Describing Motion in More Dimensions

Pre-Class Questions

Problem Set #6

Lecture Outline

1. Going from One to Two Dimensions
2. Position and Displacement Vectors
3. Displacement and Velocity Vectors
4. Velocity and Acceleration Vectors

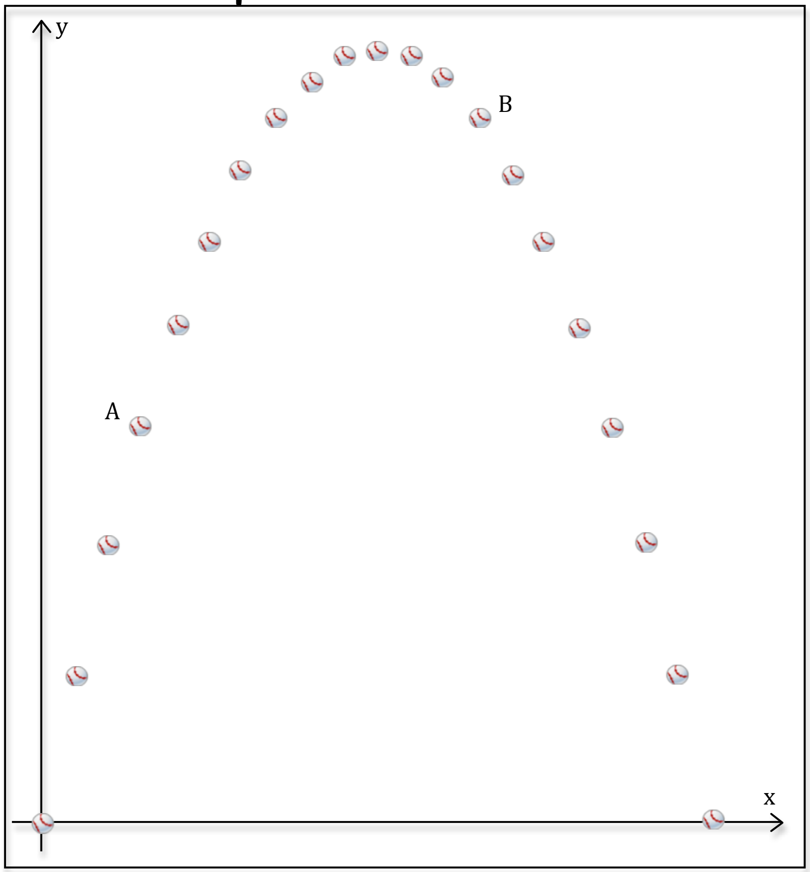
Pre-Class Summary:

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system.	x
Displacement	A change in position.	$\Delta x \equiv x_f - x_i$
Average Velocity	The average rate of displacement.	$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$
Speed	The magnitude of the velocity.	v = |v|
Average Acceleration	The rate of change of velocity.	$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

Pre-Class Summary:

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system.	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
Displacement	A change in position.	$d\vec{r} \equiv \vec{r}_f - \vec{r}_i = dx\hat{i} + dy\hat{j} + dz\hat{k}$
Velocity	The average rate of displacement.	$\vec{v} \equiv \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
Speed	The magnitude of the velocity.	$v \equiv \lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2 + v_z^2}$
Acceleration	The rate of change of velocity.	$\vec{a} \equiv \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Position and Displacement

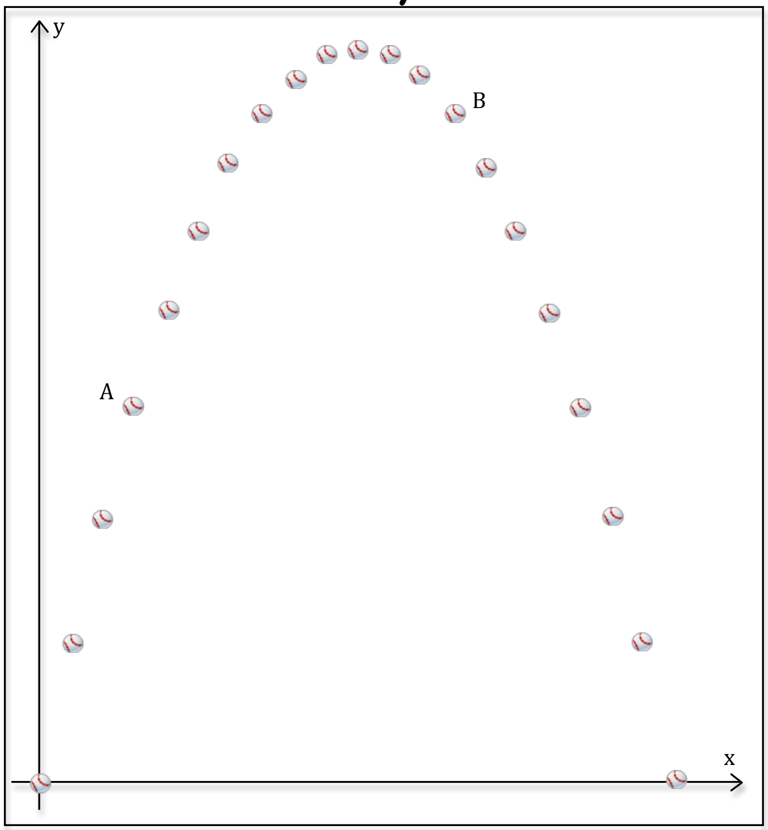


Using the coordinates shown:

1. Draw the position vector for the ball when it is at A. Label it $\vec{r}_A$.
2. Draw the position vector for the ball when it is at B. Label it $\vec{r}_B$.
3. Draw the displacement vector between A and B. Label it $\Delta\vec{r}$.
4. Explain why you know that $\vec{r}_A + \Delta\vec{r} = \vec{r}_B$. Solve for $\Delta\vec{r}$.

Example 1: A baseball is tossed into the air. After 0.30s it is at the position (2.2m, 2.5m). After 1.3s it is at the position (9.6m, 4.5m). (a)Draw these two position vectors and (b) the displacement vector. (c)Find the magnitude and direction of the displacement vector.

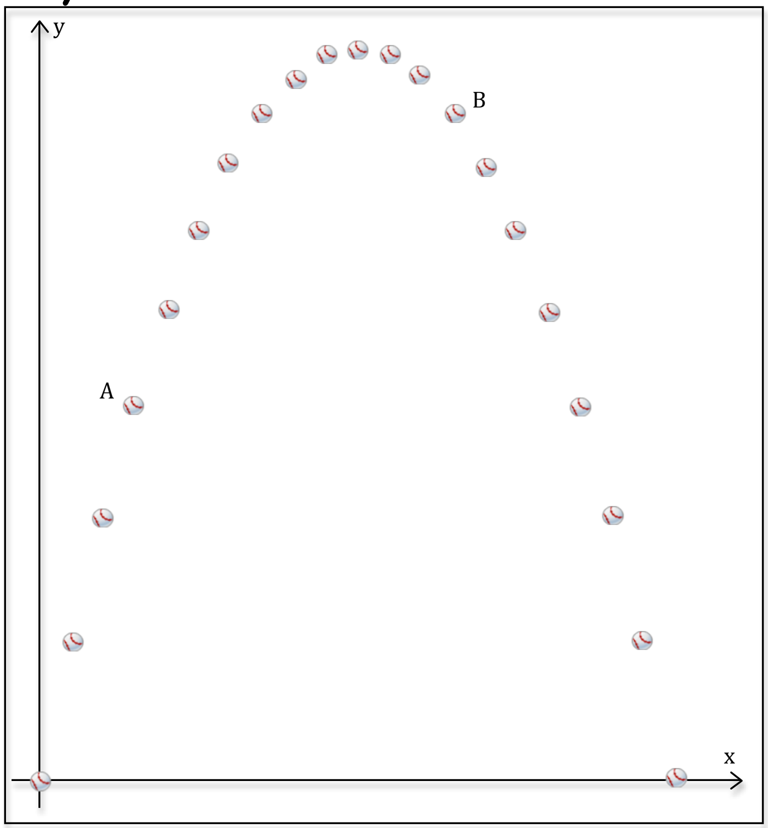
Displacement and Velocity



1. Draw the displacement vector for the ball from one image before A to one image after A. Label it $\Delta\vec{r}_A$.
2. Draw the displacement vector for the ball from one image before B to one image after B. Label it $\Delta\vec{r}_B$.
3. Explain why the velocity vector at A must point in the direction of the displacement vector at A (This is also true at B).
4. Sketch the velocity vectors at A and B.

Example 2:The position vector of the ball at 0.20s has the components (1.47m, 1.76m) and the position vector at 0.40s is given (2.94m, 3.14m). During this interval, find (a)the components of the average velocity vector, (b)the average speed of the ball, and (c)the direction of the average velocity vector.

Velocity and Acceleration

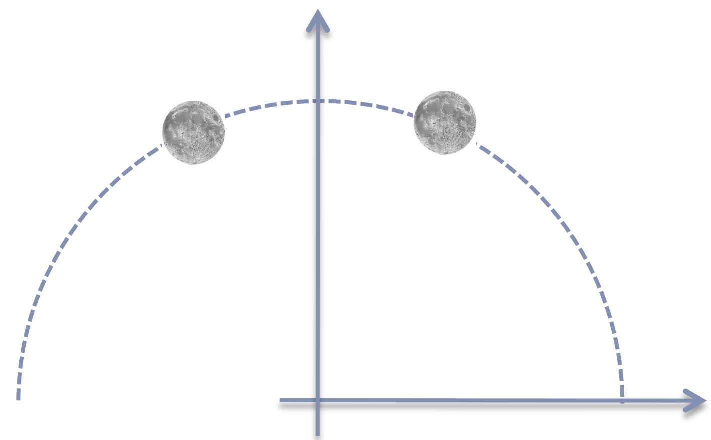


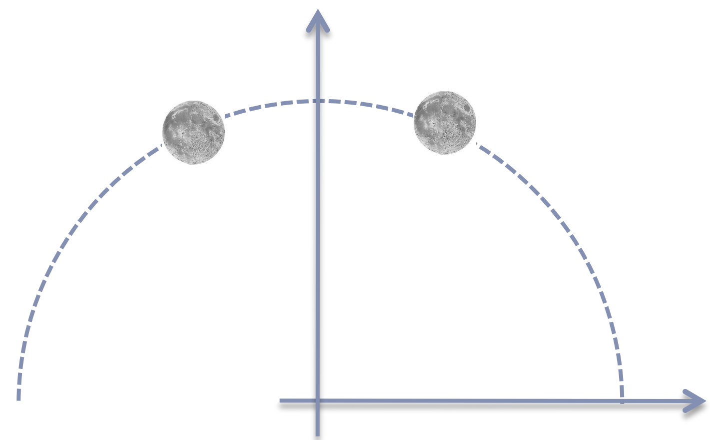
1. Draw the velocity vector for the ball at A. Label it $\vec{v}_A$.
2. Draw the velocity vector for the ball at B. Label it $\vec{v}_B$.
3. Redraw the two velocity vectors with their tails at the origin.
4. Draw the change in velocity vector. Label it $\Delta\vec{v}$.
5. Explain why $\Delta\vec{v}$ must point in the direction of the acceleration vector.
6. Explain why the acceleration vector points directly downward.

Example 3: At t = 0.30s a baseball has a velocity of (7.35m/s, 6.86m/s). At t = 1.3s its velocity is (7.35m/s, -2.94m/s). Find the magnitude and direction of the average acceleration vector.

The Orbit of the Moon

1. *In the upper drawing at the right, add the position vectors at the two points shown.*
2. *Now, add the displacement vector assuming the moon is orbiting clockwise.*
3. *In the lower drawing, add the velocity vectors at the two points shown.*
4. *Move the two velocity vectors so their tails are at the origin.*
5. *Now add the vector to represent the change in velocity.*





Example 4: The moon orbits every 27.4days and it is 3.84×10^5m away. Find the speed of the moon in its orbit.

Lecture 06 - Summary

To describe motion in more than one dimension, we keep the same definitions and let the mathematics of vectors deal with the resulting complications.

Quantity	Definition	Mathematical Representation
Position	The location of an object with respect to a coordinate system.	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
Displacement	A change in position.	$d\vec{r} \equiv \vec{r}_f - \vec{r}_i = dx\hat{i} + dy\hat{j} + dz\hat{k}$
Velocity	The average rate of displacement.	$\vec{v} \equiv \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
Speed	The magnitude of the velocity.	$v \equiv \lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2 + v_z^2}$
Acceleration	The rate of change of velocity.	$\vec{a} \equiv \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$