Center of Mass

Pre-Lecture Questions

Problem Set #17 (due next time)

Lecture Outline

1. The Definition of Center of Mass
2. Calculating the CM for Extended Objects

Pre-Class Summary:

The Second Law applies to a collection of objects as long as we use:

- Forces that act on the system from the outside, not forces that act between objects within the system.
- The mass is the total mass of all the objects in the system.
- The acceleration is the acceleration of the center of mass of the system

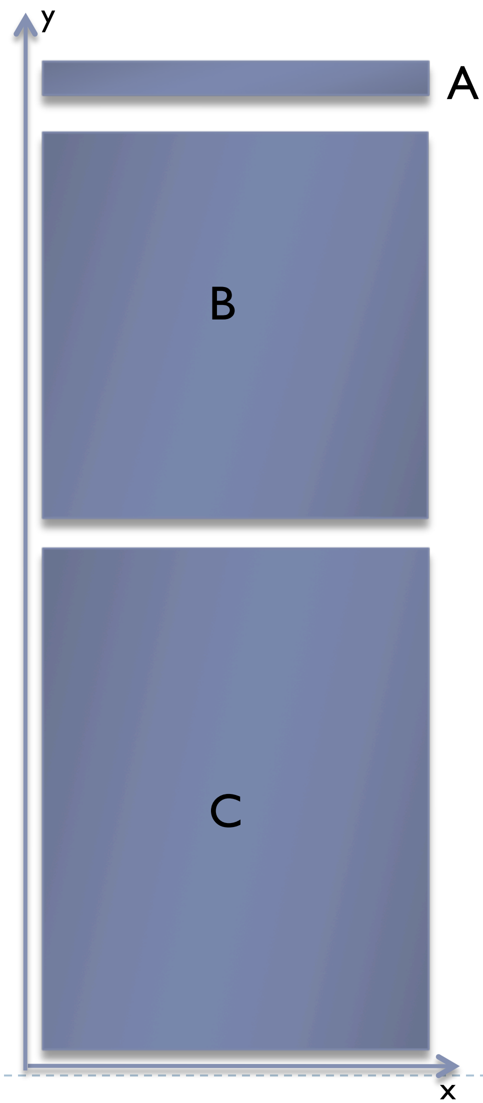
The definition of the center of mass is, $\vec{r}_{cm} \equiv \frac{1}{M}\int \vec{r}\, dm$

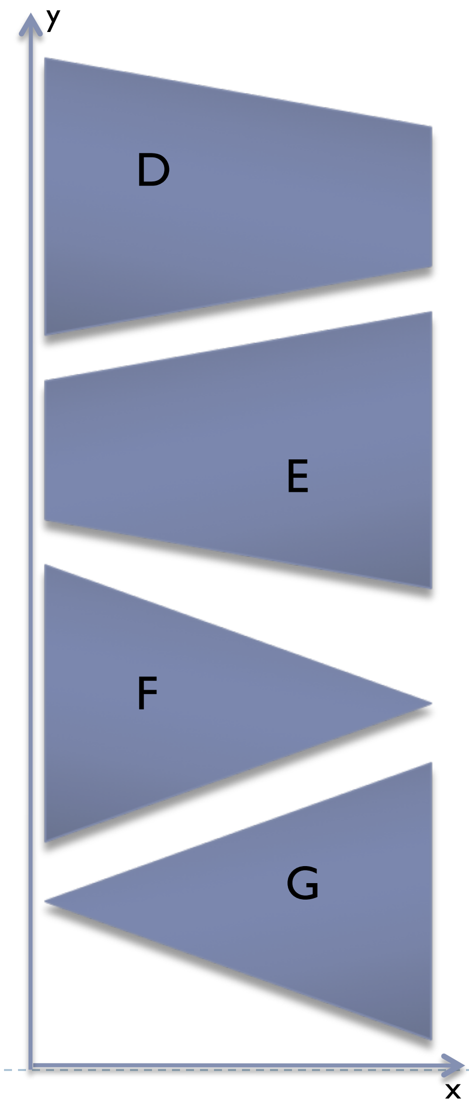
For discreet masses, $\vec{r}_{cm} \equiv \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$

The definition is a vector equation so,

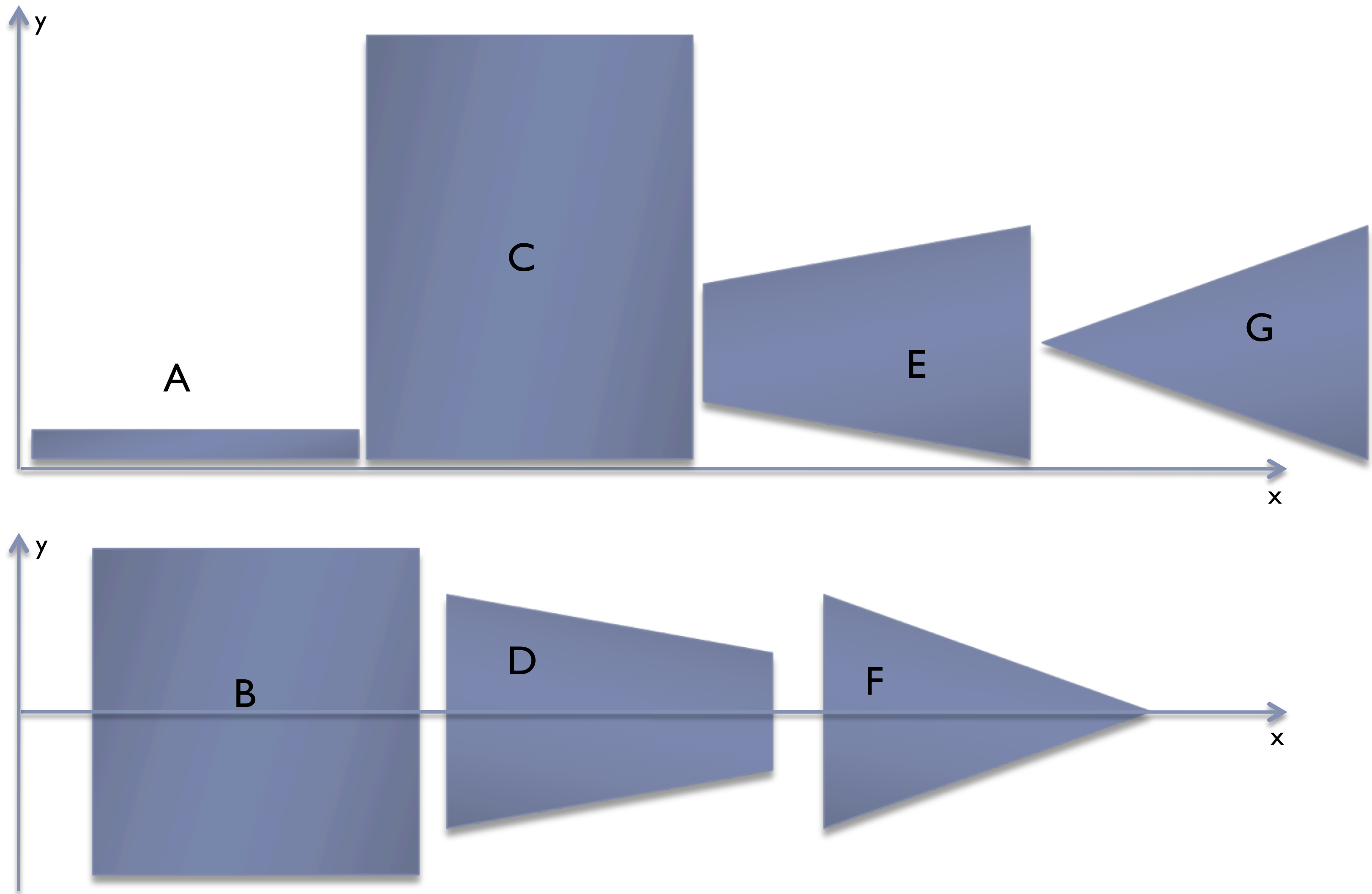
$$x_{cm} \equiv \tfrac{1}{M}\int x\, dm \qquad y_{cm} \equiv \tfrac{1}{M}\int y\, dm \qquad z_{cm} \equiv \tfrac{1}{M}\int z\, dm$$

The objects below are all the same width. Rank them from greatest to least based on the x-position of their center of mass.





The objects below are all the same width. Rank them from greatest to least based on the y-position of their center of mass.

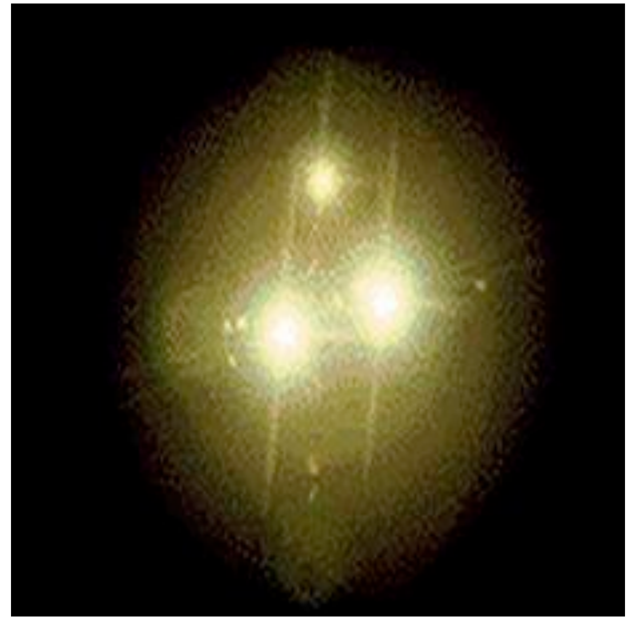


Demonstration: Find center of mass of a meter stick and bat.

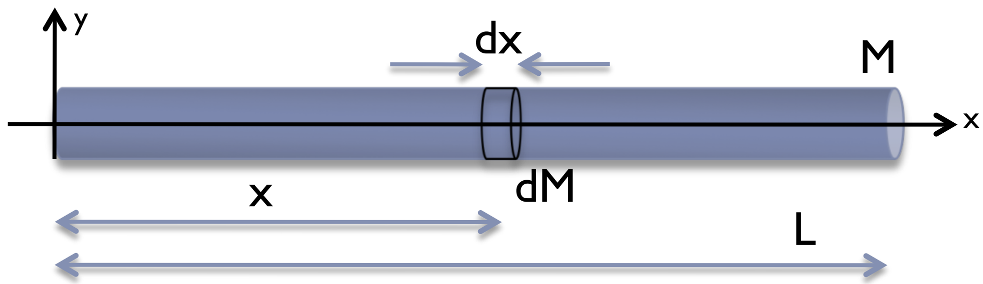
Example 1: A binary star consists of two stars that rotate about each other. Suppose one star has a mass of ten times our sun while the other has a mass of two times and that they are as far apart as the Earth and sun. Find the center of mass of the system measured from the center of the larger star.

Example 2: There are trinary star systems as well. Given the mass and positions of the three stars in the table below, find the center of mass.

mass (M)	x (AU)	y(AU)
8.0	0	0
8.0	2.0	2.0
2.0	1.0	8.0



The Center of Mass of a Uniform Stick



A uniform stick has a length L and a mass M. It can be broken down into small pieces of mass dM of length dx as shown above.

1. Find the ratio of dx to L in terms of dM and M. $\frac{dx}{L} = -$

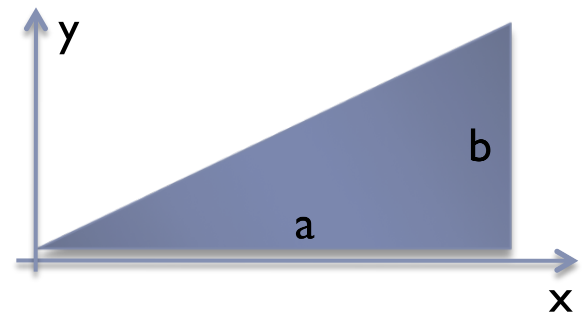
2. Solve for dM in terms of M, L, and dx.

3. Substitute the expression for dM into the definition of center of mass.

$$x_{cm} = \frac{1}{M}\int x\,dm =$$

4. Integrate from zero to L to find x_{cm}.

Example 3: Find the center of mass for the triangle shown at the right.



Lecture 17 - Summary

The Second Law applies to a collection of objects as long as we use:

- Forces that act on the system from the outside, not forces that act between objects within the system.
- The mass is the total mass of all the objects in the system.
- The acceleration is the acceleration of the center of mass of the system

The definition of the center of mass is, $\vec{r}_{cm} \equiv \frac{1}{M}\int \vec{r}\, dm$

For discreet masses, $\vec{r}_{cm} \equiv \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$

The definition is a vector equation so,

$$x_{cm} \equiv \frac{1}{M}\int x\, dm \qquad y_{cm} \equiv \frac{1}{M}\int y\, dm \qquad z_{cm} \equiv \frac{1}{M}\int z\, dm$$