is zero. So, equation 2 becomes,

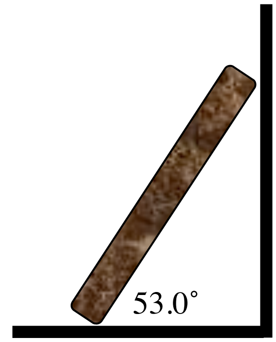
$$F_{n1} + F_{n2} = mg \Rightarrow F_{n1} = mg = (375)(9.80) \Rightarrow \boxed{F_{n1} = 3680N}. \quad (3)$$

Next, apply the Second Law for Rotation. By choosing the coordinates as we have, we have lever arms we need to write the torques easily,

$$\Sigma\tau_{cm} = I\alpha \Rightarrow F_{n1}d - F_f h = 0 \Rightarrow F_f = \tfrac{d}{h}F_{n1}, \quad (4)$$

where we have ignored the torque due to the normal force on the front wheel which we set to zero.
Plugging the numbers into equation (4) we get the frictional force,

$$F_f = \tfrac{d}{h}F_{n1} = \tfrac{d}{h}mg = \tfrac{0.800}{1.00}(375)(9.80) \Rightarrow \boxed{F_f = 2940N}.$$

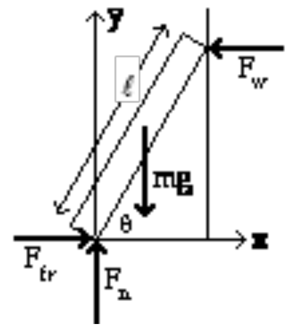


Example 33.2: A 1.00kg book leans against the smooth side of a bookshelf making a 53.0° angle with the horizontal. Find the forces that act on the book.

Given: m = 1.00kg and θ = 53.0°.
Find: F_g = ?, F_n = ?, F_w = ?, and F_{fr} = ?.

The weight can be found from the mass/weight rule,

$$F_g = mg = (1.00)(9.80) \Rightarrow \boxed{F_g = 9.80\,\text{N}}.$$



Note that there is no frictional force between the wall and the book because the wall is "smooth." There are four forces that act on the book. Choosing the origin to be where two unknown forces act will simplify the torque equation.

Applying the Second Laws for Rotation and Translation using the coordinates shown,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_w = 0, \quad (1)$$
$$\Sigma F_y = ma_y \Rightarrow F_n - F_g = 0, \quad (2)$$
$$\text{and } \Sigma\tau_o = I\alpha \Rightarrow -\tfrac{\ell}{2}F_g\cos\theta + F_w\ell\sin\theta = 0. \quad (3)$$

There are three equations for the three remaining unknowns. Solving equation 2 for the normal force,

$$F_n - F_g = 0 \Rightarrow F_n = F_g \Rightarrow \boxed{F_n = 9.80N}.$$

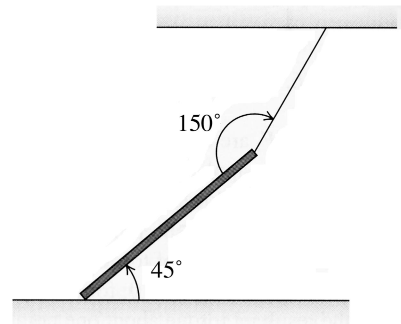
Now we can solve equation 3 for the force from the wall,

$$-\tfrac{\ell}{2}F_g\cos\theta + F_w\ell\sin\theta = 0 \Rightarrow F_w = \frac{F_g\cos\theta}{2\sin\theta} = \frac{(9.80)\cos 53.0^\circ}{2\sin 53.0^\circ} \Rightarrow \boxed{F_w = 3.69N}.$$

Plugging this back into equation (1) from the x-direction,

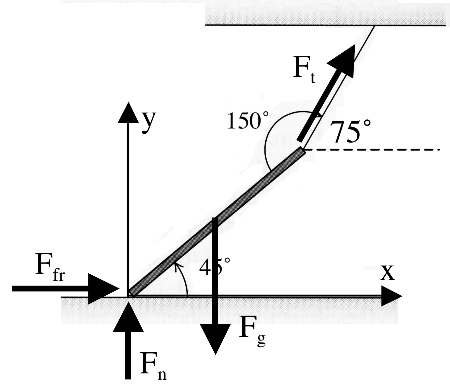
$$\Sigma F_x = ma_x \Rightarrow F_{fr} - F_w = 0 \Rightarrow F_{fr} = F_w \boxed{F_{fr} = 3.69\text{N}}.$$

Notice that if we included friction with the wall, we would have more unknown forces than the three conditions on equilibrium and the problem would have been statically indeterminate.



Example 33.3: The uniform 200kg beam show at the right is supported by a cable connected to the ceiling while the lower end rests on the floor. Find the forces that act on the beam.

Given: m = 200kg
Find: F_t, F_{fr}, F_n, and F_g.



The angle the cable makes with the horizontal is 75°. There is 30° between the cable and the direction of the beam plus another 45° between the beam and the horizontal. We'll choose the origin where two of the forces act so that the torque equation is simplified.

The weight can be calculated using the mass/weight rule,

$$F_g = mg = (200)(9.80) \Rightarrow \boxed{F_g = 1960N}.$$

Applying the Second Law for Translation and Rotation,

$$\Sigma F_x = ma_x \Rightarrow F_{fr} + F_t \cos 75° = 0 \Rightarrow F_{fr} = -F_t \cos 75°, \quad (1)$$

$$\Sigma F_y = ma_y \Rightarrow F_t \sin 75° - F_g + F_n = 0, \quad (2)$$

$$\text{and } \Sigma\tau_o = I\alpha \Rightarrow -\tfrac{\ell}{2} F_g \cos 45° + \ell F_t \sin 30° = 0 \Rightarrow F_t \sin 30° = \tfrac{1}{2} F_g \cos 45°. \quad (3)$$

The torque equation needs a bit of explaining. First we used ℓ for the length of the beam and it cancels out in the end. The torque due to gravity is clockwise and negative. The torque doe to the tension is counter-clockwise and positive. The definition of torque is,

$$\vec{\tau} \equiv \vec{r} \times \vec{F} \Rightarrow \tau_t = \ell F_t \sin\theta,$$

where $\vec{r}$ points from the origin to the end of the beam and θ is the angle between $\vec{r}$ and the tension which is just 30°. We have three equations for the three unknowns so we can solve. We can solve equation 3 for the tension,

$$F_t = \tfrac{F_g \cos 45°}{2\sin 30°} = \tfrac{(1960)\cos 45°}{2\sin 30°} \Rightarrow \boxed{F_t = 1390N}.$$

The frictional force can be found with equation 1.

$$F_{fr} = -F_t \cos 75° \Rightarrow \boxed{F_{fr} = -360N}.$$

Notice that the frictional force is negative. Therefore, it is pointing the wrong way in our sketch. It is great that the math can tell us this because we don't have to worry as much about getting the direction correct in the sketch. Finally, we can use equation 2 to get the normal force,

$$F_t \sin 75° - F_g + F_n = 0 \Rightarrow F_n = F_g - F_t \sin 75° = 1960 - (1390)\sin 75° \Rightarrow \boxed{F_n = 1340N}.$$

Section Summary

We didn't develop any new concepts of physics in the last two sections. However, we did learn to apply our knowledge of what objects do and why they do it to systems in static and dynamic equilibrium. That is, in situations where $\Sigma\vec{F} = 0$ and $\Sigma\vec{\tau}_P = 0$. The systems to which we applied Newton's Laws of Motion involved at most four forces and gave us three equations to be solved. You can imagine now the challenge faced by a civil engineer trying to design a stable building where there might be millions of forces and millions of equations to be solved!