Motion and the Calculus

Pre-Class Questions:

Problem Set #3 (due next time)

Lecture Outline

1. The Calculus of Motion
2. Mass on a Spring

We need to advance or understanding of these ideas by using our knowledge of the calculus.

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system	x
Displacement	A change in position	$\Delta x = x_f - x_i$
Average Velocity	The average rate of displacement	$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$
Speed	The magnitude of the velocity	$v = \mid v \mid$
Average Acceleration	The average rate of change of velocity	$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

All of the changes become infinitesimal so Δ's become d's.

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system	x
Displacement	A change in position	dx
Velocity	The rate of displacement	$v \equiv \frac{dx}{dt}$
Speed	The magnitude of the velocity	$v = \lvert v \rvert$
Average Acceleration	The rate of change of velocity	$a \equiv \frac{dv}{dt}$

Let's summarize our understanding of the definition of velocity: average $\bar{v} \equiv \frac{\Delta x}{\Delta t}$ and instantaneous $v \equiv \frac{dx}{dt}$

1. Algebraically, it can be thought of as an equation:

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i}$$

2. Graphically, it can be thought of as the slope of the position versus time graph:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{rise}{run} = slope \qquad v \equiv \frac{dx}{dt} = \text{slope of tangent line}$$

3. It can be rearranged and thought about in terms of the area under the velocity versus time graph:

$$\bar{v} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v}\,\Delta t = area \qquad v = \frac{dx}{dt} \Rightarrow \int dx = \int v\,dt = area$$

Let's summarize our understanding of the definition of acceleration: average $\bar{a} \equiv \frac{\Delta v}{\Delta t}$ and instantaneous $a \equiv \frac{dv}{dt}$

1. Algebraically, average acceleration can be thought of as an equation: $\bar{a} = \frac{v_f - v_i}{t_f - t_i}$

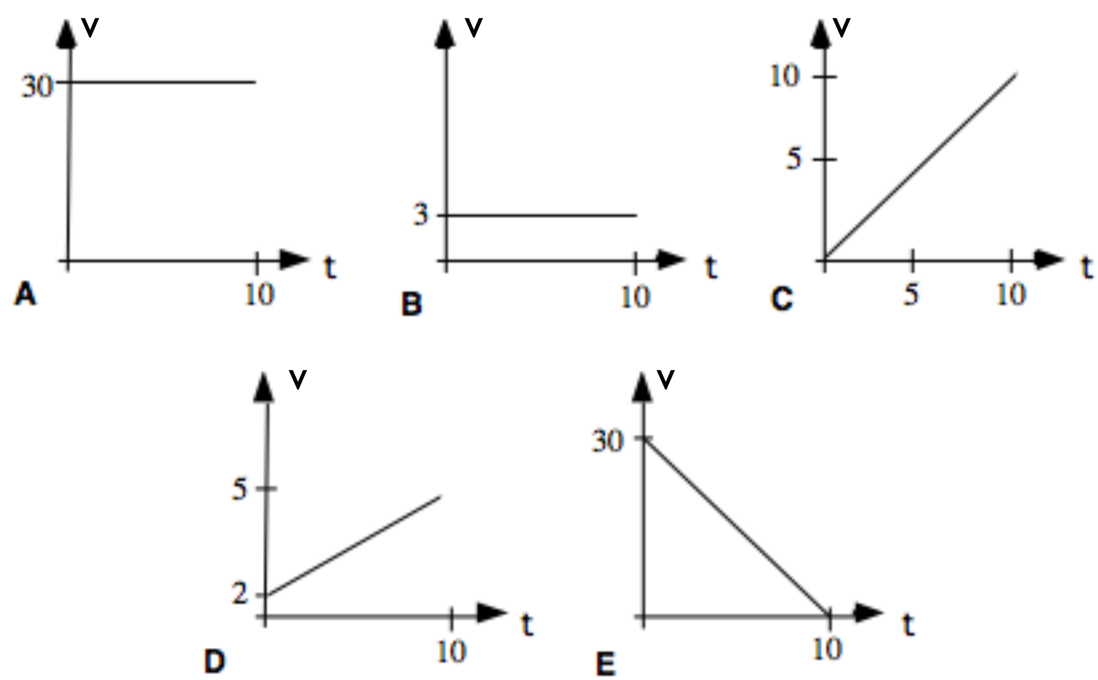
2. Graphically, acceleration can be thought of as the slope of the velocity versus time graph:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{rise}{run} = slope \qquad a \equiv \frac{dv}{dt} = \text{slope of tangent line}$$

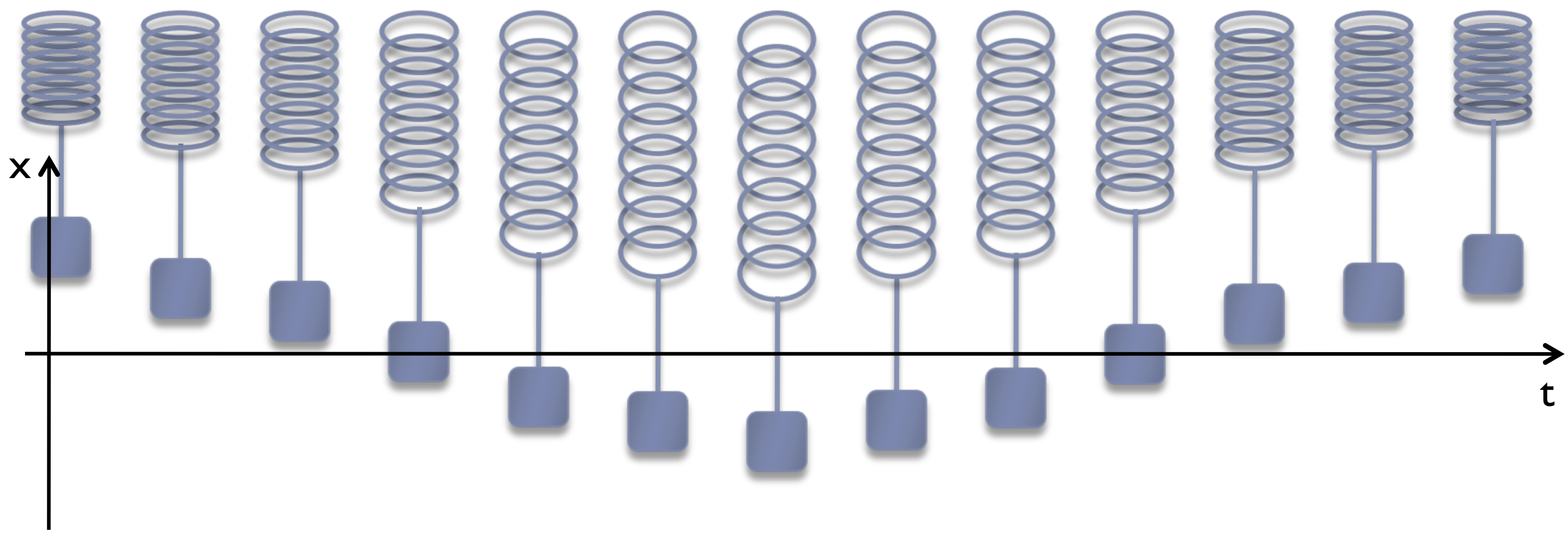
3. It can be rearranged and thought about in terms of the area under the acceleration versus time graph:

$$\bar{a} = \frac{\Delta v}{\Delta t} \Rightarrow \Delta v = \bar{a}\,\Delta t = area \qquad a = \frac{dv}{dt} \Rightarrow \int dv = \int a\,dt = area$$

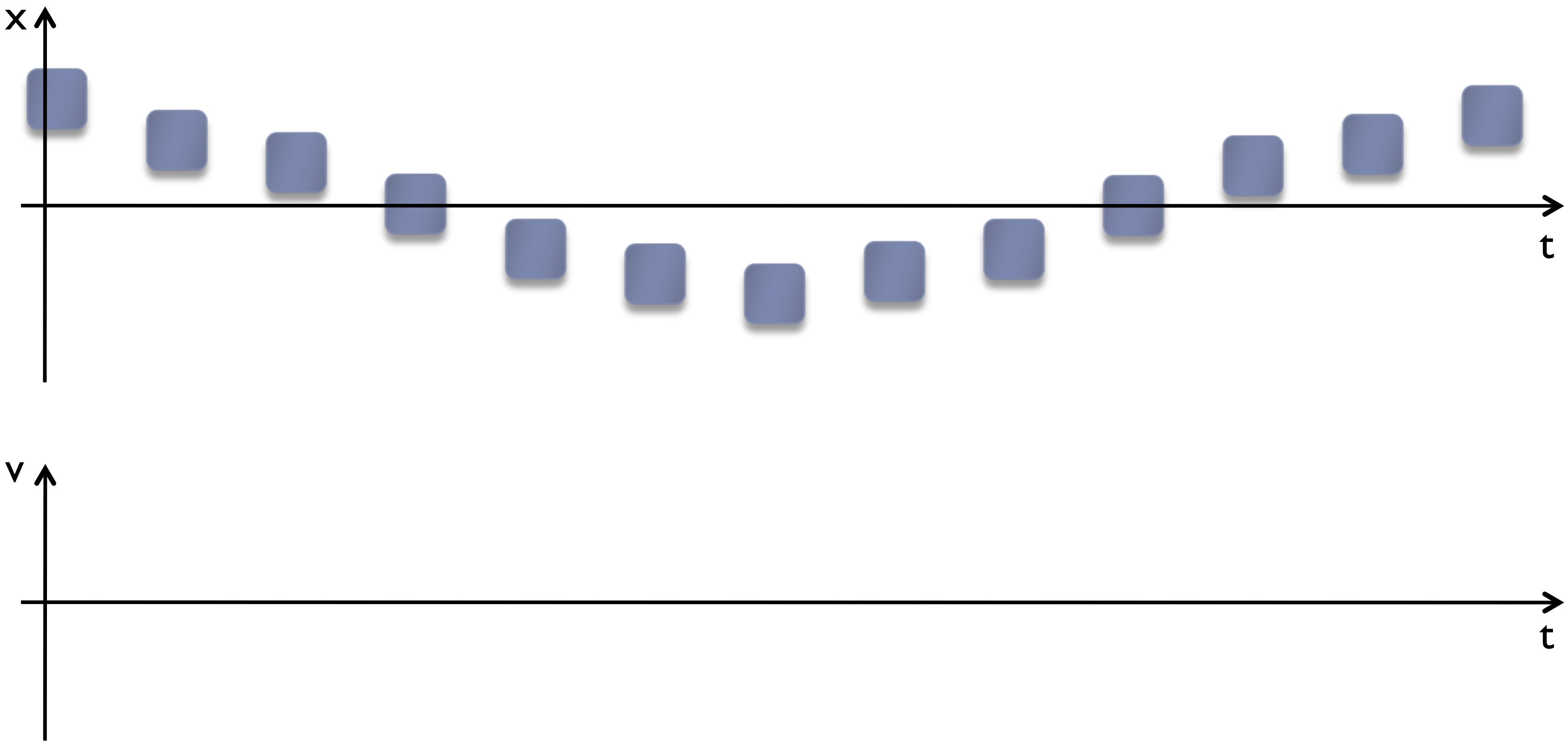
Below are five graphs of velocity of a car versus time where velocity is in meters/second and the time is in seconds. Rank them from greatest to least based upon the displacement of the car.



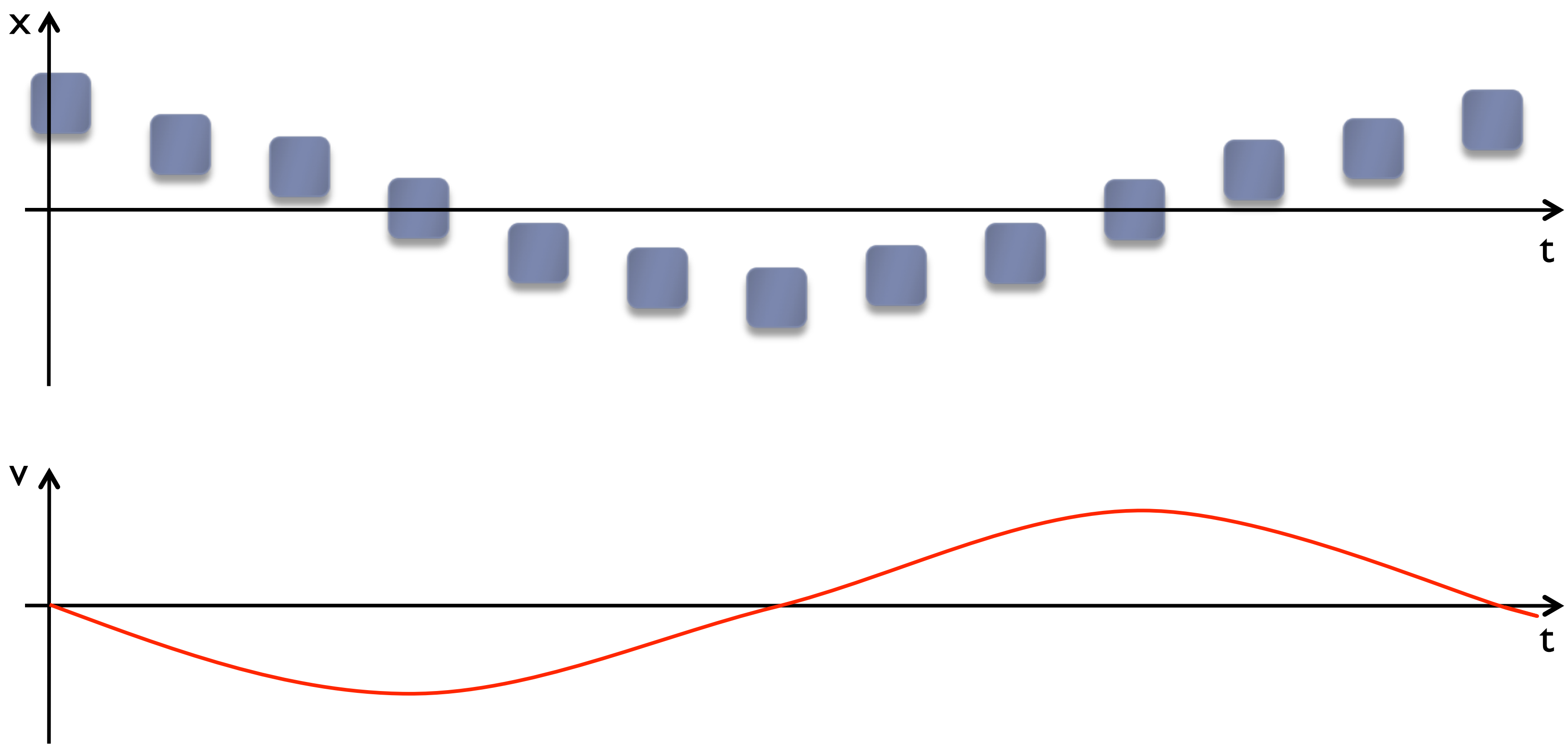
Example 1: The position versus time for the last pitch of Matt Cain's perfect game was given by $x = 50.0 - 137t + 13.5t^2$ where x is the distance from home in feet and t is the time in seconds. Find (a)the velocity as a function of time, (b)the acceleration as a function of time, (c)the initial speed of the ball, (d)the time to reach home plate, and (d)the speed when it the ball gets there.



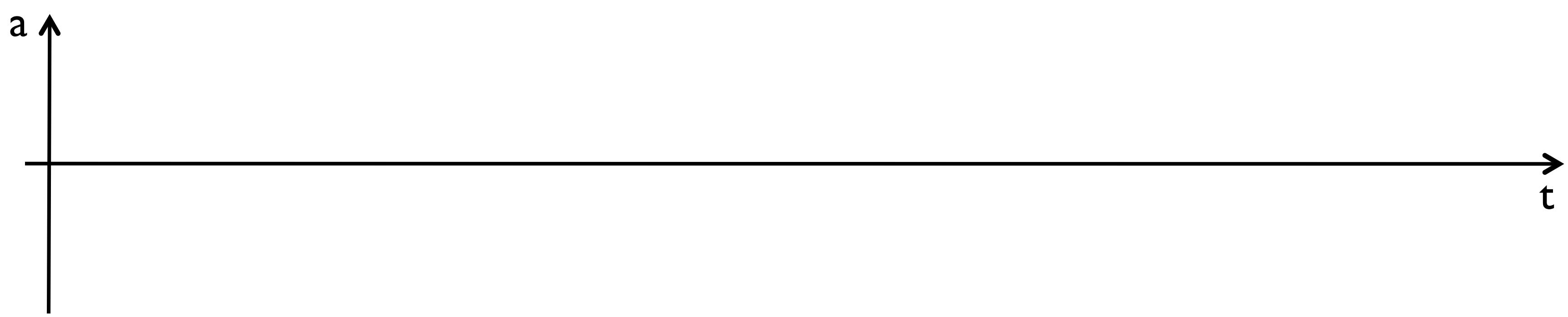
A mass oscillates at the end of the spring. Shown above are images of the system at equal time intervals. Sketch the position versus time.



Sketch the velocity versus time.



Sketch the acceleration versus time.



Example 2: The equation for the acceleration of the mass is something like a = - A cos wt. Find the velocity as a function time and the position as a function of time.

Lecture 03 - Summary

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Speed	The magnitude of the velocity	v = | v |
Average Acceleration	The rate of change of velocity	$a \equiv \frac{dv}{dt}$