Uniform Circular Motion

Pre-Lecture Questions

Problem Set #9 (Solutions posted after class)

Lecture Outline

1. Uniform Circular Motion
2. Centripetal Acceleration
3. Review of the Description of Motion

Pre-Class Summary:

Tangential Speed $v_t = \frac{2\pi r}{T}$

Centripetal Acceleration $a_c = \frac{v^2}{r}$

Centripetal acceleration points toward the center of the circle.

Equations of Motion for Circular Motion

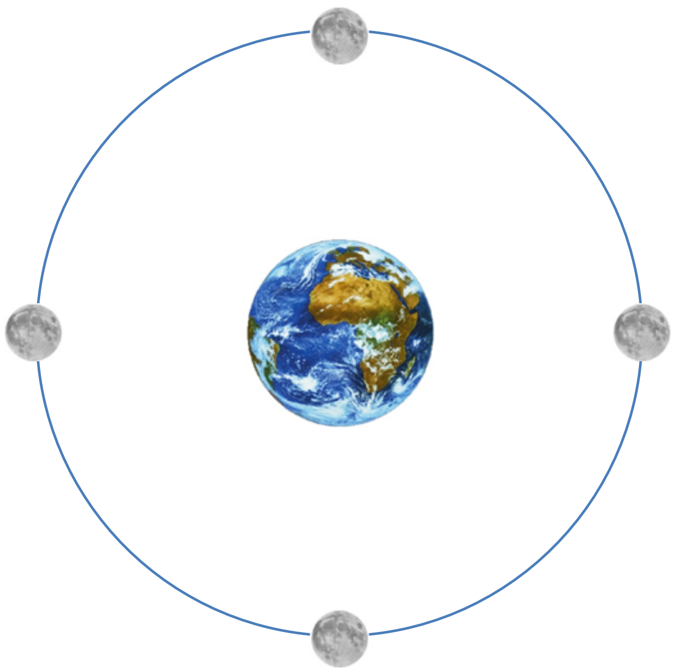
$$a_x(t) = -\omega^2 r\cos\omega t \qquad a_y(t) = -\omega^2 r\sin\omega t$$

$$v_x(t) = -\omega r\sin\omega t \qquad v_y(t) = \omega r\cos\omega t$$

$$x(t) = r\cos\omega t \qquad y(t) = r\sin\omega t$$

$$a_x(x) = -\omega^2 x \qquad a_y(y) = -\omega^2 y$$

$$v_x(x) = \pm\omega\sqrt{r^2 - x^2} \qquad v_y(y) = \pm\omega\sqrt{r^2 - y^2}$$

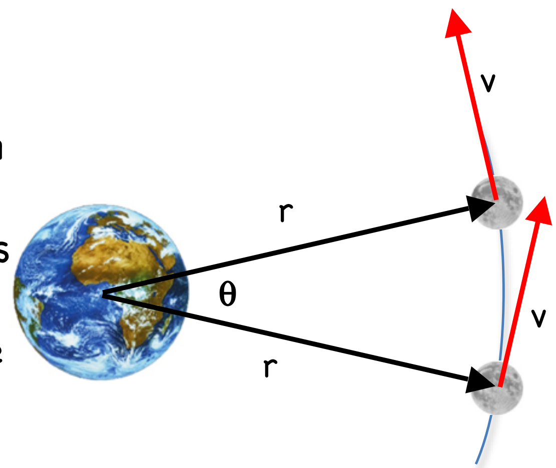


At the left is a sketch of the moon orbiting Earth. Use the center of Earth as the origin. For each of the four images of the moon:

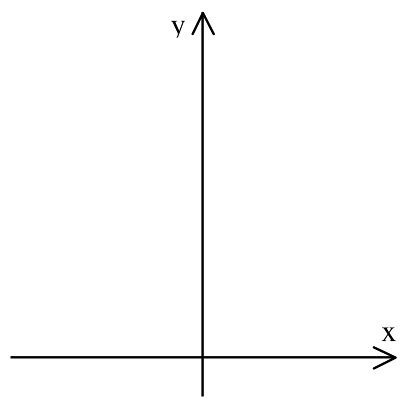
1. draw the position vector.
2. draw the velocity vector.
3. compare the lengths of each position vector?
4. compare the lengths of each velocity vector?
5. does the moon accelerate in its orbit? Explain.

Example 1: The moon is $3.84x10^8$m from Earth and it takes 27.3 days or $2.36x10^6$s to complete one orbit. Find (a)the magnitude of the position vector and (a)the magnitude of the velocity vector using the center of Earth as the origin.

At the right is a sketch of the moon orbiting Earth. The position and velocity vectors are shown at two times Δt apart during which the position and velocity vectors both rotate through an angle θ:



1. Draw the displacement vector $\Delta\vec{r}$. Label the triangle formed by the r vectors and Δr with an A.
2. Redraw the two velocity vectors on the coordinate systems at the right with their tails at the origin. Include the angle θ.
3. Draw the change in velocity vector $\Delta\vec{v}$. Label the triangle formed by the v vectors and Δv with a B.
4. Explain why A and B are similar triangles.
5. Find the ratio $\frac{\Delta v}{\Delta r}$ in terms of r and v. Solve for Δv.
6. Divide Δv by Δt to get the magnitude of the acceleration vector.
7. Show the result is, $a = \frac{v^2}{r}$



Example 2: A physics professor twirls a ball overhead in a circle of radius 50cm. The ball completes 5.0 revolutions per second. Find (a)the period, (b)the frequency, (c)the angular frequency, (d)the speed, and (e)the acceleration of the ball.

Example 3: A physics professor twirls a ball overhead in a circle of radius 50cm. The ball completes 5.0 revolutions per second. Assume the velocity vector points in the y-direction at $t = 0$. Find the components of the velocity vector of the ball when $t = 0.025s$.

Question: You come upon two friends arguing. The first one says that the moon orbits Earth at a constant velocity and therefore the moon is not accelerating. The second one says that since the moon in moving in a circle it must have a centripetal acceleration even though it has a constant velocity.

Choose one answer:

A. Both are right.
B. The first is right and the second is wrong.
C. The second is right and the first is wrong.
D. Both are wrong.

Correct the wrong statements.

Lecture 09- Summary

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Review of the Description of Motion

- **Classical Mechanics**
  - **Translation** — The motion of objects as a whole
    - **Kinematics**
    - **Force**
    - **Momentum**
    - **Energy**
  - **Rotation** — The spinning motion of objects
    - **Kinematics**
    - **Torque**
    - **Angular Momentum**
    - **Energy**

Kinematics

Laws:

none

Definitions:

Position $\vec{r}$

Displacement $d\vec{r}$

Velocity $\vec{v} \equiv \frac{d\vec{r}}{dt}$

Acceleration

$\vec{a} \equiv \frac{d\vec{v}}{dt}$

Useful Relations:

Kinematic Equations

$x = x_o + v_o t + \frac{1}{2} a t^2$

$v = v_o + at$

$v^2 = v_o^2 + 2a(x - x_o)$

$x - x_o = \frac{1}{2}(v + v_o)t$

Tangential Speed

$v_t = \frac{2\pi r}{T}$

Centripetal Acceleration

$a_c = \frac{v^2}{r}$