Work and Kinetic Energy

Pre-Lecture Questions

Problem Set #19 (due next time)

Lecture Outline

1. The Work Done by a Constant Force
2. Review of the Vector Dot Product
3. The Work Done by a Varying Force

Pre-Class Summary:

The definition of work is the product of the component of the force along the direction of motion times the distance, $\Delta W \equiv F_{\parallel}\Delta s$.

The vector dot product $\vec{A} \bullet \vec{B} \equiv AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$,

will take the component automatically, $\Delta W \equiv \vec{F} \bullet \Delta\vec{s}$.

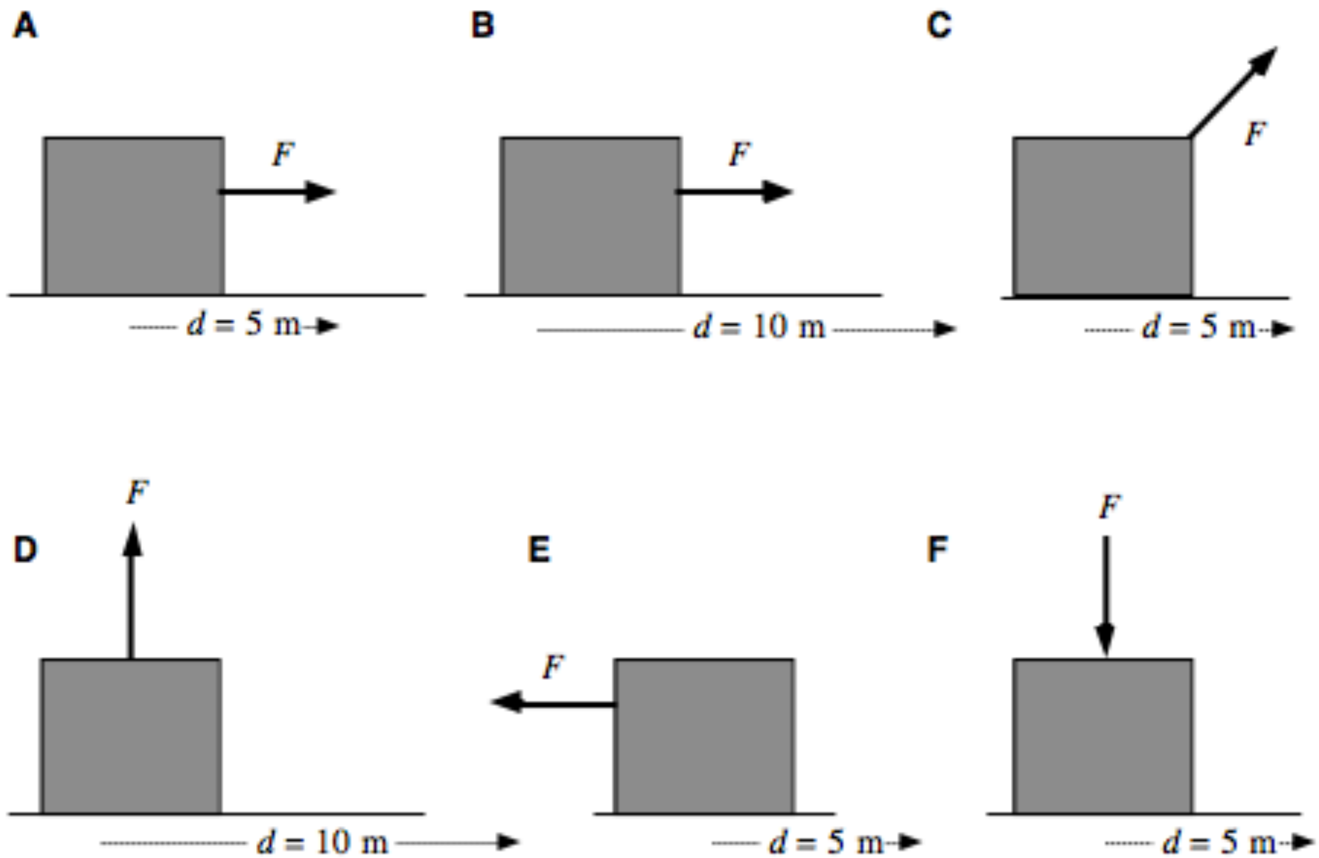
If the force is not constant then we need to integrate $W \equiv \int \vec{F} \bullet d\vec{s}$.

The advantage of using the idea of work is that it is a scalar quantity as opposed to force that is a vector.

Along the way we introduced the force exerted by a spring described by Hooke's Rule, $\vec{F}_s = -k\vec{x}$.

In the figures below, identical boxes of mass 10 kg are moving at the same initial velocity to the right on a flat surface. The same magnitude force, F, is applied to each box for the distance, d, indicated in the figures.

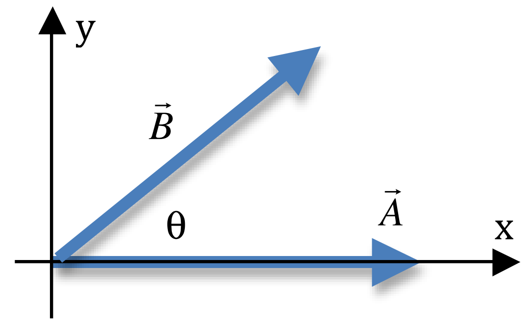
Rank these situations in order of the work done on the box by F while the box moves the indicated distance to the right.



Example 1: A 200N crate is pushed up a 37° ramp at a constant speed for 4.0m by exerting a 150N force parallel with the ramp. Find (a)the work done by each force that acts on it and (b)the total work done by all the forces.

Example 2: The same 200N crate is pushed up the 37° ramp for 4.0m by the same applied force of 150N. This time small wheels are placed under the ramp so that there is no friction. Find (a)the work done by each force that acts on it and (b)the total work done by all the forces.

Vector Components and the Dot Product



1. Draw the component of vector $\vec{B}$ along vector $\vec{A}$. Label it $B_{||}$.
2. Express $B_{||}$ in terms of the length B and the angle θ.

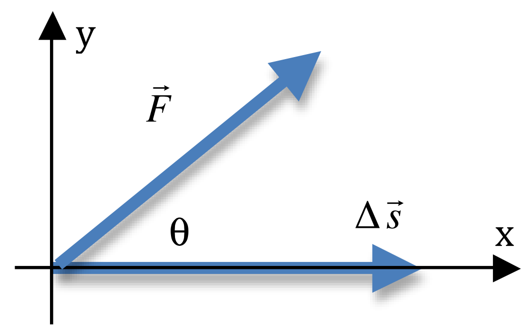
   $B_{||} =$

3. Write the dot product of vector $\vec{B}$ along vector $\vec{A}$ in terms of the lengths A and B as well as the angle θ.

   $\vec{A} \bullet \vec{B} =$

4. Rewrite the dot product using the result from part 2.

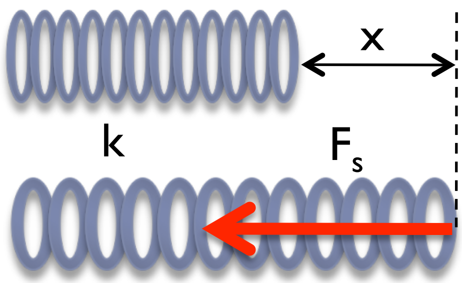
   $\vec{A} \bullet \vec{B} =$



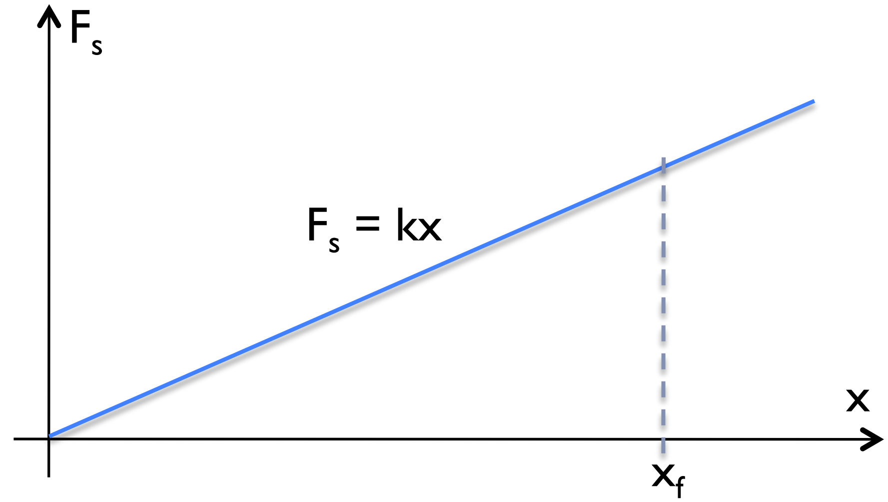
5. At the right a constant force $\vec{F}$ is exerted on an object as it moves through a distance $\Delta\vec{s}$. Express the work done on the object in terms of the dot product using the result from part 4.

   $W \equiv F_{||} \Delta s =$

Example 3: In the last example, the gravitational force was $\vec{F}_g = -(120N)\hat{i} - (160N)\hat{j}$*, while the displacement vector was* $\Delta\vec{s} = (4.0m)\hat{i}$ *. Find the work done by the gravitational force using the dot product.*



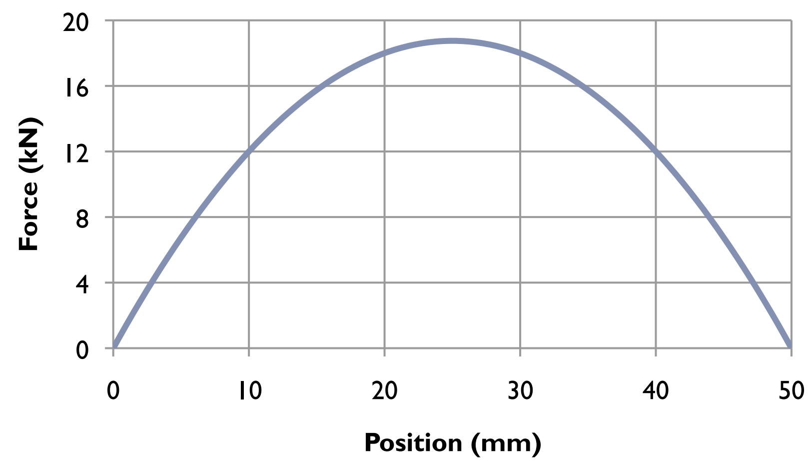
Hooke's Rule $F_s = kx$



Consider finding the work is done to stretch the spring from zero to x_f. The graph above shows the force as it changes with the position.

Given that work is $F_{||}\Delta s$. What does the product of force and distance represent in the graph above? Discuss this with your neighbors:

Example 4: The force exerted on a baseball by a bat is given by $F = -0.03x^2+1.5x$ where F is in kN and x is in mm. This is plotted at the right. Find the work done on the ball.



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